OBSERVATIONS OF NONRADIAL p-MODE OSCILLATIONS ON THE SUN*

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Received 1977 January 27; accepted 1977 May 19

ABSTRACT

The solar velocity field is analyzed in space and time for an array of size \((x, y, t) = (256", 256", 256 \text{ minutes})\) that has sampling intervals of \((\Delta x, \Delta y, \Delta t) = (2", 2", 15 \text{ s})\). A \((k_h, \omega)\) distribution of power spectral density is derived from these data. The observed power density is concentrated in bands whose frequencies agree with eigenfrequencies for nonradial p-mode oscillations. Comparison of the theoretical and observed frequencies places a lower limit on the depth of the convective layer in the solar envelope. We have determined this lower limit from a masking operation which objectively tests the goodness of fit of the theoretical eigenfrequencies to the observed power. The limiting model is adopted for which the observed power inside the mask is the same as would be found if the actual power distribution were random inside a broad-band mask. The surface abundance of Li places an upper limit on the convective envelope depth. The limits we find are:

\[0.011 < \frac{M_s}{M_\odot} < 0.095; \quad 0.62 < \frac{r_c}{R_\odot} < 0.75; \quad 1.7 \times 10^6 K < T_b < 3.2 \times 10^6 K; \]

\[0.10 \text{ g cm}^{-3} < \rho_b < 0.73 \text{ g cm}^{-3}\]

where \(T_b\) and \(\rho_b\) are the temperature and density at the inner convective-radiative interface.

Subject headings: Sun: atmospheric motions — Sun: general — Sun: interior

1. INTRODUCTION

Since their discovery by Leighton, Noyes, and Simon (1962) the solar 5-minute oscillations have been studied intensively. Most studies have found the period of the 5-minute oscillations to undergo apparently random fluctuations within a range of about 3–7 minutes. It has been pointed out by Ulrich (1970, hereafter Paper I) and by Deubner (1975) that this apparent randomness is a result of inadequately selecting oscillations of a determined horizontal wavelength. Calculations have shown (Paper I; Wolff 1972a, b; Ando and Osaki 1975; Ulrich and Rhodes 1977, hereafter Paper II) that p-mode eigenfrequencies of the solar envelope are increasing functions of the total horizontal wavelength, \(k_h\), of the oscillations. The apparently random variations in frequency are a consequence of interference among a large number of modes and may also be a consequence of variations in the degree of excitation of p-modes with different wavelengths and frequencies. By properly resolving the oscillatory motion in wavenumber and frequency \((k_h, \omega)\), we should be able to observe distinct bands of power. The recent work by Deubner (1975) has found power in bands as predicted. We describe here a series of observations made at the Sacramento Peak Observatory which also resolve the power into bands.

The 5-minute oscillations have often been analyzed theoretically as a purely atmospheric phenomenon (Noyes and Leighton 1963; Souffrin 1966; Kahn 1961; Whitaker 1963; Uchida 1967; Thomas, Clark, and Clark 1971). These analyses have treated the photosphere as a rigid boundary or as a layer with an imposed turbulent boundary condition. Within the atmosphere both acoustic waves and gravity waves have comparable frequencies, although the spatial characteristics are distinct. Because most observational work has emphasized only the frequencies, some theoretical papers (the first three listed above) have interpreted the oscillations as acoustic modes, while others (the last three) have treated the oscillations as gravity modes. The early work by Frazier (1968b) clearly established that a major portion of if not all the oscillatory power is in the form of acoustic modes. The role of the subphotospheric layers has been unclear until recently. The observations by Deubner (1975, 1976) show that the oscillations can be resolved into frequencies which have the character of nonradial p-mode eigenfrequencies of the solar envelope. At the time of Deubner’s work, existing eigenfrequency calculations disagreed systematically.

* Research support at UCLA by NSF AST 75-19750 and NASA NSG 7158 and at Caltech by PHY76-02724.
† Operated by AURA, Inc., under contract with the National Science Foundation.

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II. PREVIOUS OBSERVATIONS

In order to clarify the relationship between our observations and previous ones, we list in Table 1 some characteristics of previous analyses of the oscillations. Although there is considerable variation in the way that the oscillations have been studied, most previous studies of solar oscillations including Deubner's (1975, 1976) suffer from the problem that data are taken sequentially along a line in only one horizontal direction. Many different orientations of the line have been studied, but only the observations by Frazier (1968b), Deubner (1972), and Musman (1974) have simultaneously studied the dependence on both horizontal dimensions and on time. However, these three earlier studies of the velocity in x, y, and t were of too limited scale to distinguish the predicted power bands. The more common and more extensive studies in one horizontal direction and time can be used to obtain a measure of the power density on the \((k_h, \omega)\)-plane, but this requires the assumption that the actual power on the solar surface is isotropic in \(k_x\) and \(k_y\). This assumption is good for a long time average or for a spatial average over a portion of the solar surface larger than \(2\pi/k_x\). However, for more limited measurements, statistical fluctuations about isotropy will cause a smearing of the bands on the \((k_h, \omega)\)-plane. Only the recent work by Deubner (1975, 1976) appears extensive enough for the isotropy assumption to have been valid.

We have avoided the need to assume isotropy by simultaneously observing in x, y, and t. However, our present data processing does not utilize a complete three-dimensional analysis but instead corresponds to observing in x and t. In this analysis mode our observations are equivalent to measuring the velocity with a 256\(\times\)2\(\times\) slit where the long dimension of the slit is in the y direction. No previous x, t observations have been made with such a slit geometry. This procedure is only a preliminary stage in the data reduction; we plan a full three-dimensional analysis later. In addition to the observations reported here, there are available two additional 256\(\times\)2\(\times\) data sets obtained on different days, and six independent 512\(\times\)512 data sets. Questions concerning the stability, reproducibility, and isotropy of the oscillations can be settled by the analysis, which is currently in progress, of these additional data.

Although our spectral analysis treats only one spatial dimension and time, the fact that we average over the other spatial dimension imposes a strong filter on the modes which can contribute to our signal. Only those modes which are described by a \(k_h\) vector...
nearly exactly perpendicular to the slit survive the averaging process. No oblique waves can contribute to our power spectrum. Thus we do not need to perform an Uberoi transform (Uberoi 1955) in order to obtain the spectral power density as a function of $k_x$. The measured value of $k_x$ is immediately equal to $k_h$.

These ideas are made quantitative in § IVû.

The measured value of $k_h$ is immediately equal to $k_x$.

obtain the spectral power density as a function of $k_x$.

The observations were made on 1975 January 8 from 15:29 UT to 19:45 UT with the diode array attached to the echelle spectrograph of the vacuum tower telescope at the Sacramento Peak Observatory. This instrument has been described by Worden and Simon (1976). Line-of-sight velocity measurements were made in a quiet region at disk center, using 256 diodes arranged in pairs in opposite wings of magnetically insensitive ($g = 0$) line Fe I $\lambda 5576.10$. Each set of 128 diodes was positioned in the steepest portion of the line profile, 76 mÅ from line center, and had a spectral bandpass of 76 mÅ. Each diode received light from a $2'' \times 2''$ area of the solar surface. Data were collected from all 128 diode pairs every 0.08 s. The data collected at each scan position are the sums of two separate readings of the diodes. Then the solar image was moved a distance of $2''$ across the entrance slit and the measurement repeated. In this way a $128 \times 128$ array of measurements was built up. In Figure 1 we indicate schematically the observing procedure. The scan direction is shown by the arrow. The raster origin (diode pair 1, scan 1) at the northwestern corner was located at the beginning of the run at $3\degree 33\arcmin$ N heliographic latitude and $0\degree 00'$ longitude. By the end of the run the raster origin was at $3\degree 33\arcmin$ N and $2\degree 11\arcmin$ W. At the beginning of the run the scan direction was oriented at an angle of $77\degree 2$ with respect to the solar equator, and at the end this angle was $79\degree 6$. On January 8 the latitude of the disk center was $3\degree 8\arcmin$ S. Thus the raster origin was $7\degree 1$ north of the disk center, and the $256''$ ($=15\degree$ in latitude) north-south edge of the raster was approximately centered as shown. Each array required about 11 s to complete, including the time necessary to reposition the slit to its original location. We repeated the observations at a 15 s rate. Data were also collected with two other strings of diodes in the chromospheric line Ca II $\lambda 8542.14$ (bandpass 0.3 Å in line center) and in the continuum ($\lambda 6578$). While these additional data have not yet been analyzed as time series, they have been used to independently assess the atmospheric “seeing” which existed during each run, and they will eventually be used to register successive Doppler frames.

As an example of all three of these digital pictures, a single frame from the January 8 run is shown in Figure 2 (Plate 23). Here the 16384 digital measurements in each image have been converted into a conventional gray-level display, and show a $\lambda 5576$ Dopplergram, a $\lambda 8542$ Ca II spectroheliogram, and a $\lambda 6578$ continuum picture. All three images are co-spatial. In the orientation shown in the figure, the slit direction was vertical, while the direction of scan of the slit was horizontal and toward the right. The orientation of the data on the Sun is shown by the symbol at bottom right.

Owing to the discrete nature of the 128 diode pairs, the character of the data is not the same in the two spatial directions. In what follows, we will refer to each set of velocity measurements made by the complete string of 128 diode pairs at one scan position as a column of data (i.e., a vertical slice in Fig. 2), and each set of 128 separate measurements by a single diode pair as a row of data (i.e., horizontal in Fig. 2). Each entire $128 \times 128$ array will be called a frame. This terminology also applies to the $\lambda 8542$ and $\lambda 6578$ pictures.

Before discussing the method of analysis and the results obtained from these data, we will first briefly describe the sources of instrumental (mechanical, electrical, and optical) noise in the data of which we are currently aware. We also emphasize that we do not believe that any of these noise sources have seriously affected the results of our analysis.

The noise is due to:

1. **Spectrograph drift.**—This is a slow drift in the measured velocity of all points in the frame caused by temperature changes at the spectrograph. Over the duration of a several-hour-long observing run, this effect can cause the measured mean velocity of the frame to change by up to 2 km s$^{-1}$. To compensate for this drift, the mean frame velocity is computed in real time, and at the end of each frame the control computer adjusts the grating angle or rotates a Doppler compensating glass plate to recenter the $\lambda 5576$
line on the two columns of diodes prior to the beginning of the succeeding 15 s frame.

2. Spectrograph seeing.—Air currents within the spectrograph often cause the measured column average velocity to fluctuate with an amplitude of about \( \pm 0.2 \, \text{km s}^{-1} \) on a time scale of 5–15 s. Because this time scale is comparable with the time required to measure a frame, the spectrograph seeing can introduce large, spurious velocity gradients from the beginning column to the ending column. A few higher spatial harmonics of noise may also be introduced by the spectrograph seeing.

3. Guider “jitter.”—At random intervals a mechanical resonance within the telescope can develop as the solar image is being guided and stepped across the entrance slit of the spectrograph. This can apparently produce a spurious up-and-down (columnwise) velocity signal with a spatial scale of about four stepping intervals (=8") per cycle. The wavenumber of this spurious signal is about 1.10 Mm.

4. Optical path changes.—Although each diode has a linear response to varying light levels, diode-to-diode velocity variations (row effects) are observed during an observing run, despite a careful calibration procedure carried out before the start and at the end of each run. Each diode’s response depends critically on a delicate and precise alignment of the diode, its field lens, and the direction of the incident light. It is suspected, but not yet proved, that changes in telescope orientation, heating, polarization, focus, etc., during the day may disturb this optical alignment enough to produce the observed variations. While these row variations are primarily a fixed-pattern type of noise, the temporal variations and trends of each of the individual diodes are somewhat different, so that a very careful analysis is necessary for the removal of these row effects. The pictures of the data shown in Figure 2 have already had a simple approximation to these diode-to-diode fluctuations suppressed by “destreaking,” i.e., removing the average of each row from each measurement within that row. However, the raw data have not been destreaked, and the more detailed time-dependent analysis indicated above is in progress.

5. Frame misregistration.—At the beginning of each observing run the location of the first frame was positioned slightly to the east of the central meridian. Then, at the conclusion of each successive frame, this location was moved slightly to the west by the amount necessary to compensate for solar rotation. Thus, nominally, the same region of oscillating elements was observed for the duration of each run centered near the central meridian. However, owing to the change in the scan direction mentioned above and possibly other effects, such as atmospheric dispersion, some drift of the raster area did occur during the 256-minute run. The magnitude and direction of this drift were determined by first identifying several chromospheric emission features which were present in all of the A8542 pictures. The image coordinates (i.e., pixel positions) of these features were then measured on a few frames early in the run, on a few frames midway through the run, and on a few frames near the end of the run. From these measurements we determined that the dominant image drift during this run was in the direction of solar rotation with a magnitude of roughly 4-5 pixels per hour (or \( \sim 8'\)–10") hour\(^{-1}\)). However, because this motion was not constant during the 256-minute run, the net pixel shift over the whole run was only about 10–12 pixels. Because of the scanning geometry of these observations, as shown in Figure 1, this net east-west image motion was almost entirely along the spectrograph slit, or along the column direction of the individual frames (i.e., vertical in Fig. 2). Because of the alignment of the direction of image drift with the column direction for this particular run, we chose to analyze this data by creating column averages of the velocity within each frame. By averaging in this direction, we found that the net effect of the image drift was only a shift of less than 10% of the length of each column of data (or 10–12 out of 128 measurements per column). This shift resulted in some randomization of the column-average amplitudes. However, this method of averaging ensured that no frequency shifts were introduced into the resulting power spectrum due to the image drift.

Eventually, the individual frame-to-frame registration differences will be corrected by the two-dimensional digital cross-correlation of successive Ca \( \equiv \) A8542 spectroheliograms, Fe i A5576 Dopplergrams, and (in the runs where active regions are present in the images) the continuum pictures. However, the fixed-pattern component of the noise from effect (4) must be removed from each frame before such correlations can be performed.

Thus far we have removed effects (1) and (2) by subtracting from the measured velocities within each frame a linear trend in the velocities as a function of the column number fitted with the least-squares procedure described in §IVa. The analysis of the data in the row direction and the full three-dimensional analysis will both require the removal of both effects (4) and (5). Thus, so far we have completely analyzed only the column-average velocities. However, preliminary calculations in which the image drift has been removed from the row averages suggest that accurate spectra can be computed in this manner as well.

IV. ANALYSIS AND RESULTS


The data analyzed here consist of 1024 frames obtained during the 256-minute run on 1975 January 8. We first calculated the mean velocity within each column of every frame. Then, for each frame, the least-squares linear trend through the 128 column-mean velocities was calculated and subtracted from each column-mean velocity. This procedure gave each set of 128 column means, which constituted a single frame, an overall velocity of zero. Figure 3 shows the detrended column-mean velocities as functions of
Fig. 3.—A display of the column-mean velocity as a function of column number and frame number. Each line represents the run of column-mean velocity across a frame. The displacement of the line relative to a line between the ticks gives the velocity at that column. Frames are separated in time by 15 s. The displacement of the line which would result from a velocity of 250 m s\(^{-1}\) is shown by the line segment near the middle of the figure.
frame number (time, along abscissa) and column number (space, along ordinate). The horizontal displacement of each line shown then displays the column-mean velocity for that point in space and time. A particularly strong example of the effect of the guider resonance [noise of type (3) above] can be seen in frame 114. After the linear detrending, the column-average velocities of the 1024 frames were averaged together in groups of four frames (forming 60 s averages). These average velocities formed a 128 × 256 array which had an overall zero mean velocity. This array was then analyzed in two dimensions by utilizing a fast Fourier transform. The total number of measurements contained in this analysis was thus 16,777,216.

The output of the calculations (which included a two-dimensional cosine tapering of the first and last 10 rows and columns of the array) is the distribution of power spectral density in the \((k, \omega)\)-plane. By virtue of our use of the column-mean velocities we are assured that \(k_y < \Delta k_y\), where \(y\) is along the column direction and \(\Delta k_y = 2 \pi / (128 \Delta y)\). For our observations \(\Delta \omega = (2^\circ) \times (0.714 \text{ Mm arcsec}^{-1})\) and \(\Delta k_x = 3.44 \times 10^{-4} \text{ Mm}^{-1}\). In the orthogonal direction we are able to determine \(k_x\) to within \(\Delta k_x = 3.49 \times 10^{-4} \text{ Mm}^{-1}\) (the frames are not precisely square because a stepping interval is slightly smaller than \(2^\circ\)).

To a good approximation, \(k_x = k_y = k\) for all but the smallest values of \(k_x\). From the record length of 256 minutes it follows that the bandwidth in \(\omega\) is \(\Delta \omega = 4.09 \times 10^{-4}\) s\(^{-1}\). The resultant power which we determine is

\[
\Delta P = \frac{\partial P}{\partial \omega \partial k_x \partial k_y} \Delta \omega \Delta k_x \Delta k_y, \tag{1}
\]

where \(\Delta \omega\), \(\Delta k_x\), and \(\Delta k_y\) have the values given above. The power we determine is the mean-square amplitude of the oscillation which is one-half the square of the zero-to-peak amplitude. In order to estimate the total power at any value of \(k\), by using the assumption of isotropy, we must multiply \(\Delta P\) by \(2 \pi k_x \Delta k_x\). This factor automatically accounts for modes with negative values of \(k_x\) and the factor of 2 often introduced for these modes in the computation of absolute wavenumber spectra has not been applied to our values for \(\Delta P\).

Over most of the \((k, \omega)\)-plane the level of spurious noise is roughly \(\Delta P_{\text{noise}} \approx 0.05 \text{ m}^2 \text{s}^{-2}\). Especially noisy portions of the \((k, \omega)\)-plane occur for \(k\) near 0 and for \(k\) near 1.1 Mm\(^{-1}\) (due to the guider "jitter"). The Nyquist frequencies of \(k_{Ny} = 2.23 \text{ Mm}^{-1}\) and \(\omega_{Ny} = 5.24 \times 10^{-2}\) s\(^{-1}\) are above the regions of interest in both space and time.

The results are shown in Figure 4. This power spectrum is unsmoothed. The lowest two contour levels shown in Figure 4 are 0.125 and 0.250 m\(^2\) s\(^{-2}\). Between 0.25 and 7.25 the remaining 14 contours are spaced 0.50 m\(^2\) s\(^{-2}\) apart. Expressed in percentages, in order to aid in a comparison with Deubner's (1975) observed power spectrum, these levels are 1.63%, 3.26%, 9.78%, 16.30%, . . . up to 94.78%. Thus the lowest contour level (2.8%) displayed by Deubner (1975) should be compared with the second contour level of Figure 4.

In order to facilitate such a comparison, we have superposed brackets showing the location of Deubner's (1975) observed power spectrum upon our observed spectrum. Spaced 0.1 Mm\(^{-1}\) apart, these brackets outline the extent in \(\omega\) of the 2.8% level of Deubner's spectrum. The mark near the center of each bracket has been placed at the middle of the highest contour level encountered on Deubner's spectrum at that location. From these marks the position of the peaks of his ridges may easily be seen to lie almost exactly along our observed ridges. Furthermore, the comparison of Deubner's 2.8% level and the 3.3% level of our spectrum shows that in almost all cases the frequency resolution of our observed ridges is finer than is Deubner's, even though our observations are for a single 4 hour observing run with a horizontal scale only \(1/2\) as long as that Deubner used.

Deubner has recently determined best-fit lines which represent the eigenfrequencies (1976, private communication). He obtained these lines by averaging together six different observing runs. To locate the frequencies accurately, he averaged the frequency dependence of the observed power spectrum over a band of width \(\Delta k_h = 0.1\) Mm. At each \(k_h\) value within the band, a frequency offset was applied to compensate for the frequency shift expected on the basis of Ando and Osaki's theoretical eigenfrequencies. In this way Deubner built up well-defined frequency peaks with half-widths at half-maximum of roughly \(4.5 \times 10^{-4}\) s\(^{-1}\). The position of the peaks defined in this way agrees well with the position of the marks on Figure 4.

Because the two-dimensional fast Fourier transform routine which was used to compute the power spectrum of Figure 4 was a "one-sided" transform in the second dimension, \(\omega\) (see Braule and White 1971 for a discussion of "one-sided transforms"), the raw power spectral estimates for negative frequencies were located symmetrically above the Nyquist frequency, \(\omega_{Ny}\). Consequently, in order to obtain estimates of the total power within each absolute frequency interval \(d[\omega]\), the raw power spectrum was folded about \(\omega_{Ny}\). When this was done, a two-dimensional spectrum of size 65 × 129 resulted. In Figure 4 we show power estimates for the first 49 values of \(k_h\) and for the first 96 values of \(\omega\). The remaining portion of the spectrum, out to the spatial Nyquist wavenumber, \(k_{Ny}\), and to \(\omega_{Ny}\), was not shown there because, with the exception of the two vertical bands of white noise near \(k_h = 0\) and 1.1 Mm\(^{-1}\), the remaining power estimates were all below the lowest contour level shown.
Fig. 4.—Contour diagram showing the distribution of power on the \((k_h, \omega)\)-plane for the full 256-minute observing run. Our observations (contours) are compared with the observations by Deubner 1975 (large dots with error bars). Our contours are for 0.125, 0.25, 0.75, 1.25, 1.75, etc., \(\text{m}^2 \text{s}^{-2}\). This power is integrated over the bin of size \((\Delta \omega, \Delta k_h)\) = \((4.09 \times 10^{-4} \text{ s}^{-1}, 3.49 \times 10^{-2} \text{ Mm}^{-1})\). The dots for Deubner's 1975 observations are center frequencies for his highest contour level. End points for the error bars are for Deubner's 1975 lowest contour level. This level roughly corresponds to our second contour interval. No end for the error bar is shown when Deubner's power did not fall below his lowest level. Deubner's 1976 dispersion lines lie above the 1975 points by 0.0 to 0.0003 \text{s}^{-1}. 
Fig. 5—Vertical slices through the contour diagram at constant values of $k_h$. These slices compare a power spectrum from a 192-minute subset of data (light line) to the power spectrum for the full 256-minute data set (dark line). Fig. 5a indicates that the closely spaced peaks are inadequately resolved and strongly dependent on the duration of the observations. Fig. 5b indicates that the well-resolved peaks are not greatly dependent on the duration of observation.
b) Comparison of the 256-Minute Spectrum with a 192-Minute Subset

A 192-minute subset of the data analyzed in the preceding section has been analyzed in a manner similar to that described above. In this analysis the column-average frames were averaged together in groups of six to form a 128 × 128 array. This array was then analyzed as above. The resultant power spectrum, when displayed on a contour diagram, appears very similar to Figure 4 and will not be shown. Instead, in Figure 5 we show two slices at constant $k_1$ through the two-dimensional power spectrum. These figures show $\Delta P$ as defined in equation (1) as a function of $\omega$. The choice of which slices to display was made with a random number generator. The 192-minute subset is shown as the lighter line in these figures. Because the definition of $\Delta P$ depends on the record length, for spectral lines with a resolved intrinsic shape, the ratio of equivalent width from the long run to equivalent width in the short subset should be 1:1.3. In fact, we see that the peak value of $\Delta P$ does not change in most lines and often increases. Figure 5a shows an example where adjacent spectral lines are not resolved from each other. The multiple peaks in the 192-minute subset are not stable and are a spurious result of our frequency resolution at such long wavelengths. Figure 5b is a contrasting example at a very short wavelength.

c) Comparison of Observed and Theoretical Frequencies

The eigenfrequencies of nonradial $p$-modes as functions of the number of nodes in the radial direction and of the mass of the convective envelope have recently been published in Paper II. Previously published $p$-mode frequencies with appropriate spatial characteristics were those of Paper I and of Ando and Osaki (1975). The frequencies of Paper I and of Ando and Osaki, which were all found by Deubner (1975, 1976) to be systematically above the frequencies of his observed power, were consistent with the frequencies computed from an envelope having $M_e/M_\odot = 7.5 \times 10^{-4}$ in the context of Paper II. This systematic disagreement of the earlier theoretical frequencies with the observed frequencies reported by Deubner was such as to suggest the use of model envelopes having larger values of $M_e/M_\odot$. Because these earlier theoretical frequencies were also above our observed frequencies, we wished to directly compare our observed power spectrum with the eigenfrequencies computed from the additional model envelopes of Paper II. Such a comparison is shown in Figure 6.

In Figure 6 the observed 256-minute power spectrum of Figure 4 is repeated. However, instead of Deubner’s observed ridges as in Figure 4, Figure 6 compares the observed power distribution with the predicted position of the $p_0, p_1, \ldots, p_6$ ridges computed from model envelopes of Paper II which corresponded to $M_e/M_\odot = 0.003$ (upper light lines) and to $M_e/M_\odot = 0.087$ (lower dark lines). The set of ridges corresponding to $M_e/M_\odot = 0.087$ may be seen to fit the observed location of the ridges more closely than the ridges for $M_e/M_\odot = 0.003$. In nearly every case the strong peaks shown in the observed power spectrum fall on or slightly below one of the computed ridges from the $M_e/M_\odot = 0.087$ set.

The excellent agreement between the observations and the theory is shown in a different way in Figure 7. In this figure the one-dimensional vertical slices of the 256-minute, two-dimensional observed power spectrum of Figures 4 and 6 are shown at a series of constant values of $k_1$ as in Figure 5. Every fourth possible value of $k_1$ with $k_1 \leq 1.371$ Mm$^{-1}$ is shown. Larger values of $k_1$ are not shown because the signal-to-noise ratio is unsatisfactory. We chose the set of slices in Figure 6 randomly by tossing a coin. The theoretical eigenfrequencies of those $p$-modes with $\omega < 0.035$ s$^{-1}$ are shown as functions of the logarithm of the relative mass of the convective envelope, $\log (M_e/M_\odot)$, and are compared with these $\Delta P$ versus $\omega$ spectra. The function of $\log (M_e/M_\odot)$ is used in this figure because the eigenfrequencies are almost linearly dependent on this variable. Also shown in these figures are the observed central frequencies and half-widths at half-maximum for those distinct peaks which exceed twice the noise levels as described in § 1Vb. Peaks which fall in regions that are predicted to be poorly resolved (e.g., $k_1 \leq 0.3$ Mm$^{-1}$) were not identified. Also, we have not identified as distinct any peak in $\Delta P$ which has an adjacent minimum greater than half the maximum of $\Delta P$.

d) The Long-Period Signal

A close inspection of Figure 6 reveals an appreciable signal at small values of $\omega$. Apart from the corner where both $k_1$ and $\omega$ are small, we do not know of any instrumental effects which would produce this signal. The signal between 0.06 Mm$^{-1}$ and 0.2 Mm$^{-1}$ is almost certainly due to supergranulation. The amplitude of this motion is between 0.5 and 2 m s$^{-1}$ for all these values of $k_1$. Assuming isotropy and multiplying the power by $2\pi k_1/\Delta k_1$, we find the vertical velocity amplitude of the supergranulation to be roughly 10 m s$^{-1}$.

We also find a signal at large values of $k_1$ which appears to be due to a long-period oscillation. We believe that this signal could result from combining a small-scale component of the supergranulation velocity field and a gradual drift in the guiding of the telescope. The drift during the 41 hours of observing need only be on the order of 4" to generate an apparently varying signal from a stationary velocity field. The observed amplitude of this signal is on the order of 30 cm s$^{-1}$ and is a small fraction of the amplitude of the supergranulation velocity. This explanation of the long-period signal can be tested by registration of the frames.

e) Masking Test of Envelope Models

We wish to test the quality of the fit of the theoretical eigenfrequencies to the observed distribution of power within the region of the $(k_1, \omega)$-plane where our data provide adequate resolution. This test should
Fig. 6.—Comparison of our contour diagram with theoretical eigenfrequencies for two envelope models. The contour intervals and bin size are exactly the same as for Fig. 4. The envelope masses for the two theoretical models are shown in the lower right of this figure.
FIG. 7.—Vertical slices through the \((k_\lambda, \omega)\)-contour diagram compared with theoretical eigenfrequencies as a function of envelope mass. Model numbers are indicated on the eigenfrequency curves. The frequencies and half-widths at half-maximum (HWHM) are given for those peaks which exceed twice the noise level. The half-widths are the least significant figures of the frequencies.
measure the goodness of fit in a quantitative way and account for the power distribution throughout the chosen region in a uniform manner. Because of the finite bin size of our measured power spectrum, we do not have available smooth and continuous data. We have not smoothed our data beyond the cosine tapering because smoothing decreases the resolution. Instead, we have used a masking procedure which chooses bins on the basis of the proximity of their central frequencies to the eigenfrequencies. When the bin frequency is sufficiently close to the eigenfrequency, that bin is included as part of the mask. The total power selected by the mask depends both on the number of mask bins and on the power within the bins. The number of bins within the mask depends primarily on the restrictiveness of the mask definition.

The average power per selected bin is a good measure of the quality of the fit of the eigenfrequencies to the observed power. When the theoretical model provides the best fit, only bins with a large observed power will be selected and a maximum in the average power per bin will be achieved. The degree of concentration of the observed power to the theoretical frequencies is measured by comparing the average power per bin obtained with a restrictive mask and the average power obtained with a broad mask which selects every bin in the appropriate part of the \((k_h, \omega)\)-plane. If the observed power is distributed randomly, the average should be independent of mask width. On the other hand, if the observed power is highly concentrated to the theoretical frequencies, the average should be inversely proportional to the number of bins in the mask.

As an application of the above ideas we have developed the following numerical procedure: at each value of \(k_i\) and \(\omega_i\), where \(k_i\) and \(\omega_i\) are grid values in our spectral analysis, we define a mask function \(M(k_j, \omega_i)\) which has the value 1 when

\[
\omega_i - \omega_{k,n} < a (\omega_{k,n+1} - \omega_{k,n})/2, \quad (2a)
\]

or

\[
\omega_{k,n} - \omega < a (\omega_{k,n+1} - \omega_{k,n-1})/2, \quad (2b)
\]

and has the value 0 when the inequalities (2) are not satisfied. Here \(\omega_{k,n}\) is the calculated frequency of \(p_n\), the \(p\)-mode having \(n\) radial nodes in the displacement eigenfunction and a horizontal wavenumber \(k_i\). The quantity \(a\) determines the fraction of the area in the \((k_h, \omega)\)-plane covered by the mask function. For \(a = 1\), adjacent bands touch, and \(M = 1\) throughout the region occupied by the \(p\)-modes. For \(n = 0\), we have used the right-hand side of inequality (2a) in place of the right-hand side of inequality (2b). We have considered only those modes with \(\omega < 0.035 \text{ s}^{-1}\) to have power of solar origin. Consequently, any mode with \(\omega_{k,n} > 0.035 \text{ s}^{-1}\) is ignored and \(M = 0\) for \(\omega\) greater than the value given by inequality (2a) for the last mode with \(\omega_{k,n} < 0.035 \text{ s}^{-1}\). In the present analysis we consider only the first six modes.

We have used the mask function to evaluate the noise in our observations from the following formula:

\[
\delta P_{\text{Noise}}(k_i) = \frac{\sum_{j=1}^{65} [1 - M(k_j, \omega_i)] \delta P(k_j, \omega_i)}{\sum_{j=1}^{65} [1 - M(k_j, \omega_i)]}. \quad (3)
\]

The power due to solar motion is then

\[
P_{\odot} = \sum_{j=1}^{65} \left[ \sum_{i=1}^{65} M(k_j, \omega_i) \delta P(k_j, \omega_i) \right] - \delta P_{\text{Noise}} \sum_{j=1}^{65} M(k_j, \omega_i). \quad (4)
\]

We summed over those values of \(k_i\) for which we find a high level of power and for which the predicted eigenfrequencies should be well resolved. These limits are \(0.211 \text{ Mm}^{-1} \leq k \leq 1.090 \text{ Mm}^{-1}\) or \(j_9 = 6\) and \(j_9 = 31\). The total number of points \(N\) which contribute to the solar motion power is

\[
N = \sum_{j=1}^{65} \sum_{i=1}^{65} M(k_j, \omega_i), \quad (5)
\]

and the average power per point is

\[
\langle \delta P_{\odot} \rangle = P_{\odot}/N. \quad (6)
\]

For each value of mask width \(a\), we have examined the behavior of \(\langle \delta P_{\odot} \rangle\) as a function of envelope mass. When \(a = 1\), the mask covers a broad band on the \((k_h, \omega)\)-plane and includes essentially all the power due to the 5-minute oscillations. This broad mask is shown on Figure 8 as the region of lightest shading for the eigenfrequencies of the \(M_e/M_\odot = 0.087\) model. The number of points in this mask is 1150. For other sets of eigenfrequencies, the location of the mask shifts slightly but, because of the complete coverage, the average power per point is independent of the choice of theoretical model. This value of \(\langle \delta P_{\odot} \rangle\) is thus expected if the power is distributed randomly. For smaller values of \(a\) the mask takes on the shapes shown in Figure 8 by successively darker shades. Since the mask does not cover the entire \((k_h, \omega)\)-plane, different theoretical models pick up a different amount of the power. Thus \(\langle \delta P_{\odot} \rangle\) as a function of \(M_e/M_\odot\) provides a measure of the fit of the theory to the observations. A value of \(\langle \delta P_{\odot} \rangle\) greater than that obtained with \(a = 1\) indicates that the power is concentrated in the mask region to a higher degree than expected from a random distribution. The narrowest mask has the best resolution, but it also has the smallest number of points \((N \approx a)\) and is subject to numerical noise. We have used masks with \(a = 0.5\), 0.25, and 0.1 to determine the strictest possible limit on \(M_e/M_\odot\) without introducing excessive numerical noise. The result of this masking operation is shown in Figure 9. The noise in the curve for \(a = 0.1\) is noticeably greater than for \(a = 0.25\) or 0.5, but does not appear to be large enough to render the result questionable. This curve crosses the curve for \(a = 1\) at \(M_e/M_\odot = 0.011\).
We take this crossing point to be the lower limit to the envelope mass. We have carried out this procedure with a Gaussian mask function instead of a step function and have reached the same conclusion. The result in Figure 9 indicates that we cannot choose between models with \( M_e/M_0 > 0.03 \) on the basis of the observations.

Since we are interested primarily in the frequencies of the eigenmodes, the distribution of power along the modes is potentially useful in determining the mechanism of excitation for the oscillations. Figures 4 to 7 indicate this distribution of power in ways which are not completely satisfactory for this purpose. If the eigenfrequencies were completely resolved and perfectly sharp, the height of the peaks in Figures 5 and 7 would indicate the power per mode. Because of the spreading of the frequency peaks, we have integrated the power over a fixed bandwidth. Specifically, we used the procedure of § 1Ve to determine the power in a bandwidth of \( \delta \omega = 0.0024 \). When this bandwidth leads to overlapping bands for small
Fig. 9.—The dependence of the average power per point on the choice of theoretical model for masks chosen with the value of $a$ indicated next to each line. We have adopted as the lower limit to the envelope mass the point where the $a = 0.1$ curve crosses the $a = 1.0$ curve. This limit is indicated by the vertical line.

values of $k_x$, the inequalities (2) with $a = 1$ are used instead. The power due to noise was subtracted from the signal in the way described above. The eigenfrequencies for the model with $M_e/M_\odot = 0.087$ were used.

After integration over $\delta \omega$, the result at each grid point is the rms power for all those eigenmodes with spatial characteristics with $-\delta k_x/2 < k_x < \delta k_x/2$ and $k_y - \delta k_y/2 < k_y < k_y + \delta k_y/2$. The values of $\delta k_x$ and $\delta k_y$ for our observations indicate that each grid point represents the result of roughly 582 eigenmodes (the bin size in $k_x$ and $k_y$ does not include an integral number of spherical harmonics). In order to obtain a smooth result, we added together the power for three grid points. Each such summed power point then is the result of roughly 1745 modes. We show in Figure 10 the summed power as a function of $\omega$ and mode number determined by this procedure. Most evident in this figure is the fact that the peak power is very similar for the modes $p_2$ to $p_4$. Also the modes $p_3$ and $p_4$ seem to have significantly less power. The procedure was also followed for $p_5$ and $p_6$, but the results are not shown in Figure 10 in order to avoid complicating the drawing. The shape of these additional curves was essentially identical to the shape for $p_3$ and $p_4$, except that the resolution was poorer. The peak power of the value declined to $\log (v^2) = 1.2$ and 1.0 for $p_3$ and $p_4$, respectively. The typical peak power of 20 m$^2$s$^{-2}$ indicates that the power per mode reaches a maximum of 115 cm$^2$s$^{-2}$. The effective mass for these modes is roughly $5 \times 10^{34}$ g, according to Paper II. Thus the energy in the form of oscillations is roughly $6 \times 10^{20}$ ergs per mode.

Fig. 10.—The total power in a $\delta \omega = 0.0012$ s$^{-1}$ band centered on the theoretical eigenfrequencies as a function of the central frequency for the modes $p_0$ to $p_6$. Three bins in $k_x$ are summed to obtain the total power. This power is the result of 1745 modes.

V. DISCUSSION

The overall agreement between the observed frequencies of peaks in the power spectrum of the solar oscillations and the frequencies for nonradial $p$-mode pulsations indicates that the subphotospheric layers play the dominant role in determining the frequency of oscillation. Each $p$-mode of oscillation involves coherent motion over the solar surface. However, the identification of the 5-minute oscillations as $p$-mode oscillations of the Sun does not require large-scale coherence of the motion. As discussed above, the peaks we observe near this frequency are doubled, which may indicate that the interior and chromosphere behave as coupled oscillators. The doubling could also be due to noise in our data or frame misregistration. The frequency of trapped gravity waves in the chromosphere depends sensitively on $k$. At $k = 0.5$ Mm$^{-1}$ the trapped gravity wave has $\omega_p$ near $4 \times 10^{-3}$ s$^{-1}$, and for $k = 1.5$ Mm$^{-1}$ the value is about $1.1 \times 10^{-2}$ s$^{-1}$. We find no indication of power near these gravity wave frequencies. We conclude that the 5-minute oscillations are the photospheric result of nonradial $p$-mode oscillations of the Sun.

On the basis of our identification of the 5-minute oscillations, we can use the observed power spectra as a probe of the solar envelope. The results of § IV showed that envelope models with $M_e/M_\odot < 0.011$ are inconsistent with our present data. As far as we
can determine from the power spectra, convection can be arbitrarily efficient and the Sun could be adiabatic from the photosphere to near \( r = 0.5 R_\odot \). However, it is clear from Table 1 of Paper II that the Li destruction rate becomes very high for large envelope masses.

Further observations of this type should serve to refine this important new constraint on solar models.

We gratefully acknowledge several helpful discussions with Dr. Franz Deubner, who read the manuscript critically and also gave his permission to refer to his latest unpublished results in Figure 4. We wish to thank Frank Hegwer, Dick Mann, and Horst Mauer of the Sacramento Peak Observing Staff for their assistance in modifying and operating the diode array hardware to accommodate the requirements of this experiment. The extensive numerical calculations necessary for this analysis were performed on the IBM 360/91 of the UCLA Campus Computing Network, on the 370/158 of the Caltech Computing Center, and on the XDS 35 computer at Sacramento Peak Observatory. The cooperation and assistance of these computing center staffs are greatly appreciated. The VICAR image-processing software of the Jet Propulsion Laboratory and image-processing programs supplied by Dr. A. Tescher of Aerospace Corporation were also useful in this work. Discussions with Drs. E. Frazier, G. Chapman, R. Teske, and E. Mayfield at Aerospace were also helpful. We thank Professor R. B. Leighton for suggesting the masking procedure used in §1Ve. Also, we gratefully acknowledge preliminary support from the Aerospace Corporation while one of us (E. J. R.) was on the staff of the San Fernando Observatory.

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