THE SENSITIVITY OF NONRADIAL $p$ MODE EIGENFREQUENCIES TO 
SOLAR ENVELOPE STRUCTURE*

ROGER K. ULRICH
Astronomy Department, University of California at Los Angeles; and W. K. Kellogg Radiation Laboratory,
California Institute of Technology, Pasadena

AND
EDWARD J. RHODES, JR.
Astronomy Department, University of California at Los Angeles; and Jet Propulsion Laboratory,
Pasadena, California

Received 1977 January 27; accepted 1977 May 19

ABSTRACT

Eigenfrequencies are calculated for nonradial $p$ mode oscillations in the Sun. These frequencies 
are shown to depend on the adiabat of the solar convective envelope. The frequency dependence 
is greatest for the $p_2$ and $p_3$ modes described by spherical harmonics with $l = 500$ to 1000. The 
solar envelope model includes a realistic representation of the chromosphere out to the base 
of the corona. For the chromospheric model of Vernazza, Avrett, and Loeser chromospheric 
modes with a distinct locus of eigenfrequencies exist near $\omega = 0.026 \text{s}^{-1}$. The actual frequencies 
of these eigenmodes depend on the thickness of the chromosphere. Because of the difference 
in the dependence of $\omega$ on $l$ for the chromospheric modes as compared to the $p$ modes, the two 
types of modes can be distinguished observationally. If detected, the chromospheric modes 
would provide a good way of determining the thickness of the chromosphere.

Subject headings: stars: pulsation — Sun: atmospheric motions — Sun: chromosphere — 
Sun: interior

I. INTRODUCTION

The five-minute oscillations discovered by Leighton, 
Noyes, and Simon (1962) are best described as $p$ modes 
of the Sun of high nonradial order (Ulrich 1970a, 
hereafter Paper I [the present paper is Paper II in the 
series]; Leibacher and Stein 1971). These $p$ modes are 
classified according to the number of velocity nodes, $n$, in the radial direction and the order, $l$, of the 
spherical harmonic describing the nonradial motion. The $p_n$ mode is often referred to as the $f$ mode because 
it is intermediate between the $p$ and $g$ modes. For the 
high-order modes studied here, the frequency of this 
mode is much closer to the $p$ modes than to the $g$ 
modes, and we feel that the $p_0$ designation is appro-
propriate. Observationally, the horizontal wave number, $k_h = \frac{\sqrt{l(l + 1)}}{R}$, is determined instead of the 
order $l$. The calculations in Paper I showed that the 
eigenfrequencies $\omega$ depend on $n$ and $l$ in such a way 
that power on the $k_h$-$\omega$ plane should lie along distinct 
branches. Recent work by Deubner (1975, 1976) and 
Rhodes, Ulrich, and Simon (1977, hereafter Paper III) 
has verified this prediction. It was pointed out, but not 
demonstrated, in Paper I that the location of the power 
ridges on the $k_h$-$\omega$ plane should depend on the en-
velope structure of the solar model. In this paper we 
report calculations of the theoretical $k_h$-$\omega$ plane for 
several model envelopes and show that strong limits 
can be placed on the entropy of the adiabatic zone. 

Although we use the mixing-length parameter as the 
label for each of our envelope models, we wish to 
emphasize that the quantity upon which the theoretical 
dispersion plane depends is the envelope entropy. 
Uncertainties in the mixing-length theory influence 
only the upper layers of the convective zone directly 
below the photosphere where $7000 < T < 15,000$ K. 
Similarly, the large temperature fluctuations associated 
with convective motions occur in this region. How-
ever, these layers occupy too small a volume of the 
envelope to influence the frequencies of the normal 
modes. At the present, the $k_h$-$\omega$ plane does not appear 
to provide a test of the mixing-length theory of 
convection.

Although the entropy is the fundamental quantity 
which defines the structure of the adiabatic portion of 
the convective envelope, it is not convenient to evaluate 
this quantity directly because of the necessity of 
determining an additive constant which depends on the 
state of matter of solar composition near 0 K. In fact, 
this additive constant is not of interest to us, and the 
structure of the convective envelope is adequately 
described by the entropy difference $\Delta S$ between 
the point of marginal stability and the adiabatic regime. 
We have calculated this entropy difference from

$$\Delta S = \int_{r_1}^{r_2} C_p (\nabla - \nabla_{ad}) d \ln P, \quad (1)$$

where $C_p$ is the heat capacity at constant pressure per 
average atom of solar matter, and $\nabla$ and $\nabla_{ad}$ are the

* Research supported at UCLA by NSF AST 75-19750 and 
NASA NSG 7158 and at Caltech by PHY76-02724.
structural and adiabatic logarithmic temperature gradients with respect to pressure.

As is well known, the radius of the Sun has not yet been an important parameter in defining a proper solar model because the entropy of the convection zone has previously been a completely unknown quantity. The envelope portion of a solar model contains a small fraction of the solar mass (0.001 to 0.1 $M_\odot$), yet it occupies a significant fraction of the solar volume (40–80%). The pressure at each depth within the envelope is relatively insensitive to the envelope structure so that the actual volume of the convective envelope is largely determined by the envelope entropy and temperature.

Thus the envelope entropy is connected to the construction of solar models through the following logical sequence: (1) the adoption of a set of assumptions and a computational procedure which define the solar model, (2) the adjustment of the hydrogen abundance and the mean molecular weight $\mu$ until the model luminosity $L = L_\odot$ in the solar model, (3) the adjustment of the envelope entropy until the model radius $R = R_\odot$. Even though the last step in the sequence does not influence the interior structure significantly, the fact that this step is possible permits a wide degree of flexibility in the two preceding steps. In fact, many different solar models have been computed recently because of the discrepancy between the rate of neutrino capture reported by Davis and co-workers (see most recently Bahcall et al. 1973) and the rate of capture predicted by solar models. Models based on standard assumptions yield a capture rate of $5.6 \pm 2$ SNU ($1 \text{ SNU} = 10^{-36}$ captures target atom$^{-1}$ s$^{-1}$), according to Bahcall et al. (1973) and Ulrich (1974). An envelope fitted to this model requires the mixing-length parameter $l/H$ to be 2.25. This value of $l/H$ is in good agreement with the value obtained by Torres-Peimbert, Simpson, and Ulrich (1969) with an independent solar interior code.

One type of solar model which previously has been difficult to rule out is called the low-Z model. This model involves the assumption that the abundances of the elements heavier than He are reduced. Such a model has reduced interior opacities and a neutrino capture rate of 1.0 and 1.4 SNU, according to Bahcall et al. (1973) and Abraham and Iben (1971). This type of model also has a low He abundance, and the Bahcall et al. model requires $l/H = 1.0$ to fit the solar radius. We show in this paper that the diagnostic $k_h-\omega$ plane is capable of distinguishing between $l/H$ of 1.0 and $l/H$ of 2.25. Thus a test of the low-Z solar model is possible.

II. ENVELOPE MODELS

Model envelopes for this paper were computed with a modified version of the mixing-length code described by Henyey, Vardya, and Bodenheimer (1965). Some of the modifications of this code have been discussed by Ulrich (1970) and by Ulrich and Scalo (1977). A major additional modification of the code involves the inclusion of a chromospheric temperature distribution. We have adopted an approximation to

![Fig. 1.](image_url)

Comparison of the chromosphere structure used in the calculations (UR) to the model of Vernazza et al. (1976). The abscissa is the altitude relative to the level where $\tau = 1$. 

© American Astronomical Society • Provided by the NASA Astrophysics Data System
the model of Vernazza, Avrett, and Loeser (VAL) (1973, 1976). An approximation is used here rather than an exact fit because the envelope model code utilizes a $T$-$r$ relation rather than a $T$-altitude relation as in the published models. The appropriate $T$-$r$ relation has been determined in an iterative manner. This approximation faithfully reproduces the adopted model except that the chromospheric thickness is 2650 km instead of the 2400 km of the model. Figure 1 compares the adopted model to the VAL (1976) model. The parameters other than $l/H$ needed in the treatment of convection as discussed and defined by Henyey, Vardya, and Bodenheimer (1965) were $v = 8$, $y = \frac{1}{3}$, and $p = 1$. The gradient in turbulent pressure was ignored in the definition of $\nabla_{ad}$ for reasons discussed by Ulrich (1970b). In the present calculations no nonlocal convective effects have been considered. The composition for all but one model was $X = 0.75$, $Z = 0.017$, where $X$ and $Z$ are the mass fractions of hydrogen and elements heavier than helium and the breakdown of $Z$ was taken from Ross and Aller (1976). The one model with a different composition had $Z = 0.65$ and $Z$ distributed as above. One additional model was computed without the chromospheric temperature rise.

Some of the structural properties of the computed models are given in Table 1. All derived quantities were evaluated at the base of the convective envelope. The entropy difference $\Delta s$ was computed from equation (1). The quantity $\tau_{ad}$ is the lifetime of lithium in the convective envelope against proton capture. This lifetime must be roughly greater than $10^9$ yr, which thus requires that $l/H \leq 3$. The models with $l/H \geq 2$ have convection which is nearly as efficient as possible and are relatively less sensitive to $l/H$ than are models with $l/H < 2$. The value of $X$ and the chromospheric structure play only a minor role in governing the envelope structure.

Iben and Mahaffy (1976) have recently reported eigenfrequencies for nonradial modes of spherical harmonic order 0 to 6. Their results do not overlap ours and therefore no comparison can be made to their acoustical spectrum. However, their envelope models can be compared to ours. The parameters at the base of the envelope given in their Table 1 are in excellent agreement with our results. The values of $l/H$ which they use to obtain each of the envelope models is a factor of 1.5 to 2 smaller than the values we use. However, when we interpolate among our models to find the properties of the envelope models which have the same base temperatures as do their models, the agreement of other properties is much better than it is for models having the same values of $l/H$. For the three models given by Iben and Mahaffy, our interpolated values of the density and the radial and mass extent of the convective envelope agree to within 30%, 35%, and 10%. These discrepancies are an indication of the reliability with which we can calculate properties of the convective envelope.

Any of the parameters given in Table 1 can be used to describe the one-parameter family of envelope models. As a matter of convenience in this paper, we use $l/H$ as the parameter. This choice is appropriate here because $l/H$ is the theoretical parameter which was used to construct the models. However, for an observational test of these calculations, as will be made in Paper III, this choice would not be as appropriate because the eigenfrequencies depend on $l/H$ in a non-linear fashion. On the other hand, the log of the envelope mass has roughly the same $l/H$ dependence as the eigenfrequencies and thus the eigenfrequencies are nearly linear functions of $\log (M_e/M_0)$. The transformation between this parameter and any other parameter can be made using Table 1.

The physical variables which are most important in determining the eigenfrequencies of the solar envelope are the temperature and sound velocity. These functions are shown in Figures 2 and 3 for the models presented in Table 1 with $l/H \leq 2$. Most of the differences between the models occur at depths of 10,000-30,000 km. The differences between the models for $4.0 \geq l/H \geq 2.0$ are so slight as to be unplottable on the scales of Figures 2 and 3.

### III. MODAL ANALYSIS

a) Analysis Procedure

The eigenfrequency analysis of the envelope models follows closely the procedures described in Paper I.

#### TABLE 1

Properties of Envelope Models

<table>
<thead>
<tr>
<th>$l/H$</th>
<th>$\log T$</th>
<th>$\log P$ (dynes cm$^{-2}$)</th>
<th>$\log r$ ($R_\odot$)</th>
<th>$M - M_e$ ($M_\odot$)</th>
<th>$\tau_{ad}$ (yr)</th>
<th>$\Delta s$ ($10^6$ ergs g$^{-1}$ K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.8</td>
<td>5.692</td>
<td>10.930</td>
<td>0.912</td>
<td>7.15 x 10$^{-5}$</td>
<td>...</td>
<td>6.55</td>
</tr>
<tr>
<td>1.0</td>
<td>5.934</td>
<td>12.025</td>
<td>0.858</td>
<td>7.48 x 10$^{-5}$</td>
<td>...</td>
<td>5.07</td>
</tr>
<tr>
<td>1.2</td>
<td>6.088</td>
<td>12.703</td>
<td>0.812</td>
<td>3.04 x 10$^{-3}$</td>
<td>...</td>
<td>4.08</td>
</tr>
<tr>
<td>1.5</td>
<td>6.233</td>
<td>13.338</td>
<td>0.757</td>
<td>1.08 x 10$^{-2}$</td>
<td>3.9 x 10$^{-3}$</td>
<td>3.11</td>
</tr>
<tr>
<td>2.0</td>
<td>6.371</td>
<td>13.932</td>
<td>0.694</td>
<td>3.36 x 10$^{-2}$</td>
<td>1.4 x 10$^{-1}$</td>
<td>2.20</td>
</tr>
<tr>
<td>3.0</td>
<td>6.493</td>
<td>14.447</td>
<td>0.630</td>
<td>8.65 x 10$^{-2}$</td>
<td>2.4 x 10$^{-2}$</td>
<td>1.36</td>
</tr>
<tr>
<td>4.0</td>
<td>6.557</td>
<td>14.708</td>
<td>0.589</td>
<td>1.37 x 10$^{-1}$</td>
<td>1.1 x 10$^{-1}$</td>
<td>0.99</td>
</tr>
<tr>
<td>2.0*</td>
<td>6.422</td>
<td>14.166</td>
<td>0.687</td>
<td>5.67 x 10$^{-2}$</td>
<td>9.9 x 10$^{-2}$</td>
<td>1.89</td>
</tr>
<tr>
<td>2.0†</td>
<td>6.349</td>
<td>13.964</td>
<td>0.691</td>
<td>3.60 x 10$^{-2}$</td>
<td>9.1 x 10$^{-2}$</td>
<td>2.13</td>
</tr>
</tbody>
</table>

* $X = 0.65$.  † No chromosphere.
The following modifications have been made: (1) Since the envelope models now include a realistic chromospheric temperature distribution, it is now appropriate to impose a zero boundary condition at the outer mesh point. Above the chromosphere, the temperature rises rapidly and the waves become non-propagating. This zero boundary condition was used in the analysis described in this paper. As a check on the error introduced by neglecting the corona, one additional set of eigenmodes was calculated with a zero boundary condition at a radius of 1.04 $R_\odot$ in a realistic coronal model. The eigenfrequencies were altered less than 1% by the more accurate boundary condition. (2) The frequency has been treated as a complex instead of a purely real number as in Paper I. In all cases the imaginary part of the frequency was much smaller than the real part. (3) Small correction terms have been added to the continuity equation to account for sphericity effects. Other sphericity effects which enter through $g$ were already included in Paper I. The sphericity corrections introduced in the above modifications had a negligible effect on the eigenfrequencies.

The treatment of the energy equation in both this paper and in Paper I is not as satisfactory as the treatment used by Ando and Osaki (1975). The inaccuracy of the method used here can influence the eigenfrequency only in optically thick layers where the motion is nonadiabatic. The formulation is accurate in the optically thin layers. In these layers, however, a new problem arises; the source of energy for the temperature rise in the chromosphere is not known and cannot be modeled. If this chromospheric energy source responds to the oscillations and liberates a variable amount of energy during an oscillatory cycle, the character of the wave could be modified. All these inaccuracies which are associated with the energy equation are likely to influence the growth or decay of an eigenmode. However, the thickness of the problem layers is sufficiently small that the eigenfrequencies are not influenced. We can see that the eigenfrequencies are independent of the uncertainties in the energy equation by comparing the model shown in the last line of Table 1, which lacks a chromosphere entirely, to models with a chromosphere. The chromosphere involves a layer thicker than the layer where the energy equation is uncertain. Consequently, we expect the presence or absence of a chromosphere to cause larger changes in the $k$-$\omega$ plane than could be induced by the energy equation. Most of the derived eigenfrequencies differed from those in the model with a chromosphere and with the same value of $l/H$ by less than 1%. At the
highest frequencies which are studied here, the chromospheric layer can have a trapped oscillation which is in resonance with the oscillation in the envelope below. For those frequencies, the eigenfrequencies in the models with and without a chromosphere behaved quite differently as functions of $k$. This difference is discussed in § IIIa below.

The nonradial oscillations for which $k_R \gg 1$ do not penetrate past the outer envelope except for frequencies much higher than those studied here. This fortunate result arises because the nonradial oscillations are in effect a superposition of oblique sound waves. The increase in sound velocity caused by the rising internal temperature of the Sun ultimately reflects the oblique sound wave back toward the solar surface. Interior to this reflection layer, the amplitude of the mode decreases exponentially with a scale length of $(\alpha T)^{-1}$. This reflection layer was included in the envelope model for all modes reported on in this paper. The eigenfrequencies of the modes are determined by the temperature distribution above the reflection layer and are completely independent of the temperature interior to the reflection layer. Thus the nonradial oscillation eigenfrequencies are sensitive to the envelope structure alone. Uncertainties in the interior structure do not influence the comparison between observed and calculated frequencies.

\section*{b) The $k$-$\omega$ Plane}

Figure 4 shows the results of the modal analysis of the envelope models given in Table 1. The real part of the frequency is $\omega$ and the horizontal wavenumber is $k_h = (k_x^2 + k_y^2)^{1/2}$, where $k_x$ and $k_y$ are the wavenumbers in the $x$ and $y$ directions. Each family of eigenfrequencies is labeled on the left by $p_n$, where $n$ is the number of nodes in the radial direction. Each line within the group of lines with a given value of $n$ corresponds to the locus of eigenfrequencies in a model envelope with a specific value of the adiabatic entropy. Following Table 1, these envelope models and loci of eigenfrequencies are designated by the value of $\Pi/H$ used in the model computation. It is worth emphasizing again that this designation is not fundamental; the fundamental model envelope parameter is the entropy of the adiabatic zone. Within each group of eigenfrequency loci, the value of $\Pi/H$ decreases through the set (4.0, 3.0, 3.0, 1.5, 1.2, 1.0, 0.8) as frequency increases. For the $p_0$ modes all eigenfrequencies are independent of $\Pi/H$, while for other $p_n$ the $\Pi/H = 4.0$ and 3.0 loci are nearly indistinguishable. The eigenfrequency calculations are terminated at high frequencies because the resonance with the chromosphere introduced considerable uncertainty, and at small values of $k_h$ because the modes began to
penetrate below the region included in the envelope models.

In an important check of the computational details, the eigenfrequencies computed here can be compared to the frequencies computed by Ando and Osaki (1975). The model they used was generated by the Paczyński (1969) envelope code with \( l/H = 1.0 \). The values used for the additional convection parameters were not stated. Presumably the values given by Paczyński were adopted, and these are in agreement with the values used here, except for \( y \) which was \( 4 \) instead of \( 3 \). The eigenfrequencies obtained by Ando and Osaki (1975) agree almost precisely with the frequencies shown in Figure 4 for \( l/H = 1.0 \). Their frequencies are generally lower than ours by \( 2 \times 10^{-4} \) s\(^{-1}\), which corresponds to an error in \( l/H \) of about 0.1. Thus our computational procedure is sufficiently reliable that conclusions about the structure of the solar envelope can be drawn from a comparison of the observed \( k_\lambda - \omega \) plane to the theoretical calculations.

c) Additional Properties of the Modes

In addition to the relationship between \( k_\lambda \) and \( \omega \), other properties of the mode calculations are of interest. One such quantity is the effective mass \( m_{nk}(r) \), defined to be

\[
m_{nk}(r) = \frac{4\pi}{\rho(r)^2} \int_0^r \left( \rho \frac{\partial u}{\partial r} + \frac{c^2 \rho}{\rho} \right) r^2 dr,
\]

where \( \rho' \) is the Eulerian density perturbation. Table 2 gives the values of \( m_{nk} \) evaluated at the grid point where \( r = 0.084 \) of the envelope model for \( l/H = 2.0 \). This grid point was chosen because it corresponds to a level in the model which is easily observable. The effective mass relates the energy in a mode of oscillation to the velocity amplitude of that mode. Another quantity which is potentially observable is the phase lag of the velocity between the photosphere and low chromosphere. The velocity is represented as

\[
v = |v| e^{i(\omega t + \phi)}.
\]
<table>
<thead>
<tr>
<th>$K_{n+1}$</th>
<th>Mode</th>
<th>$w_\nu$</th>
<th>$\omega$</th>
<th>Per</th>
<th>$K_{Mn}$</th>
<th>$\Delta P$</th>
<th>$\eta$</th>
<th>$\eta_{nc}$</th>
<th>$\eta_{AO}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2</td>
<td>P0</td>
<td>0.75</td>
<td>12.9</td>
<td>2.9(27)</td>
<td>-2.5°</td>
<td>4.1(-7)</td>
<td>-1.1(-6)</td>
<td>-1.6(-7)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P1</td>
<td>1.07</td>
<td>9.6</td>
<td>2.2(26)</td>
<td>-2.5°</td>
<td>7.7(-5)</td>
<td>5.3(-5)</td>
<td>5.0(-5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P2</td>
<td>1.51</td>
<td>8.0</td>
<td>5.1(26)</td>
<td>-2.2°</td>
<td>3.2(-4)</td>
<td>2.6(-4)</td>
<td>4.1(-5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P3</td>
<td>1.52</td>
<td>6.9</td>
<td>2.0(26)</td>
<td>-1.6°</td>
<td>7.5(+4)</td>
<td>6.0(+4)</td>
<td>1.0(+4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P4</td>
<td>1.72</td>
<td>6.1</td>
<td>1.2(25)</td>
<td>-1.1°</td>
<td>1.1(-3)</td>
<td>9.7(+4)</td>
<td>1.5(+4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P5</td>
<td>1.92</td>
<td>5.5</td>
<td>8.0(24)</td>
<td>-0.2°</td>
<td>1.4(-3)</td>
<td>1.4(-3)</td>
<td>1.8(-4)</td>
<td></td>
</tr>
<tr>
<td>0.4</td>
<td>P0</td>
<td>1.06</td>
<td>9.9</td>
<td>2.0(26)</td>
<td>-3.1°</td>
<td>2.4(-6)</td>
<td>-9.3(+6)</td>
<td>-1.1(+6)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P1</td>
<td>1.40</td>
<td>7.5</td>
<td>2.6(25)</td>
<td>-2.4°</td>
<td>5.0(+4)</td>
<td>3.8(+4)</td>
<td>2.7(+5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P2</td>
<td>1.71</td>
<td>6.1</td>
<td>8.2(24)</td>
<td>-1.5°</td>
<td>1.6(-3)</td>
<td>1.5(-3)</td>
<td>2.0(+4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P3</td>
<td>1.99</td>
<td>5.5</td>
<td>4.7(24)</td>
<td>0.0°</td>
<td>2.1(-3)</td>
<td>1.9(-3)</td>
<td>3.1(-4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P4</td>
<td>2.26</td>
<td>4.6</td>
<td>3.2(24)</td>
<td>1.9°</td>
<td>2.0(-3)</td>
<td>2.1(-3)</td>
<td>3.2(-4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P5</td>
<td>2.91</td>
<td>4.2</td>
<td>2.2(24)</td>
<td>5.2°</td>
<td>1.6(-3)</td>
<td>2.2(-3)</td>
<td>3.6(-4)</td>
<td></td>
</tr>
<tr>
<td>0.6</td>
<td>P0</td>
<td>1.30</td>
<td>6.1</td>
<td>5.3(25)</td>
<td>-5.4°</td>
<td>6.0(-6)</td>
<td>-5.7(+5)</td>
<td>-5.6(+6)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P1</td>
<td>1.65</td>
<td>6.3</td>
<td>8.6(24)</td>
<td>-2.2°</td>
<td>1.1(-3)</td>
<td>9.2(+4)</td>
<td>1.1(+4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P2</td>
<td>2.00</td>
<td>5.2</td>
<td>5.6(24)</td>
<td>-0.4°</td>
<td>2.5(+3)</td>
<td>2.2(+3)</td>
<td>7.0(+4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P3</td>
<td>2.54</td>
<td>4.5</td>
<td>3.2(24)</td>
<td>2.4°</td>
<td>2.8(+3)</td>
<td>2.7(+3)</td>
<td>1.0(+5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P4</td>
<td>2.67</td>
<td>4.9</td>
<td>1.6(24)</td>
<td>8.9°</td>
<td>7.3(+4)</td>
<td>2.5(+3)</td>
<td>7.4(+4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P5</td>
<td>2.95</td>
<td>3.6</td>
<td>1.0(24)</td>
<td>10.0°</td>
<td>2.1(-4)</td>
<td>1.0(-3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.8</td>
<td>P0</td>
<td>1.49</td>
<td>7.0</td>
<td>1.0(25)</td>
<td>-5.6°</td>
<td>1.2(-5)</td>
<td>-1.0(+4)</td>
<td>-5.6(+6)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P1</td>
<td>1.67</td>
<td>5.6</td>
<td>4.3(24)</td>
<td>-1.7°</td>
<td>1.8(-5)</td>
<td>1.5(-5)</td>
<td>1.1(+4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P2</td>
<td>2.26</td>
<td>4.6</td>
<td>2.2(24)</td>
<td>1.1°</td>
<td>3.1(-3)</td>
<td>2.0(+3)</td>
<td>7.0(+4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P3</td>
<td>2.63</td>
<td>4.0</td>
<td>1.2(24)</td>
<td>10.9°</td>
<td>-2.2(+4)</td>
<td>2.7(+3)</td>
<td>1.0(+5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P4</td>
<td>2.97</td>
<td>3.5</td>
<td>6.0(23)</td>
<td>10.1°</td>
<td>-5.7(-5)</td>
<td>1.2(-3)</td>
<td>7.4(+4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P5</td>
<td>3.42</td>
<td>3.1</td>
<td>3.9(24)</td>
<td>107.0°</td>
<td>-8.4(-3)</td>
<td>-3.5(-3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td>P0</td>
<td>1.67</td>
<td>6.3</td>
<td>6.3(24)</td>
<td>-3.7°</td>
<td>1.9(+5)</td>
<td>-2.2(-4)</td>
<td>-5.2(+6)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P1</td>
<td>2.08</td>
<td>5.0</td>
<td>2.7(24)</td>
<td>-1.0°</td>
<td>2.5(+3)</td>
<td>2.1(-3)</td>
<td>2.2(+4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P2</td>
<td>2.50</td>
<td>4.2</td>
<td>1.4(24)</td>
<td>3.5°</td>
<td>2.9(+3)</td>
<td>3.1(-3)</td>
<td>1.0(-3)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P3</td>
<td>2.89</td>
<td>3.6</td>
<td>6.4(23)</td>
<td>7.7°</td>
<td>-7.8(-4)</td>
<td>2.0(-3)</td>
<td>1.5(-5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P4</td>
<td>3.21</td>
<td>3.3</td>
<td>6.0(23)</td>
<td>15.7°</td>
<td>-1.7(+2)</td>
<td>-1.5(-3)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.2</td>
<td>P0</td>
<td>1.85</td>
<td>5.7</td>
<td>4.5(24)</td>
<td>-3.3°</td>
<td>2.4(+5)</td>
<td>-4.0(-4)</td>
<td>-1.1(+5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P1</td>
<td>2.28</td>
<td>4.6</td>
<td>1.9(24)</td>
<td>0.0°</td>
<td>3.0(-3)</td>
<td>2.6(-3)</td>
<td>4.2(+4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P2</td>
<td>2.71</td>
<td>3.9</td>
<td>8.0(23)</td>
<td>17.1°</td>
<td>-6.7(-5)</td>
<td>5.0(-3)</td>
<td>1.4(-3)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P3</td>
<td>3.11</td>
<td>3.4</td>
<td>5.6(23)</td>
<td>11.9°</td>
<td>-9.7(-3)</td>
<td>-6.4(-5)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>P4</td>
<td>3.58</td>
<td>2.9</td>
<td>7.1(23)</td>
<td>15.9°</td>
<td>-6.2(-3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.4</td>
<td>P0</td>
<td>1.97</td>
<td>5.3</td>
<td>2.1(24)</td>
<td>-3.3°</td>
<td>2.0(+5)</td>
<td>-7.1(-4)</td>
<td>-1.2(-5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P1</td>
<td>2.47</td>
<td>4.2</td>
<td>1.3(24)</td>
<td>1.4°</td>
<td>3.5(-3)</td>
<td>3.0(-3)</td>
<td>7.1(-4)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P2</td>
<td>2.96</td>
<td>3.6</td>
<td>7.0(23)</td>
<td>7.0°</td>
<td>-5.6(-4)</td>
<td>2.0(-3)</td>
<td>1.0(-3)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>P3</td>
<td>3.28</td>
<td>3.2</td>
<td>5.6(23)</td>
<td>58.1°</td>
<td>-2.1(-4)</td>
<td>-3.9(-5)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 2 gives $\Delta \phi = \phi(\tau = 0.084) - \phi(\tau = 0.0011)$. The level $\tau = 0.0011$ was chosen because it is near the temperature minimum. Thus $\Delta \phi$ is the phase angle by which the photospheric velocity leads the velocity in the low chromosphere.

Although the growth rates calculated here are not reliable, it is of interest to show how the presence of a chromospheric temperature rise affects the stability of the modes. All modes were assumed to have time dependence of the form $\exp(it + \sigma t)$. Table 2 gives the growth parameter $\eta = -\sigma/\omega$ for two models with $l/H = 2$—first, the model with a chromosphere, and second, in the column headed by $\eta_{nc}$, for the model without a chromosphere. Surprisingly, the chromosphere has a slight destabilizing effect for the longer period modes. For the shorter period modes the chromosphere has a strong stabilizing effect. Finally, Table 2 gives the stability parameter calculated by Ando and Osaki (1975) from the more accurate theory in the column headed by $\eta_{a0}$. The pattern of stability shown by the no-chromosphere calculations here is very close to the pattern found by Ando and Osaki, although several modes, according to our calculations, are 5 to 10 times as unstable as was found by Ando and Osaki. The temperature fluctuation associated with the oscillations is of some interest because it can influence the transparency of the solar limb. This fluctuation can be conveniently compared to $(V_{ad} - V)\delta r/H$, the adiabatic temperature fluctuation. In nearly all cases the actual temperature fluctuation is less than one-fifth as large as this adiabatic temperature fluctuation. The only exceptions occurred near the velocity nodes in those modes which had frequencies higher than the chromospheric resonance frequency.

Detailed tables of the modes and the envelope models are available on request.

d) The Effect of the Chromospheric Resonance on the Eigenfrequencies

If the photosphere and base of the corona are treated as rigid boundaries, a standing wave can be set up. The frequency of this wave can be found from the vertical wavenumber $k_r$, where

$$k_r \approx \left( \frac{\omega^2}{c^2} - \frac{\omega_0^2}{c^2} - k_r^2 \right)^{1/2}$$

and the condition that

$$\int_{\text{photo}}^{\text{corona}} k_r dr = n\pi.$$

A rough evaluation of $k_r$ from the characteristics of the model chromosphere yields $\omega_{cr} \approx 0.026 \text{ s}^{-1}$ for

![Fig. 5.—A detail of the $k_r$-$\omega$ plane showing how the chromospheric mode interacts with the interior $p$ modes. The envelope model was for $l/H = 2.0$.](image-url)
$n_C = 1$ and $\omega_C \approx 0.035 \text{s}^{-1}$ for $n_C = 2$. These frequencies depend on the detailed structure of the chromosphere and could be affected by inhomogeneities. The dependence of $\omega_C$ on $k_h$ is much smaller than is the dependence of the frequencies of the envelope $p$ modes on $k_h$. At several points the envelope $p$ mode frequencies and the chromospheric mode frequencies cross. At these points the pattern of eigenfrequencies on the dispersion $k-\omega$ plane is shown in Figure 5. This is an expansion of part of Figure 4 and shows the mode pattern for a single-envelope model. The mode-interchange pattern shown in Figure 5 should have been included in Figure 4 but was suppressed for the sake of clarity. The numerical value of $\omega_C$ for $n_C = 1$ given above is confirmed by the detailed model integrations shown in Figure 5.

Although inhomogeneities below the photosphere are not large enough in amplitude to influence the eigenfrequencies, the known inhomogeneities in the chromosphere are large enough to alter $\omega_C$ by a large factor. The model of the chromosphere does not account for these inhomogeneities, and the calculated value of $\omega_C$ may not be appropriate. In fact, it is not clear that any single chromospheric frequency can describe the response of the chromosphere to an oscillatory forcing function. It is probably necessary for the structure of the chromospheric model to be guided by the observed behavior of power in the $k-\omega$ plane.

IV. CONCLUSIONS

We have shown that the eigenfrequencies of acoustic normal modes in the Sun depend on the entropy of the adiabatic portion of the solar convective envelope. The sensitivity is great enough to be detectable, especially for $I/H \approx 1.5$. The calculations agree well with earlier calculations by Ulrich (1970a) and by Ando and Osaki (1975). Although we find the instability of the acoustic modes to be stronger than found by Ando and Osaki, we find the same pattern of instability that they found as long as we neglect the chromosphere. When we include a chromosphere, the band of instability shifts toward longer periods. The agreement between these two very different and independent investigations encourages us to believe that the results are not sensitive to the computational procedure.

We thank G. W. Simon and R. F. Stein for reading this manuscript and for several helpful suggestions. One of us (R. K. U.) gratefully acknowledges a number of stimulating discussions with Peter Goldreich. The technical assistance provided by the staff of the Kellogg Radiation Laboratory in preparing the manuscript for publication has been very helpful.

REFERENCES


Edward J. Rhodes, Jr. and Roger K. Ulrich: Department of Astronomy, University of California, Los Angeles, Los Angeles, CA 90024