THE EMISSION SPECTRUM OF THE HYDROGEN BALMER SERIES
OBSERVED ABOVE THE SOLAR LIMB FROM SKYLAB
I. A QUIET SUN AND A POLAR CORONAL HOLE

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ABSTRACT

The hydrogen Balmer emission-line spectrum (Hα [3835 Å] to the series limit at 3646 Å) above
the limb of the quiet Sun and above the north polar coronal hole is discussed. The data were
obtained by the NRL XUV spectrograph aboard Skylab, with the slit tangent to the limb at
2" (1450 km) and at 4" (2900 km) above the limb. Electron densities of ~2 × 10¹¹ cm⁻³, 2" above
the limb of both the quiet Sun and coronal hole, are calculated from the Stark broadening of the
higher series member lines, and the related merging of the higher member lines. The widths of
the lines with principal quantum number m < 15 are broadened by opacity, and the opacities are
estimated from the line widths. The widths of lines of m ≈ 15 are not appreciably affected by
either opacity or Stark broadening. The combined ion temperature and nonthermal mass motion
determined from the widths of these lines are consistent with previously determined values. The
intensities of the lines indicate that the upper levels (≥9) are populated in statistical equilibrium
with each other. Absolute intensities are given, as well as the decrease of the intensity of the lines
as a function of height above the limb. The Balmer emission-line spectrum above the limb over three
active regions is discussed in the following paper.

Subject headings: Sun: corona — Sun: spectra

I. INTRODUCTION

A series of previous papers have discussed XUV limb spectra above a quiet Sun and polar coronal hole region
(Doschek, Feldman, and Tousey 1975; Doschek et al. 1976, hereafter Paper I; Feldman, Doschek, and Tousey
1975; Feldman et al. 1976, hereafter Paper II). The XUV spectrum of a chromospheric network region observed
near Sun center was also analyzed (Feldman, Doschek, and Patterson 1976). These spectra covered the wavelength
range from 1175 to 1940 Å, and were recorded by the NRL normal incidence spectrograph on Skylab. Recently,
we have examined spectra recorded by this instrument in the 1940–3940 Å region. These spectra are from the same
quiet Sun and coronal hole regions discussed in Papers I and II. Limb-brightening curves for chromospheric ions
and a list of lines for the spectrum recorded at +4" above the limb of the quiet Sun are given in Doschek, Feldman,

In this paper we discuss the hydrogen Balmer emission-line spectrum (from H9 to the series limit at 3646 Å)
above the limb of the quiet Sun and the north polar coronal hole. In a following paper we discuss the Balmer
spectrum above the limb over three active regions. The data discussed in this paper were obtained with the slit
tangent to the limb, 2" (1450 km), 4" (2900 km), and 6" (4350 km) above the limb. At 6" the hydrogen series for
lines above H10 becomes too weak for accurate intensity measurements to be made. This is due to the compara-
tively high level of scattered light in this spectral region. Below, we discuss the intensities and line widths of the
Balmer lines from principal quantum number m = 9 (3835 Å) to the Balmer limit. Electron densities are calculated
above the coronal hole and quiet Sun from the Stark broadening of high series member lines. The widths of the
lines of m < 15 are broadened by opacity. The opacity is estimated from the line widths. The widths of lines of
m ≈ 15 are not appreciably affected by either opacity or Stark broadening. The combined ion temperature and
nonthermal mass motion can be determined from the widths of these lines. The results are compared with previous
observations by Redman and Suemoto (1954).

II. THE INSTRUMENT

The NRL slit spectrograph on Skylab consists of two main systems, a telescope and the spectrograph. The
instrument can operate in either of two wavelength ranges: 970–1940 Å, or 1940–3940 Å. Below 1100 Å the ef-
ficiency is very low. An off-axis paraboloidal mirror images the Sun on the entrance slit of the spectrograph. The

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projected slit dimensions on the Sun are $2^\circ \times 60^\prime$ ($1450 \times 43,500$ km). The reflection of the Sun's image from the exterior of the slit plate is fed into an image dissector tube that drives the telescope mirror, so that the image of the solar limb can be set at predetermined locations relative to the slit. A 150 line mm$^{-1}$ predisser is used to disperse the radiation in the 1940–3940 Å region toward the main grating. The radiation from the predisser is diffracted by the 600 line mm$^{-1}$ main grating, and focused onto Kodak 104 photographic film. All the gratings are Bausch and Lomb aluminum replicas. The main grating is coated with a layer of MgF$_2$; the predisser is coated with layers of Al + ZnS + Al + ZnS. The spectral resolution in the long-wavelength region is 0.12 Å, and the dispersion at the film plane is 8.32 Å mm$^{-1}$. A complete description the the instrument is given by Bartoe et al. (1974).

III. DATA REDUCTION

We have obtained intensities and half-widths of the hydrogen Balmer lines from $m = 9$ (3835 Å) to the Balmer continuum at 3646 Å. The spectra were scanned with a Grant microdensitometer. Exposures of 2.5, 10, and 40 s were recorded, from which a characteristic curve for the film in this wavelength region was derived. A detailed description of this stage of data reduction is given in Papers I and II.

Line-intensity ratios between the same lines in different exposures agree to within an accuracy of better than 15%. The principal source of error is the background due to scattered light. The main source of this background is believed to be light from the disk of the Sun, scattered at the surface of the telescope's primary mirror, that then enters the instrument. The scattered light intensity as a function of height above the limb is discussed in Paper II. We were unable to find a satisfactory method for subtracting the background completely. Uncertainty in the true background levels prevents our use of the intensity of the Balmer continuum for electron temperature determinations.

The full widths at half-maximum (FWHM) of any line for which two exposures are usable normally agree to better than 10%. In a few instances, the widths of neighboring Balmer lines may disagree by 20%. The cause of these inconsistencies is blending with other emission lines or absorption lines present in the scattered continuum. Previous researchers discussing data obtained from eclipse expeditions either have omitted lines which were believed to be blended or have attempted to measure the width of a blended line after applying a correction to the line profile. From the data presented here, it is normally not obvious from one exposure to another that a line is blended; nor are the data good enough (except for the stronger lines, where the effects of blending are small) to justify resolving a blended line into two components. Therefore all the data measured will be used, unless lines are obviously blended or their intensities fall on either the shoulder or the toe of the film-characteristic curve. For example, the H14 line is obviously blended in the exposures $2^\circ$ above the limb, but is not blended $4^\circ$ above the limb (see Figure 3).

IV. RESULTS AND ANALYSIS

Figures 1 and 2 show the spectrum between 3640 and 3720 Å for slit positions of $2^\circ$ and $4^\circ$ above the quiet Sun and a coronal hole, respectively. Note that, in each figure, the intensity scale differs by a factor of 10 between the $2^\circ$ and $4^\circ$ exposures. Different scales are used for the quiet Sun and coronal hole data. Figure 3 shows the intensity profiles of lines H11–H14 plotted with higher dispersion, again with a factor of 10 difference between the $2^\circ$ and $4^\circ$ exposures. However, in Figure 3, the zero point refers to the level of the background.

Absolute intensities of the Balmer emission lines are given in Table 1. Sun center absolute intensities tabulated by Allen (1973) were used to calibrate the instrument between 3600 and 3900 Å using the technique discussed by Doschek, Feldman, and Cohen (1977). The accuracy of the intensities is estimated to be better than a factor of 2. The small but consistent difference in intensities between the coronal hole and quiet Sun is not significant. The intensity decline of the hydrogen lines is so steep that even a small displacement of the slit with respect to its nominal position would cause these differences in intensities. Between $2^\circ$ and $4^\circ$ the intensities fall by about a factor of 8. Between $4^\circ$ and $6^\circ$ the intensities decrease by about a factor of 5. The pointing accuracy of the instrument is estimated as better than 0.5 with a jitter of less than 0.5.

At a given temperature, density, turbulence, and emitting path length, the widths of the hydrogen lines are determined by different physical processes, depending on the value of $m$. The widths (corrected for instrumental resolution) are tabulated in Table 1, and plotted in Figure 4. Under solar conditions and observing above the limb, the lines from $m < 16$ are broadened mainly by Doppler and opacity effects. The Doppler broadening may be composed of a thermal component and a nonthermal component, due to either microscopic or macroscopic turbulence. The broadening due to opacity increases rapidly for the lower series members. The optical depth at line center scales as the absorption oscillator strength, which for hydrogen is proportional to $m^{-3}$. For higher series members, i.e., $m > 16$, broadening by the Stark effect becomes increasingly important. Quasi-static ion broadening increases as $m^2$, and electron impact broadening scales as $m^4$. For intermediate $m$ values, i.e., $m \approx 16$, the FWHM remains almost constant, being determined primarily by Doppler broadening. However, for the low electron densities encountered on the Sun, the Doppler broadening remains significant even up to the highest series members observed. For all values of $m$, the instrumental width of 0.12 Å does not significantly widen the lines.

Stark broadening of hydrogen is now well understood (e.g., see the references in Griem 1974), and most laboratory experiments show good agreement between experimental and theoretical profiles (e.g., Bengtson and Chester 1976; Himmel 1976). Detailed tabulations of Stark profiles are available up to $m = 4$ (Kepple and Griem, 1976).
1968; Smith, Cooper, and Vidal 1973). We have found it convenient to use the compilation of Stark broadening theory given by Kurochka (1969) and Kurochka and Maslennikova (1970). (See also Minaeva 1968.) These authors have extended the Stark broadening theory to high series members specifically for application to low-density solar plasmas. They treat the electron impact broadening according to Griem (1960, 1967), and consider the effects of Stark and Doppler broadening.

The FWHM, $\Delta \lambda$, due to Stark and Doppler broadening but not including opacity broadening, is given by (Kurochka 1969)

$$\Delta \lambda = 2\left(\frac{k_{nm}F_0(4 + 0.8\gamma)}{\lambda} + (0.833 \Delta \lambda_D)^{1.5} \right)$$

for $\lambda \leq 15$

$$\Delta \lambda = 2(0.833 \Delta \lambda_D)$$

for $\lambda > 15$ (1)

where

$$\beta_D = \frac{\Delta \lambda_D}{k_{nm}F_0}, \quad \Delta \lambda_D = \frac{\lambda}{c} \left(\xi^2 + 2RT\right)^{1/2}, \quad k_{nm} = 5.5 \times 10^{-5} \frac{(mn)^4}{m^3 - n^3},$$

$n = 2$ for Balmer lines,

$$\gamma = 8.4 \times 10^{-6} \frac{N_e^{1/3}I(n, m)}{T_e^{1/3}(m^3 - n^3)} \log \left[ \frac{4 \times 10^{13} T_e^2}{N_e I(n, m)} \right],$$

where

$$I(2, m) = \frac{3}{4}(m^4 - 9m^2 + 12); \quad F_0 = e\left(\frac{\pi N_e}{3}\right)^{1/3} = 1.25 \times 10^{-9} N_e^{2/3}.$$
In equations (1) and (2), \( m \) and \( n \) are the upper and lower level principal quantum numbers of the transition, respectively, \( T_e \) is the electron temperature in K (assumed equal to the ion temperature \( T_i \)), \( N_e \) is the electron density in \( \text{cm}^{-3} \), \( \xi \) is the nonthermal velocity, and \( \Delta \lambda, \Delta \lambda_0 \) are in Angstrom units. The factor 0.833 has been inserted by us into equation (1) in order that the values for the FWHM calculated using the equation actually agree with the values tabulated by Kurochka (1969). (Note also that the factor \( 8.4 \times 10^{-6} \) was incorrectly printed in Kurochka and Maslennikova as \( 8.4 \times 10^{-6} \)).

The value for \( \Delta \lambda_0 \), the Doppler broadening component, was initially determined from the \( m = 16 \) line. The justification for this is that, at any reasonable solar electron densities, the Stark broadening of lines near the \( m = 16 \) line is negligible compared with the observed width of the line. Furthermore, inspection of Table 1 shows that opacity does not significantly broaden the lines for \( m > 11 \). The parameter \( \Delta \lambda_0 \) in equation (1) is related to our measured FWHM at \( m = 16 \) by \( \Delta \lambda_0 = (\text{FWHM})/2\ln(2) \). For both the quiet Sun and coronal hole, \( (\text{FWHM})_{16} = 0.31 \text{ Å} \), and therefore \( \Delta \lambda_0 = 0.18 \text{ Å} \). If this broadening is interpreted as due entirely to thermal Doppler broadening, the effective ion temperature \( T_i \) is 18,000 K. However, as shown in the following paper on active regions, most of the broadening is due to mass motions. The widths of the hydrogen lines increase with height above the limb, as is also observed for intersystem lines of Si ii and C ii (Doschek, Feldman, and Cohen 1977). An electron temperature \( T_e \) of 8000 K was used for the Stark broadening calculations. The results are not too sensitive to \( T_e \) because the temperature enters equation (1) explicitly only through \( \gamma \). A variation in \( T_e \) can be compensated for by a smaller variation in the density \( N_e \), since the term involving \( \gamma \) is \( \propto N_e/T_e^{1/2} \).

Values of the line widths (excluding opacity broadening) were calculated starting with a value of 0.18 Å for \( \Delta \lambda_0 \) in equations (1) and (2). The value of \( \Delta \lambda_0 \) was varied slightly around 0.18 Å in an attempt to improve the fit. The
TABLE 1

PROPERTIES OF HYDROGEN BALMER LINES ABOVE THE SOLAR LIMB

<table>
<thead>
<tr>
<th>m</th>
<th>$\lambda$(Å)</th>
<th>Quiet Sun</th>
<th>Coronal Hole</th>
<th>Quiet Sun</th>
<th>Coronal Hole</th>
<th>Quiet Sun</th>
<th>Coronal Hole</th>
<th>Quiet Sun</th>
<th>Coronal Hole</th>
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<td>$\Delta \lambda$ Å</td>
<td>$\tau_0$</td>
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<td>61.0 0.41 1.4</td>
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<td>10.0 0.55BL†</td>
<td>3.0 0.66BL†</td>
<td>1.0 0.36</td>
<td>1.0 0.44</td>
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<td>5.3 0.41</td>
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<td>1.6 0.52BL†</td>
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* Line intensities in units $10^8$ ergs cm$^{-2}$ s$^{-1}$ sr$^{-1}$ at the Sun.
† BL = blend.
Fig. 4.—Measured FWHM of the Balmer lines, 2" above the limb, for the quiet Sun and a coronal hole. Curves show predicted line widths (not including opacity broadening) for electron densities between $5 \times 10^{10}$ and $4 \times 10^{11}$ cm$^{-3}$, with $\Delta \lambda_0 = 0.19$ Å and $T_e = 8000$ K.

The best fit was achieved with $\Delta \lambda_0 = 0.18$ Å or 0.19 Å. The results for $\Delta \lambda_0 = 0.19$ Å are shown in Figure 4 for values of the electron density $N_e$ ranging from $5 \times 10^{10}$ cm$^{-3}$ to $4 \times 10^{11}$ cm$^{-3}$. Although there is scatter in the half-width values, an electron density of $2 \pm 1 \times 10^{11}$ cm$^{-3}$ fits the data reasonably well for values of $m \geq 17$. For $m < 17$, the scatter in the data and opacity broadening cause most of the data points to fall outside the theoretical curves.

Another method for determining the electron density is based on the quantum number of the last resolved line of the Balmer series. Two neighboring lines are considered to become unresolved when the widths of the lines become equal to their wavelength separation. Using this criterion and ignoring Doppler broadening lead to the well-known Inglis-Teller (1939) relation,

$$N_e = 1.82 \times 10^{20} m_s^{-7.5},$$  
(3)

where $m_s$ is the number of the last resolved series number. Kurochka (1975) modified the Inglis-Teller relation by including Doppler broadening and more accurate Stark profiles and derived the expression

$$N_e = 8.0 \times 10^{22}(m_{s+D} + 1)^{-7.5} - 1.6 \times 10^6(m_{s+D} + 1)^{-3} \delta^{1/2},$$  
(4)

where $m_{s+D}$ is the number of the last resolved series number broadened by the combined Stark and Doppler effects. Before using equation (4) to calculate electron densities, we must point out that in Figures 1 and 2 it is not obvious which line is the last observed line. The lines near the series limit are clearly not merging due to their increasing widths. They are simply difficult to see, because their decreasing intensity causes them to merge with the background noise. Also, blending with other lines is a problem. The difficulty of observing faint lines has been pointed out before by Ivanov-Kholodnyi and Nikol'skii (1961). They estimated that a correction of $\delta = 3$ or 4 must be added to the apparently last resolved line, depending upon the resolution of the spectrograph, the amount of background, and the value of $m_{s+D}$. From the trend in line intensities, the last observable line in Figures 1 and 2 would be approximately 30. Without the correction $\delta$, a value of $m_{s+D} = 30$ gives an electron density of $4.2 \times 10^{11}$ cm$^{-3}$. If values for $\delta$ of 3 or 4 are used, the calculated density is $1.9 \times 10^{11}$ cm$^{-3}$ or $1.4 \times 10^{11}$ cm$^{-3}$, respectively. These values agree well with the value calculated from the line widths. However, the densities derived from the line widths are clearly more reliable.

The best previously available slit spectra of hydrogen emission lines known to the authors are those by Redman and Suemoto (1954) taken at the total solar eclipse of 1952 February 25. Their slit width was 1/1, and the instrument had a dispersion of 2.4 Å mm$^{-1}$. Their results are similar to ours: $\log N_e \approx 11.5$ and $6000$ K $\leq T_e \leq 10,000$ K, with a minimum FWHM of 0.27 Å at an apparent height of 1150 km. Their height was determined partly from a comparison of line intensity ratios with those measured by J. Houtgast on slitless spectrograms obtained at the same eclipse, and partly by using the motion of the Moon relative to the Sun. Redman and Suemoto consider their height measurements to be estimates only. The somewhat higher densities and smaller minimum line width reported by Redman and Suemoto are consistent with the assumption that their observations are representative of a slightly lower effective height than 2".
The lines from \( m < 15 \) are broadened by opacity. The intensity in these lines is given by

\[
I_m(\tau, x) = b \int_0^{\tau_m} S_m(\tau') \exp\left[ -\tau' \phi(x) \right] \phi(x) d\tau',
\]

where \( b \) is a constant, \( S_m \) is the source function representing the number of excited atoms in the upper level of the transition, and \( \phi(x) \) is the Voigt function. The dimensionless frequency \( x \) is defined as

\[
x = \frac{(\lambda - \lambda_m)}{\Delta \lambda_p},
\]

where \( \Delta \lambda_p \) is the width due to Doppler broadening and \( \lambda_m \) is the wavelength of the \( m \)th line at line center. Assuming a constant source function, equation (5) becomes

\[
I_m(\tau, x) = b S_m(1 - \left[ \exp\left( -\tau_0 \exp\left( -x^2\right)\right) \right]),
\]

where we also take \( \phi(x) = \exp(-x^2) \). Equation (7) can be written in terms of the observed FWHM \( \Delta \lambda_p \) and the quantity \( \tau_0 \). A relationship involving only FWHM \( \Delta \lambda_p \) and the \( \tau_0 \) quantity \( \tau_0 \) can be obtained, i.e.,

\[
\frac{1 - \exp\left( -\tau_{0m} \exp\left( -\left(\ln 2/r_0\right)^2\right)\right)}{1 - \exp\left( -\tau_{0m}\right)} = 0.5,
\]

where \( \tau_0 = \frac{1}{2}(\ln 2) \Delta \lambda_p/\text{FWHM}_m \). Taking \( \Delta \lambda_p \) from the H16 line and using equation (8), the line widths for H9-H15 give the values of \( \tau_{0m} \) listed in Table 1. An error in \( r \) of \( \pm 10\% \) causes an error of a factor of about 1.6 in the corresponding value of \( \tau_{0m} \). For example, for \( r = 0.775 \) and \( \tau_0 = 1.6 \), an error of \( \pm 10\% \) in \( r \) would give the values \( \tau_0 = 1.0 \) or \( \tau_0 = 2.6 \).

The opacity \( \tau_0 \) at line center is given by

\[
\tau_{0m} = \frac{n^{1/2} e^2 f_{2m} \lambda^2}{mc^2 \Delta \lambda_p^2} \bar{N}_2 \bar{L},
\]

where \( f_{2m} \) is the absorption oscillator strength from the second level to the \( m \)th level, \( \bar{N}_2 \) is the average number density of the second level, and \( \bar{L} \) is the total path length along the line of sight over which emission occurs. From equation (9) it is clear that \( \tau_{0m} \) should vary as \( m^{-3} \). A plot of \( \tau_{0m} \) against \( m \) is shown in Figure 5 along with a plot of \( m^{-3} \) against \( m \). The data fall quite close to the \( m^{-3} \) curve, indicating that most of the error in determining \( \tau_{0m} \) is systematic and not random.

The ratios of the average populations of the upper levels, \( \bar{N}_m \), to the population of \( \bar{N}_2 \), can be obtained by combining the absolute intensities with the opacities. That is,

\[
I_m = \frac{h \nu_m A_{m2}}{4\pi} \bar{N}_m \bar{L},
\]

where \( h \nu_m \) is the energy in ergs of the \( m \)th line, \( A_{m2} \) is the spontaneous decay rate, and \( \bar{L} \) is given in Table 1. For example, for the \( m = 16 \) line, \( \bar{N}_{16} \bar{L} = 3 \times 10^{12} \) cm\(^{-2}\). Since \( \bar{N}_2 \bar{L} = 7.7 \times 10^{14} \) cm\(^{-3}\) and \( \bar{N}_{16}/\bar{N}_2 = 4 \times 10^{-3} \),

Finally, we briefly compare our results with the one-component solar atmosphere model of Vernazza, Avrett, and Loeser (1973). They give the observed temperature, density, and nonthermal velocity as a function of height above a zero point defined by \( \tau_{0000} = 1 \). The observed solar limb, which is our zero point, is at 350 km on their scale. We can compare our results to the model at 2" above the limb, or at 1800 km on the Vernazza, Avrett, and Loeser (1973) scale. However, because our slit is 2" wide, covering the range from 1075 to 2525 km, and because the decrease of hydrogen line intensity above the limb is so rapid, the average weighted height of the slit should be closer to 1500 km than to 1800 km. At 1500 km, the temperature given by Vernazza et al. is about 6500 K. We cannot comment on the temperature, because our results are not sensitive to temperature. Lowering our nominal temperature from 8000 to 6500 K does not significantly alter our derived electron density. The electron density given by Vernazza et al. is about \( 5 \times 10^{10} \) cm\(^{-3}\), which is outside our error bar range, and about a factor of 4 less...
than our value. The nonthermal velocity given by Vernazza et al. is about 5.5 km s\(^{-1}\) at 1500 km, which is about a factor of 2 lower than the velocities we obtain from the lines near \(m = 16\). Their velocities appear to be averages of the velocities given by Linsky and Avrett (1970) and Hirayama (1971). Our results are closer to the Hirayama (1971) values, which are about 7 km s\(^{-1}\) at 1500 km.

The differences between our results and the Vernazza, Avrett, and Loeser (1973) model may be due to the presence of atmospheric inhomogeneities such as spicules. The Vernazza et al. model does not consider inhomogeneities, which become increasingly important with height above the limb. The densities in spicules are estimated to be about \(1.6 \times 10^{11} \text{ cm}^{-3}\) at 2000 km above the observed limb (Beckers 1972), which is considerably higher than in the Vernazza et al. model. We continue the discussion of the effects of spicules on our results in the following paper on active regions.

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