HIGH TIME RESOLUTION ANALYSIS OF SOLAR HARD 
X-RAY FLARES OBSERVED ON BOARD 
THE ESRO TD-1A SATELLITE

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(Received 4 September, 1975)

Abstract. The Utrecht solar hard X-ray spectrometer S-100 on board the ESRO TD-1A satellite covers 
the energy range above 25 keV with 12 logarithmically spaced channels. Continuous sun-pointing is 
combined with high time resolution: 1.2 s for the four low energy channels (25–90 keV) and 4.8 s for the 
others. It is emphasized that the instrument design and calibration yield data virtually free of 
pile-up and other instrumental defects.

A complete set of observations is presented for all well-observed flares during the period March 12,
1972 to October 1, 1973, including four from the highly active period August 1–8, 1972. Photon 
spectra are computed every 1.2 s for each event by deconvolution through the instrument response, 
rather than by fitting techniques. Using these actual photon spectra, the index γ for the best fitting 
single power law and the minimum (thick target) injection rate of electrons above 25 keV, \( F_{25} \), are 
calculated.

Results for γ and \( F_{25} \) at 1.2 s intervals are presented for each event. Examination of all these results 
tentatively suggests a real distinction between events of a purely impulsive nature and prolonged 
events.

Techniques of time series analysis are applied to the burst time profiles. Specifically:

1. The fluctuations present in the series are shown to be compatible with Poisson noise in the count 
rate.

2. It is emphasized that, without spatial resolution, the X-ray source must be characterized by the 
e-folding time scale \( \tau \) of the total count rate; examination of individual \( \tau \)'s through the event shows 
very few statistically real \( \tau \)'s as short as 1.2 s, confirming (1).

3. For all events, the series are Fourier analysed; no small events showed statistically significant 
periodicities, but the large event of August 4, 1972 exhibited real periods of 30, 60 and 120 s in both 
the flux and the spectral index.

4. Statistically real, small timing differences (~0.2 s) are shown to exist between spike peaks at 
different photon energies.

A search is made for correlations between instantaneous values of inferred parameters (e.g. \( F_{25} \), γ 
and the time scales). Most results are negative, but in the August 4 and 7, 1972 events a very well 
defined path was followed through the \( (F_{25}, \gamma) \)-plane, giving insight into the electron acceleration 
process.

Finally some general conclusions are drawn concerning the implications of our analysis for the 
physics of particle acceleration, including the possibility of two classes of event. Specifically, the severe 
problems posed by the large electron fluxes (equivalent current ~10^{17} A) demanded by the data are 
discussed in relation to flare theories. Some possibilities for getting around these problems, such as by 
reacceleration in a confinement region, are briefly considered.

1. Introduction

It is now widely accepted that acceleration of electrons is a basic feature in the 
flash phase of many solar flares (Kane, 1974; Lin, 1974a,b). These fast electrons

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emit hard X-rays by the process of bremsstrahlung and since there are no major propagation effects, it is believed that observations of hard X-rays provide the most direct clue to the spectrum and flux of fast electrons during flares, and their evolution.

In this paper we report and analyze observations made by the Utrecht solar hard X-ray spectrometer S-100 on board the ESRO TD-1A satellite in the period March 12, 1972 until October 1, 1973. The observations were made above 25 keV in 12 logarithmically spaced channels with a time resolution of 1.2 s in the four low-energy channels (25–90 keV) and 4.8 s in the others. With one exception, this paper includes only events large enough for a reasonable determination of the photon spectrum.

Observations of many smaller solar hard X-ray flares have been made by spectrometers flown on OSO-3 (Hudson et al., 1969), OGO-5 (Kane and Anderson, 1970) and OSO-7 (Datlowe et al., 1974a, b). These observations were made in the energy range from a few keV up to ~100 keV and have been used mainly to study general statistical characteristics. The scarcer large events have on the other hand been observed at higher energies (≥20 keV) by OSO-5 (Frost, 1969; Frost and Dennis, 1971).

The main assets of the present observations are the continuous high time resolution of 1.2 s together with the absence of any saturation effects during even the largest events.

The purpose of the present paper is twofold. On the one hand observations are presented in a self-contained format allowing independent usage by other workers; on the other hand a general analysis is made, avoiding specific models for specific events, but emphasizing broad issues.

In view of the direct relevance to acceleration processes, a detailed time series analysis making full use of the 1.2 s time resolution was a major aim, in addition to spectral analysis. In our opinion this aspect has never been satisfactorily pursued for any set of data.

No attempt was made to invoke other flare observations (radio-, optical-, EUV- and particle radiation) in the analysis. Previous joint data studies seem to have been scarcely fruitful in terms of quantitative physical models and we would attribute this largely to insufficient attention having been given to analysis of individual data before intercomparisons are made.

The outline of the paper is as follows. In Section 2 a few essentials on the instrument and satellite are reviewed, and in Section 3 we discuss our selection of events.

Spectral analysis is discussed in Section 4. After a few remarks on our data reduction technique, we introduce the so-called thick target parameters and review their utility as a reference device as well as their interpretation under general circumstances.

Next, the observations and corresponding thick target parameters are presented and analyzed in the context of existing observations. The possibility is pointed out
that the X-ray bursts can be classified in two groups. Breaks in the X-ray photon spectra are observed and their interpretation is discussed.

Section 5 is concerned with time series analysis of both the count rates and the inferred source parameters. In particular, the statistical properties of the count rates are investigated and related to the issue of time scales involved in the electron acceleration process. Finally, we inquire into the possible presence of periodicities in the time profiles using Fourier analysis and into the significance of timing differences between short spike peaks at different photon energies. Finally, in Section 6, the presence of correlations between various parameters is examined. A remarkable correlation is found to exist between the thick target parameters $F_{25}$ and $\gamma$ of the large August 4, 1972 flare.

In Section 7 implications for X-ray flare models are formulated. Specifically we discuss (1) possible physical differences between the two types of hard X-ray bursts. (2) The question of thermal vs non-thermal X-ray emission. (3) The MHD implications of the very large electron fluxes ($\sim 10^{36} \text{ s}^{-1}$) demanded by the data.

2. Instrument and Satellite

On March 12, 1972 the ESRO TD-1A satellite was launched into a near polar, circular orbit at a height of about 550 km. The orbital inclination is 97.6°, causing the orbital plane to precess 1° a day. Thus the satellite was sunlit along its whole orbit for a period of about seven months after launch. After a hibernation period of about three months, during which the satellite eclipsed periodically and the scientific instruments were switched off, a new observation period started early February 1973. Reference should be made to a description of the satellite by Tilgner (1971). The satellite is three-axis stabilized. One axis is kept sunpointed with an accuracy of better than 1°.

The Utrecht hard X-ray spectrometer S-100 (van Beek, 1973) has its main axis mounted parallel to this axis and hence the instrument is permanently viewing the Sun. Measurements of the solar radiation above 25 keV are made in 12 (roughly) logarithmically spaced energy channels. On May 18, 1972 the channel limits were 28, 39, 51, 71, 102, 141, 193, 266, 363, 488, 641, 834 and 1037 keV, these limits increasing very slowly with time.

The permanent sunpointing permits continuous use of the fine time resolution, set to 1.2 s for the four low energy channels and to 4.8 s for the others. Basically the detector consists of a CsI (Na) scintillation crystal (sensitive area 5 cm²; thickness 1.5 cm) optically coupled to a photomultiplier tube, followed by a pulse height analyzer (Figure 1). Passive shielding has been applied to suppress the background X-radiation generated in satellite and instrument. This shield ends in a collimator. All charged particles that enter within the acceptance cone of the collimator (geometry factor $\sim 1 \text{ cm}^2 \text{ sr}$) and penetrate into the crystal, are also detected by the two solid state detectors (SSDs) and hence are rejected in the
normal mode of operation. Once every 38.4 s, however, the number of particles detected by both SSDs is recorded during 4.8 s (particle counter).

The output of the particle counter is subcommutated with channel 1, causing regular datagaps of 4.8 s in each stretch of 38.4 s X-ray data of this channel (cf. e.g. Figure 2 and 16; datagaps occurring in all channels, e.g. Figure 16 at UT 0631, can be due to telemetry breakdown). The particle count rate is mainly due to electrons from the radiation belts with energies exceeding ~1 MeV; it provides a useful diagnostic tool to distinguish solar flares from phenomena caused by particles: hard X-rays (from a flare) will not affect the count rate of the particle counter.

It is important to note that the two SSDs (thickness: 360 μm silicon each) provide a sharp cut-off of the photon efficiency below ~20 keV. Even during the largest solar flares this allows the instrument to measure spectra free from any saturation effects, such as pulse pile-up and DC shift. The sensitivity under low background conditions, and the efficiency of the instrument, are given in Table I.

Because of data compression requirements the registers for channels 1 through 7 have a variable counting stepsize, defined as follows: 0(1)63; 64(4)316; 320(16)1328; 1344(64)5440. These numbers are matched to Poisson statistics: the induced discretisation step for a read-out $c$ is always $<c^{1/2}$ and usually $\ll_c^{1/2}$. The information content of the signal is therefore not affected (though sometimes the effect is visible in the time profile, e.g. Figure 16, channel 1, around UT 0633:30). By the same token, Poisson statistics do not apply exactly to measured count rates, but do so to a very good approximation.

For inflight calibration two radioactive sources, $^{90}$Sr and $^{241}$Am have been
mounted in front of and between the two SDDs, respectively. On ground command coincidence and anti-coincidence measurements between the SDDs and the scintillation counter can be initiated or terminated. In the calibration modes of operation, the photon spectrum of the $^{244}\text{Am}$ source or the electron spectrum of the $^{90}\text{Sr}$ source can be analyzed, thus providing an accurate inflight measurement of the efficiency, the resolution and amplification of the scintillation counter and the energy calibration of the discriminator level. These measurements are performed a few times a week. The instrument was subjected to an extensive preflight calibration program, providing detailed knowledge of all detector characteristics. For a more extensive description of the instrument, see van Beek (1973); van Beek and de Feiter (1973), and van Beek et al. (1974).

The instrument has operated successfully during the two years’ lifetime of the satellite and has observed a considerable number of solar flares. The observation periods of the instrument are: March 18–October 25, 1972 and February 15, 1973–May 5, 1974 (during the second hibernation the instrument was kept switched on and sunpointed). Due to failure of both satellite tape recorders, from May 23, 1972 on, observations were possible only during real time telemetry contact. Data coverage initially dropped to about 30%, but increased gradually up to 60% and more in August 1972 by extension of the ground station network.

3. Event Selection

The observations presented in this paper are summarized in Table II. Likely periods of hard X-ray flare emission were first located in the satellite data using the criterion that the 1–8 Å soft X-ray flux measured by SOLRAD should be above the minimum level M1 (i.e. $1–8\ Å \text{ flux} = 1 \times 10^{-2} \text{ erg cm}^{-2} \text{ s}^{-1}$; Simon and McIntosh, 1972). This turned out to be a reliable and effective method.

A flare was included in this paper when its hard X-ray emission was well covered, sufficiently free from background contamination and detectable up to channel 4 at least, so that a reasonable determination of the photon spectrum was
### TABLE II
Survey of observations

<table>
<thead>
<tr>
<th>Event No.</th>
<th>Date</th>
<th>Time of max count rate in channel 1 (UT)</th>
<th>Optical*/soft X-ray class⁴</th>
<th>Position¹</th>
<th>X-ray timeprofile (Fig. No.)</th>
<th>Thick target parameters (Fig. No.)</th>
<th>Background subtracted</th>
<th>Photon spectra (Fig. No.)</th>
<th>Power spectra (Fig. No.)</th>
<th>Timing of spikes (Fig. No.)</th>
<th>Correlation E₂₅ and γ (Fig. No.)</th>
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<tr>
<td>1</td>
<td>21-3-72</td>
<td>0112:51</td>
<td>1B/M1</td>
<td>N08E42</td>
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<td>3</td>
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<td>1B/M4</td>
<td>S15E24</td>
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* Solar Geophysical Data (Prompt Reports).

⁴ Preliminary Report and Forecast of Solar Geophysical Data (NOAA); for the C/M/X-classification see Simon and McIntosh (1972).
possible every 1.2 s. The period covered for this paper is 1972: March 18–October 25 and 1973: February 15–October 1 (i.e. up to the beginning of second hibernation). In this way the events Nos. 1–9 and 11 from Table II were found. Many more events have been observed having detectable X-ray flux in, say, channel 1 and 2 only. Event No. 10 is an example. Table II also serves as a cross reference to all figures pertaining to one and the same event. Throughout, we will consistently refer to the event number as given in Table II, first column.

4. Spectral Analysis

4.1. Data reduction

The spectral response function of the instrument was removed from the data retaining full time resolution, using a reduction method that requires much less computing time than the usual $\chi^2$-fit to two or more parameters (e.g. to a power law $a e^{-\gamma}$), and, moreover, reconstructs the photon spectrum as accurately as is possible (Hoyng and Stevens, 1974). No a priori assumption is needed on the shape of the photon spectrum. The method easily establishes, for example, deviations from a single power law behaviour, if present (cf. Figure 23). By means of inflight and preflight calibrations, the spectral response function of the instrument was taken accurately into account (including effects of efficiency, escape peak, Compton scatterings, crystal nonlinearity and crystal dead layer; see van Beek (1973), p. 43–62).

We routinely also compute the best single power law fit to the photon spectrum so obtained (not to the measured pulse height distribution). This single power law fit was needed for model calculations, such as determination of thick target parameters as described in the next section. Figure 22 and 23 give a few more details on our data reduction method.

For the smaller events Nos. 1–6 background subtraction could be achieved straightforwardly. In the long lasting events Nos. 7–9 background subtraction was complicated due to lack of directly available background data. No background was subtracted in these three events. However, neglect of background should not be serious in these cases because of the large excess of X-ray over background counts.

4.2. Thick target parameters

In the thick target model one conceives of fast electrons being injected into a “target” region having a relatively high density (Brown, 1971). These electrons loose their energy mainly by electron-electron collisions and a minor fraction ($\sim 10^{-5}$) of their initial energy goes into hard X-rays, emitted during electron-proton collisions (bremsstrahlung). If the target density is so high that the injection rate of fast electrons is virtually constant over an energy loss time, then the emerging X-ray photon spectrum is no longer a convolution of the electron
injection spectrum with respect to time, and also it becomes independent of the
target density. In the nonrelativistic range, the Bethe-Heitler cross section allows
analytic solution for the electron energy spectrum if the photon spectrum at 1 a.u.
(see Section 4.1.) is given by

\[ I(\varepsilon) = a \varepsilon^{\gamma - 1} \text{ (cm}^2 \text{s keV)}^{-1}. \]  

One then finds (Brown, 1971):

\[ F(E_0) = 4.15 \times 10^{-33} \frac{a(\gamma - 1)^2 B(\gamma - \frac{1}{2}, \frac{1}{2})}{E_0^{\gamma}} \text{ s}^{-1}, \]  

\[ P(E_0) = 1.6 \times 10^{-9} \frac{\gamma}{\gamma - 1} E_0 F(E_0) \text{ erg s}^{-1}. \]  

[B(x, y) is the beta function, and E_0 is in keV]. F(E_0) and P(E_0) are the number
flux and energy flux of fast electrons into the target region, having energy \( \geq E_0 \).
Henceforth \( E_0 = 25 \) keV is taken: a small extrapolation below the low energy limit
of channel 1, and we introduce the notation \( F_{25} \) and \( P_{25} \).

Because the photon spectra we observed are always close to a single power law,
the parameters \((a, \gamma)\) provide a sensible and complete description of an X-ray
burst. However, instead of the pair \((a, \gamma)\) we will use the parameters \((F_{25}, \gamma)\): they
are just a more convenient set than \((a, \gamma)\) and will be referred to as thick target
parameters. It is stressed that \( F_{25} \) and \( P_{25} \) are used here as purely formal
parameters, representing required number and energy fluxes into a hypothetical
target, for the electron energy loss function given by Brown (1971).

A few comments on the interpretation of \( F_{25} \) and \( P_{25} \).

If a given X-ray burst is due to fast electron injection into a target, then \( F_{25} \)
represents a lower limit to the required number flux, as is further discussed in
Section 7.

Other parameters pertaining to the X-ray emission, expressible in \( a \) and \( \gamma \), can
always be re-expressed in terms of \( F_{25} \) and \( \gamma \): the emission measure of the
(instantaneous) collection of nonthermal, hard X-rays emitting electrons with
energies above \( E_0 \) is given by

\[ n_0 N_{25} = 6.06 \times 10^{44} \frac{(\gamma - 1)^2}{\gamma - \frac{3}{2}} B(\gamma - \frac{1}{2}, \frac{1}{2}) a E_0^{-\gamma + 3/2} \]  

\[ = 1.82 \times 10^{10} \frac{F_{25}}{\gamma - \frac{3}{2}} \text{ cm}^{-3}; \]  

\( n_0 \) is the mean density of the background protons and \( N_{25} \) is the effective total
number of fast electrons with energy \( \geq 25 \) keV. It is emphasized that, whereas \( F_{25} \)
depends on the energy loss of a fast electron per unit time, \( n_0 N_{25} \) does not, so
that the second equality in (4) only holds if \( F_{25} \) is evaluated according to (2).

In the thick target model each electron is accelerated and dumped into the
target only once. Reacceleration of electrons would therefore reduce the actual
electron flux. In fact, if there is significant and continuous reacceleration, then
target and acceleration region become identical and $F_{25}$ looses its meaning: In this case the hard X-rays are produced in some region in the solar atmosphere, where, by some mechanism, a nonthermal electron population is maintained and contained, characterized by $n_0 N_{25}$. This situation will be referred to as the 'containment model' (see Section 7).

For all models (apart from an additional function of $\gamma$ of order unity), $P_{25}$ as evaluated from (2) and (3) is equal to the rate by which energy is dumped by collisions.

4.3. Discussion of Observations, Thick Target Parameters and Spectra

In the Figures 2–21 we have put together the hard X-ray observations of the flares in Table II and their thick target parameters when available. The sequence follows that of Table II:

Figure 2–19: flares Nos. 1–9 from Table II. For each event, the page to the left shows the X-ray time profile. The vertical scale is in counts/(1.2 s × 5 cm$^2$) i.e. for channel 1–4 a directly measured quantity is plotted). The page to the right shows the computed thick target parameters; to facilitate intercomparison, the time-profile of channel 1 is reproduced at the top. In each case the time integrals $\int F_{25} \, dt$ and $\int P_{25} \, dt$ over the entire event are given.

Figure 20 and 21: last two flares from Table II.

Note the following points about the figures:

(1) The behaviour of $F_{25}$ as a function of time: $F(E_0)$, cf. (2), contains the factors $a E_0^{-\gamma}$ and $(\gamma - 1)^2 B(\gamma - 1, \frac{1}{2})$. It follows that $F_{25}$ will be roughly proportional to the count rate of channel 1 ($\sim 30$–$40$ keV) and that $F_{25}$ increases with increasing $\gamma$. This is clearly visible in the figures.

(2) All datagaps in the count rates were linearly interpolated (in particular the regularly recurring gaps in channel 1) before computing the thick target parameters. This is necessary because the values of $F_{25}$ and $\gamma$ depend most strongly on the lower energies.

(3) The accuracies in $F_{25}$ and $\gamma$ at any given time are determined by the received flux. In the smaller events, the accuracies are of the order of 20–40% for $F_{25}$ and 10–30% for $\gamma$, cf. Figure 3 and 7. In large events, e.g. August 4, Figure 17, the accuracy of $F_{25}$ and $\gamma$ amounts to a few percent. the values of $\int F_{25} \, dt$ and $\int P_{25} \, dt$ are usually correct to a few units of the second decimal.

(4) All time mentioned in this paper are expressed in Universal Time, with an absolute accuracy of better than 0.1 s.

4.3.1. Discussion of Time Profiles and Thick Target Parameters

Inspection of the X-ray timeprofiles suggest that they can be classified phenomenologically in two types: impulsive and extended X-ray bursts:

(a) Impulsive X-Ray Bursts (summarily referred to as "impulsive bursts"): The hard X-ray emission lasts from $\sim 10$ to $\sim 100$ s and the time profile exhibits either

(text continues on p. 226)
Fig. 2. Event of March 21, 1972. Regular data gaps of 4.8 s in channel 1 are due to subcommutation (Section 2). The signal of channel 2 and 3 appears to be periodic, period ~9 s (Section 5.3).
Fig. 3. Thick target parameters of the March 21, 1972 event. The fluctuations in $F_{25}$ and $\gamma$ are unreal. The qualitative behaviour of $F_{25}$ ($F_{25} \uparrow$ if $\gamma \uparrow$ and if count rate channel 1 \uparrow) is clearly visible.
Fig. 4. Event of May 18, 1972. Channel 1 shows a single peak of ~30 s width, preceded by a precursor, and ending in a prolonged tail, that could be of thermal origin (Section 7). The photon spectrum of this event at UT 1406 is shown in Figure 22.
Fig. 5. Thick target parameters of the May 18, 1972 event. Fluctuation in $\gamma$ are minimal at maximal count rate in channel 1. $\gamma$ increases linearly with time throughout the event; analogous behaviour of $\gamma$ is seen in all impulsive X-ray bursts.
Fig. 6. Event of May 18, 1972. This flare occurred 2 hours after event No. 2, Figure 4, in the same active region. Both flares have quite the same emission features and in particular the tail in channel 1 can be of thermal origin.
Fig. 7. Thick target parameters of the May 18, 1972 event. Again $\gamma$ increases linearly with time and the fluctuations are minimal at maximal count rate in channel 1. The apparent decrease of $\gamma$ at UT 1618 is unreal. Upward fluctuations in $\gamma$ are seen to cause large peaks in $F_{25}$; these are unreal.
Event of August 2, 1972. The X-ray emission consists of a series of spikes with a duration of a few seconds each. This event had probably a small electron acceleration region ($\leq 10^6$ cm; Section 5.2) and also a small X-ray source region, whose position was fixed in time (Neupert et al., 1974). Datlowe and Peterson (1973) find that the hard X-ray photon spectrum for this flare preserves its power law character down to $\sim 10$ keV. The vertical arrows identify the spikes used in the analysis of Section 5.4.
Fig. 9. Thick target parameters of the August 2, 1972 event. Deviations from the linear increase of \( \gamma \) are largely real.
EVENT # 5
August 7, 1972

Fig. 10. Event of August 7, 1972.
Fig. 11. Thick target parameters of the August 7, 1972 event.
Fig. 12. Event of May 19, 1973. The X-ray emission shows a precursor and a few pronounced spikes. Note that a large spike in channel 1 was (probably) not observed, due to subcommutation. The arrows identify the spikes used in the analysis of Section 5.4.
Fig. 13. Thick target parameters of the May 19, 1973 event. All data gaps in the count rates are linearly interpolated before computation of the thick target parameters; at the vertical arrow an identical bad result is pointed out.
Fig. 14. Event of 2 August 1972. This is the first and smallest of a series of three extended X-ray bursts, all occurring in the same highly active region of August 1972. The observations are incomplete, see inset.
Fig. 15. Thick target parameters of the August 2, 1972 event.
Fig. 16. Event of August 4, 1972. This flare and that of August 7, 1972, Fig. 18, are extremely large. Data gaps occurring in all channels at the same time can be due to telemetry breakdown. The constant amplitude fluctuations in channel 1 at UT 0630:30 are of instrumental origin (Section 2). The X-ray time profiles show quite pronounced oscillations that gradually die out towards the end (Section 5.3). The X-ray source of this flare probably consists of a large, vibrating and expanding coronal trap (Section 6). Photon spectra of this event are shown in Figure 23, at times indicated by vertical arrows. Features in the time profiles are seen usually to occur progressively later at higher energies (cf. Section 6)
Fig. 17. Thick target parameters of the August 4, 1972 event. The accuracy in \( F_{25} \) and \( \gamma \) has improved to a few percent in this case. \( F_{25} \) and \( \gamma \) show a remarkable correlation on average (Figure 28); at the same time, details of \( \gamma \) and count rate of channel 1 anticorrelate in the region A-D (Section 6). The letter A-E refer to those in Figure 28. Only every fifth data point \( F_{25} \) and \( \gamma \) is shown.
Fig. 18. Event of August 7, 1972. This is the other extremely large flare from the highly active August 1972 period. Unfortunately, the end part of the flare is missing.
Fig. 19. Thick target parameters of the August 7, 1972 event. $\gamma$ is seen to decrease on the average, as it does in all extended X-ray bursts, cf. Figure 15 and 17. Again an anticorrelation between $\gamma$ and the count rate is seen to exist, cf. Figure 29 and Section 6. Only every third data point $F_{25}$ and $\gamma$ is shown.
Fig. 20. Event of May 18, 1972. It is one example of many smaller events in our data, exhibiting detectable flux in channel 1 and 2 only. This event is classified as a subflare and its hard X-ray emission consists of a single peak of \( \sim 10 \) s duration only. The X-ray emission is very hard in the sense that the ratio of hard X-ray v. soft X-ray flux is large in this event, cf. Section 4.3.3.
Fig. 21. Event of August 23, 1972. The arrow identifies the spike used in the analysis of Section 5.4.
a single peak of ~10 s duration (Figure 20) to ~30 s (Figure 6), or a series of peaks each lasting from a few seconds (Figure 2 and 12) to ~10 s (Figure 8 and 10). These impulsive X-ray bursts are by far the most common type of hard X-ray flare emission in our data. Many more were observed than included here. Sometimes a precursor is visible before the main event (Figure 4 and 12). The photon spectra of these impulsive bursts are usually compatible with simple power laws (but see Section 4.3.2.).

Observations of this type of burst have also been reported by Hudson et al. (1969) Kane and Anderson (1970), McKenzie et al. (1973) and by Datlowe et al. (1974b).

(b) Extended X-Ray Bursts (summarily referred to as "extended bursts"), comprising the three large events of August 2 (Figure 14), August 4 (Figure 16) and August 7 (Figure 18). The hard X-ray emission extends over ~10^3 s and the flux can be an order of magnitude larger than that of impulsive bursts. Because of their long duration only the August 4 event was observed completely. This type of hard X-ray emission is mostly associated with very large solar flares and occurs therefore only rarely. The three extended bursts reported here were all produced by the highly active region of August 1972 (Zirin and Tanaka, 1973). The time profile of the August 4 event shows a periodic signal, quite pronounced in the beginning and disappearing towards the end, where all three flares exhibit an almost featureless decay. For the August 4 event, there exists a marked correlation between $F_{25}$ and $\gamma$ (Section 6).

The photon spectra are near power laws throughout the event. Other observations of such extended events are scarce: a flare on March 1, 1969 (Frost, 1969) and the well-known March 30, 1969 event reported by Frost and Dennis (1971). All these extended bursts are associated with large radio bursts and, contrary to impulsive bursts, there is usually a type II burst.

We point out that care must be taken with the above classification because of the small number of events on which it is based; certainly no general validity is claimed. Yet, experience with our observational material as well as other observations published so far do give support. The difference between the two groups is of phenomenological nature and physical difference remains to be investigated. A few suggestions in this direction will be made in Section 7.

Table III lists the average thick target parameters for each of the events #1–9. Note that ratios of $F_{25}$-values of different events differ from the corresponding channel 1 count rate ratios, because the channel limits shift with time.

In Table IV we have summarized characteristics and "typical values" of impulsive and extended bursts. The basic difference between the two types are given in the first two columns in this table; all other differences can be seen as consequences:

1. Impulsive bursts have a duration of at least an order of magnitude smaller than extended bursts. As $\langle F_{25} \rangle$, like $\langle \gamma \rangle$, is not too different for both groups, the
large differences in the last two columns are mainly due to duration effects.

(2) In all impulsive bursts γ has a tendency to increase about linearly or to remain constant as a function of time through the event, whereas in all extended bursts γ decreases on the average.

This systematic softening of the spectrum of an impulsive burst from beginning to end as we observe it contradicts former results (Kane and Anderson, 1970; McKenzie et al., 1973). This may be due to the fact that these experiments suffered from instrumental problems and also because their energy range extends to lower energies than ours. All authors, however, agree that towards the end of impulsive bursts the spectrum softens.

The usually very detailed (spiky) structure in the time profiles of impulsive bursts and in parts of those of extended bursts provides evidence for the existence of some kind of continuous acceleration in these flares. There seems to be general agreement on this point. In Section 7 we will come back to the main points made above concerning impulsive and extended bursts, when possible physical differences between them are discussed.

<p>| TABLE III |
| Average thick target parameters |</p>
<table>
<thead>
<tr>
<th>Event No.</th>
<th>γ</th>
<th>⟨F_{25}⟩×10^{-36}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.1</td>
<td>5×10^{-2}</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>0.8</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>0.9</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>0.8</td>
</tr>
<tr>
<td>5</td>
<td>3.6</td>
<td>0.5</td>
</tr>
<tr>
<td>6</td>
<td>4.2</td>
<td>2</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>0.25^{a}</td>
</tr>
<tr>
<td>8</td>
<td>3.3</td>
<td>3.5</td>
</tr>
<tr>
<td>9</td>
<td>3.6</td>
<td>5^{b}</td>
</tr>
</tbody>
</table>

*a* Probably too low because main part event is missing.

*b* Too high because decay phase is missing.

<p>| TABLE IV |
| Tentative group characteristics |</p>
<table>
<thead>
<tr>
<th>Type</th>
<th>Duration (s)</th>
<th>⟨γ⟩</th>
<th>⟨F_{25}⟩ (s^{-1})</th>
<th>⟨n_{0}N_{25}⟩ (cm^{-3})</th>
<th>⟨F_{25} dt⟩ (erg)</th>
<th>⟨P_{25} dt⟩ (erg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impulsive X-ray burst</td>
<td>10^{1}–10^{2}</td>
<td>≧0</td>
<td>1×10^{36}</td>
<td>5×10^{45}</td>
<td>5×10^{37}</td>
<td>2.5×10^{30}</td>
</tr>
<tr>
<td>Extended X-ray burst</td>
<td>≈10^{3}</td>
<td>&lt;0</td>
<td>3×10^{36}</td>
<td>3×10^{46}</td>
<td>3×10^{39}</td>
<td>1.5×10^{32}</td>
</tr>
</tbody>
</table>
4.3.2. Spectra

The photon spectra that we derive for impulsive bursts are generally compatible with a single power law spectrum. In no case did we find clear evidence for a softening or steepening of the spectrum towards higher energies; note, however, that this conclusion is based on the use of 4, (in a few cases 5) energy channels, extending over less than one energy decade.

In Figure 22 a photon spectrum for event No. 2 (May 18) is shown as derived from simultaneous observations by the OSO-7 experiment of the UCSD group* and by our TD-1A experiment. This event was well within the dynamical range of both instruments and the observations show surprisingly good agreement.

In extended bursts, the spectrum often softens towards higher energies, see Figure 23, for the August 4 event. During the first part of this event the spectra are compatible with two power laws with a break around ~60 keV, and $\Delta \gamma \geq 1$. A sharp break is however not positively indicated.

Such a steepening of the spectrum towards higher energy has been reported before by Frost (1969), Frost and Dennis (1971), Kane and Anderson (1970) and by van Beek et al. (1974). Elcan (1975) reports that these spectral breaks occur regularly in the OSO-7 observations, and that breaks as large as $\Delta \gamma \geq 2$ around 50 keV have been observed.

The relevance of this break to the electron acceleration spectrum is discussed in Section 7.3.

4.3.3. Ratio of Soft and Hard X-Ray Flux from Impulsive Bursts

The soft X-rays emitted in impulsive bursts (say $\epsilon < 10$ keV) are generally interpreted as bremsstrahlung from a thermal plasma ($kT < 1-4$ keV; emission measure $Y \sim 10^{47}-10^{49}$ cm$^{-3}$; Kahler et al., 1970; McKenzie et al., 1973; Culhane and Phillips, 1970).

Attention is called to the fact that the ratio of hard X-ray flux in channel 1 (28–39 keV) to the soft X-ray flux in the 1–8 Å band can differ greatly between different impulsive bursts. In our data, the two extremes happened to be separated by only 64 hours: A flare on May 18, 1972 (Figure 20), classified as C2 (1–8 Å flux $= 2 \times 10^{-3}$ erg/cm$^2$s) and on the other hand a SN/M5 flare ($5 \times 10^{-2}$ erg/cm$^2$s) on May 16, having no detectable hard X-ray flux. Estimating an upper limit of $\sim 5$ counts/(5 cm$\times$1.2 s) in channel 1 for this flare, one finds the soft v. hard X-ray flux ratio for both events to differ by a factor 125.

It is pointed out here that these large differences must be explainable in terms of different hard X-ray source parameters. From a preliminary analysis we found that variations in hard X-ray source density would have by far the greatest effect.

* We gratefully acknowledge extensive contacts with Drs H. S. Hudson, D. W. Datlowe and M. J. Elcan on the subject of instrument intercomparision.
Fig. 22. The hard X-ray photon spectrum of event No. 2, Figure 4, as measured by TD-1A and OSO-7. The OSO-7 fit is based on five channels from 21–155 keV, the TD-1A fit on the four channels shown. The difference between both results is inconsequential. For the sake of the figure, the OSO-7 data were fitted in the same way used throughout in this paper (the log of the photon flux was least mean square fitted to the log of the energy; this method differs from that used by Datlowe et al. (1974b)).
4.3.4. Homology of the May 18, 1972 Events

Attention is drawn to the similarity in events Nos. 2 and 3, which come from the same location and were separated by only 2 hours. If they derive from regeneration of the same energy store, then an energy storage rate of $\sim 3 \times 10^{26}$ erg s$^{-1}$ prior to the second flare is implied.

5. Time Series Analysis

In this section the following issues will be discussed: Noise properties of observed count rates; time scales involved in the hard X-ray emission; the possible presence of periodicities in the observed count rates and, finally, the question of timing differences in short spikes.

All of these are found to be governed by statistical considerations for which we introduce a few notions and definitions:

The number of counts acquired in a channel register in one integration period of 1.2 s (i.e. what we have generally referred to as the count rate) is denoted by $c_i$, where $i$ labels read-out sequence. Now $c_i$ is a realization of a random variable $c$, having a Poisson distribution (cf. Section 2) with expectation value $\lambda_i$ (that is
Thus whereas the \( c_i \) are measured, \( \lambda_i \) is unknown, the two differing by \( \Delta c_i = c_i - \lambda_i \). The series \( \{ \lambda_i \} \) represents the actual time profile that one would like to measure but due to fluctuations \( \Delta_i \), only the series \( \{ c_i \} \) is achieved.

Consecutive \( c_i \) as well as \( c_i \) from different channels – are statistically independent (Hoyng and Stevens, 1974). In analyzing the time development we use the notation \( \Delta c_i = c_i - c_{i-1} \) (\( c_i, c_{i-1} \) from the same channel), likewise, for \( \Delta c_i, \Delta d_i \) and \( \lambda_i \); the integration time (1.2 s) is denoted by \( \tau_0 \). For expectation value and variance of a random variable, e.g. \( c_i \), the symbols \( E c_i \) and \( D^2 c_i \) are used.

5.1. Noise properties of the series \( \{ c_i \} \)

We wish to check to what extent the rapid fluctuations in the observed series \( \{ c_i \} \) are purely Poisson count noise superimposed on a smooth profile \( \{ \lambda_i \} \) and to what extent they are intrinsic to the Sun. To do so it is insufficient to compare \( c_i^{1/2} \) with \( \Delta c_i \) for each \( i \). Rather one must examine the distribution of all the \( \Delta c_i \). A quantity \( \chi^2 \) is introduced, the sum of the squares of the (normalized) differences of \( c_i \) and the running mean \( \mu_i = (c_{i-1} + c_i + c_{i+1})/3 \):

\[
\chi^2 = \sum_{i=1}^{n} (c_i - \mu_i)^2 / \mu_i.
\]

(5)

Following standard procedures we find:

\[
E \chi^2 = E \sum_{i=1}^{n} (c_i - \mu_i)^2 / \mu_i = \frac{2}{3} n + \frac{3}{2} \sum \left\{ \frac{\lambda_{i-1} + \lambda_{i+1}}{2} - \lambda_i \right\}^2 / \lambda_i,
\]

(6)

\[
D \chi^2 = \{ E(\chi^2 - E\chi^2)^2 \}^{1/2} \sim (2n)^{1/2};
\]

(7)

(6) differs from the usual in that \( 2n/3 \) occurs instead of \( n \) because \( c_i \) and \( \mu_i \) have a tendency to fluctuate in the same direction, lowering the value of \( (c_i - \mu_i)^2 \). Further, as the variance of \( \chi^2 \) was difficult to evaluate we just estimated in (7) its order of magnitude by the ordinary \( \chi^2 \) variance. Table V gives some results.

The time series of event Nos. 1, 2, 3, 5 and 7–10 are consistent with the hypothesis of Poisson noise superimposed on a slowly varying systematic change.* From Table V it is inferred that the second term in (6) is small compared to \( 2n/3 \), or, on the average, for any given \( i \) one expects

\[
\left| \frac{\lambda_{i-1} + \lambda_{i+1}}{2} - \lambda_i \right| \ll \lambda_i^{1/2}.
\]

(8)

In other words, an observed \( c_{i+1} \) does – on the average – not differ significantly from the previous \( c_i \); their difference are attributable to Poisson noise. This implies that these events do not have significant time scales on a 1.2 s basis, see next section. It does not imply that the events are secularly “structureless” over longer intervals; that depends on the sign distribution of the series \( \{ c_i - \mu_i \} \), which does not enter in (5) at all.

* We correct here an earlier statement that the August 4 event exhibited “intrinsic white noise in its emission at all periods from 15 s down to the Nyquist period, 2.4 s” (Hoyng et al., 1975). There is no such intrinsic white noise. 
For the events Nos. 4 and 6, one cannot decide between enhanced noise or real structure on the basis of (5) only. However, in Figure 9 and 13, the similarities between different channels show that the large values of $\chi^2$ are, very probably, due to systematic changes, that is to the second term in (6). For the extreme case of event No. 4, channel 1, one then finds:

$$\left| \frac{\lambda_{i-1} + \lambda_{i+1}}{2} - \lambda_i \right| \approx 2.4 \lambda_i^{1/2}.$$  \hspace{1cm} (9)

The conclusion is that – with the sensitivity of the present instrument – only events Nos. 4 and 6 can be said to have much real structure on a 1.2 s basis.

5.2. Time scales

Various times may be used to characterize a burst and hence the physics of electron acceleration and of the X-ray source. One relevant time is the total burst duration which indicates for how long electron fluxes are maintained (above some arbitrary threshold) in the source either against collisional losses or by continuous production (or both). The same may be true of the duration of a small feature (spike) superimposed on the general profile. It must be emphasized, however, that without spatial resolution the interpretation of this latter time is model dependent. In particular, a small spike might be due to a large change in acceleration rate over a small part of the source volume or equally to a small change in acceleration rate in the source as a whole. Since the central solar flare problem is that of how to release energy and to produce fast particles quickly enough, the most relevant (and model-independent) quantity is the $e$-folding time $\tau$ for change of the fast particle flux as a whole. In terms of the X-ray burst profile $c(t)$, this is

$$\tau = c(dc/dt)^{-1}$$ \hspace{1cm} (10)

and we are particularly interested in the minimum statistically real $\tau$ values
exhibited by the data. Examples of physical factors determining $\tau$ are given at the end of this section.

The derivative $dc/dt$ is computed from two consecutive $c_i$ giving

$$\tau_i = c_i\tau_0/\Delta c_i.$$  \hspace{1cm} (11)

In this way the best time resolution is achieved. Statistical aspects are important, however, and the question is under what conditions $\tau_i$ from (11) approximates the true time scale $\tilde{\tau}_i = \lambda_i \tau_0/\Delta \lambda_i$. It is obvious that when $|\Delta c_i| = |c_i - c_{i-1}| \leq c_i^{1/2}$ one is unable to distinguish a possible real change in the count rate from a random fluctuation [hence, from (11), $\tau_i$ can only be meaningful when $\tau_i < \tau_0 c_i^{1/2}$]. It turns out that two conditions must be met, for $\tau_i$ to be real: $c_i \gg 1$ and $c_i^{1/2}/|\Delta c_i| \ll 1$. In practice we adopted

(a) $c_i \geq 75$.
(b) $c_i^{1/2}/|\Delta c_i| \leq 0.43$, and also $\leq 0.31$ (probability $1.0 \times 10^{-1}$ and $2.15 \times 10^{-2}$, respectively).

Because (8) implies that, on the average, $c_i^{1/2}/|\Delta c_i| \gg 1$, it follows that the majority of the $\tau_i$ computed through an event – with the possible exception of events Nos. 4 and 6 – are meaningless due to fluctuations.

To further minimize the influence of fluctuations and increase the physical significance, we require (a) and (b) to hold for channel 1, 2 and 3 at the same time (probability – when (a) already holds $\sim 10^{-3}$ for $c_i^{1/2}/|\Delta c_i| \leq 0.43$ and $10^{-5}$ for the stronger requirement $\leq 0.31$). The results are given in Table VI. It includes all $\tau_i$ of all events Nos. 1–9 satisfying (a) and (b), using probability $10^{-3}$. Imposing the stronger requirement of probability $10^{-5}$, the subgroup printed in italics in Table VI is obtained.

It is seen that only the events Nos. 4 and 6 (as expected) and 8 and 9 from Table II have a few significant time scales $\tau_i$. For each event also the a posteriori probability is given (under a.p.p.) i.e. the ratio of the number of lines in Table VI for that event and the number of times $c_i \geq 75$ holds for all three channels at the same time. Values for $\tau_i$ range from $\sim 2$ s in impulsive bursts to $\sim +20$ s in extended X-ray bursts. We conclude that (especially in large events) the shortest e-folding

<table>
<thead>
<tr>
<th>Event No.</th>
<th>Time (UT)</th>
<th>$\tau_i$ (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>ch 1</td>
</tr>
<tr>
<td>4</td>
<td>1839 : 03.7</td>
<td>2.5</td>
</tr>
<tr>
<td>(August 2)</td>
<td>13.3</td>
<td>2.5</td>
</tr>
<tr>
<td>a.p.p.</td>
<td>29.0</td>
<td>2.5</td>
</tr>
<tr>
<td>0.39</td>
<td>30.2</td>
<td>2.5</td>
</tr>
<tr>
<td></td>
<td>31.4</td>
<td>2.5</td>
</tr>
<tr>
<td>Event No.</td>
<td>Time (UT)</td>
<td>( \tau_i ) (s)</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------</td>
<td>-----------------</td>
</tr>
<tr>
<td>4 (August 2)</td>
<td>1839:32.6</td>
<td>-3.9</td>
</tr>
<tr>
<td>continued</td>
<td>33.8</td>
<td>8.9</td>
</tr>
<tr>
<td></td>
<td>36.2</td>
<td>2.3</td>
</tr>
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<td></td>
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<td>5.7</td>
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<td>4.3</td>
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<td></td>
<td>47.0</td>
<td>-6.0</td>
</tr>
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<td>6 (May 19)</td>
<td>2244:07.0</td>
<td>2.4</td>
</tr>
<tr>
<td>a.p.p.: 0.16</td>
<td>28.7</td>
<td>— (^b)</td>
</tr>
<tr>
<td></td>
<td>29.9</td>
<td>— (^b)</td>
</tr>
<tr>
<td>8 (August 4)</td>
<td>0622:43.4</td>
<td>8.7</td>
</tr>
<tr>
<td>a.p.p.: 1.6 (	imes 10^{-2})</td>
<td>44.6</td>
<td>9.9</td>
</tr>
<tr>
<td></td>
<td>45.8</td>
<td>5.9</td>
</tr>
<tr>
<td></td>
<td>47.0</td>
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<td>48.2</td>
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<td></td>
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<td>11</td>
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<td>0623:07.5</td>
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<td></td>
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<td>24:16.2</td>
<td>-21</td>
</tr>
<tr>
<td></td>
<td>54.7</td>
<td>18</td>
</tr>
<tr>
<td>9 (August 7)</td>
<td>1515:22.7</td>
<td>5.2</td>
</tr>
<tr>
<td>a.p.p.: 3.5 (	imes 10^{-2})</td>
<td>17:06.3</td>
<td>7.7</td>
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<td></td>
<td>31.6</td>
<td>17</td>
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<tr>
<td></td>
<td>53.3</td>
<td>+20</td>
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<td></td>
<td>55.7</td>
<td>-19</td>
</tr>
<tr>
<td></td>
<td>20:51.6</td>
<td>18</td>
</tr>
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<td>52.8</td>
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<td></td>
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<td>17</td>
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<tr>
<td></td>
<td>58.8</td>
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<td>21:00.0</td>
<td>22</td>
</tr>
<tr>
<td></td>
<td>01.2</td>
<td>23</td>
</tr>
</tbody>
</table>

\(^a\) A line is printed in italics if \( c^{1/2}/|\Delta c| \leq 0.31 \) for all three entries.

\(^b\) Missing due to subcommutation, but very probably satisfying \( c^{1/2}/|\Delta c| \leq 0.31 \), cf. Figure 12.
time scales present in the source are well above the instrumental resolution.* A striking feature is the strict sign correlation, the August 2 event having 7 positive and 6 negative sets, spread more or less evenly over the whole duration of the event, whereas August 4 and 7 have almost only positive time scales, clustering at the steep initial rise.

It is noted that the time scales for the two extended bursts of August 4 and 7 in general decrease with increasing energy.

Two applications are discussed:

5.2.1. Length Scale for Acceleration Region

If the profile is acceleration governed one expects, on the grounds of dimensional arguments, that \( \tau \sim \frac{l}{v} \gtrapprox \frac{l}{v_A} \) where \( l \), \( v \) and \( v_A \) are equal to the length scale of, a typical velocity in, and the Alfvén velocity in the acceleration region [cf. eq. (19) and following discussion], or:

\[
l \approx 2 \times 10^{11} H n_0^{-1/2} \tau,
\]

where \( H \) and \( n_0 \) are magnetic field and density of the acceleration region (\( l \), \( H \), \( n_0 \) and \( \tau \) in c.g.s. units). Adopting e.g. \( H \sim 100 \text{ G} \), \( n_0 \sim 10^{10} \) (see second application below) and \( \tau \approx 2-4 \text{ s} \) (Table VI) for the impulsive burst of August 2, \( l \approx 4-8 \times 10^8 \text{ cm} \) is obtained, a rather small quantity. In the containment model this would be also a typical dimension of the hard X-ray source region as acceleration and source region are identical in this case.

In the initial acceleration phase of an extended burst such as August 4, the corresponding \( \tau \) equals 5–20 s, so that the acceleration region (\( l \)) could be up to an order of magnitude larger (depending on \( n_0 \) and \( H \)) than in an impulsive burst. Note that this dimension should be distinguished from the very large value \( L \sim 3 \times 10^{10} \text{ cm} \) deduced by Brown and Hoyng (1975) for the dimension of the magnetic trap postulated during the later phase of the August 4 event.

Concerning the size of the hard X-ray source, it is interesting that existing observations suggest that it be small (\( \approx 10^9 \text{ cm} \)) in small flares. In particular, Neupert et al. (1974) find that the soft X-ray source of the impulsive burst of August 2, as observed near 1.9 \( \text{ Å} \), is always smaller than one resolution element of the spectroheliograph, \( 1.5 \times 10^9 \times 1.5 \times 10^9 \text{ cm}^2 \), and that its position remains fixed in time [the size of the acceleration region was found above to be small, too: \( \sim 5 \times 10^8 \text{ cm} \)].

5.2.2. Lower Limit to the Density of the Hard X-Ray Source Region

This can be derived from the (smallest observed) negative \( \tau \), by considering the collisional decay (proton density \( n_0 \)) of fast electrons [distribution function \( f(E) \)] with no injection or acceleration. Using (a) the continuity equation, (b) collisional

* In the computations and in Table VI, actually \( \tau_i = \frac{1}{2} \tau_0 (c_i + c_{i-1}) / \Delta c \) is used instead of (11). This detail is mentioned here because it follows that \( |\tau_i| \approx \frac{1}{2} \tau_0 = 0.6 \text{ s} \). This absolute lower limit is seen not to be approached in Table VI.
losses of an electron, given by $\dot{E} \sim n_0 E^{-1/2}$ (Brown, 1971), (c) $f(E) \sim E^{-\gamma+1/2}$ where $\gamma$ is the observed hard X-ray power law index, one finds

$$\tau \approx f(\partial f/\partial E)^{-1} = \left[ \frac{\partial}{\partial E} (\dot{f}) \right]^{-1} = E/\gamma \dot{E}$$

or, numerically:

$$\tau \approx 2 \times 10^8 \frac{E_{keV}^{3/2}}{\gamma n_0}$$

(13) ($\tau$ and $n_0$ in c.g.s. units). It is essential to recall that this derivation breaks down if collisions are not the only loss mechanism (e.g. escape, betatron or collective losses).

For the impulsive burst of August 2, $\gamma \sim 3$ (Table III), $\tau \sim -2$ s at $E \sim 60$ keV, and $n_0 \approx 10^{10}$ cm$^{-3}$ is found. The time scales in Table VI for both impulsive bursts in Table VI are about constant as a function of channel number, that is they do not show any trace of an $E^{3/2}$ dependence. Subject to the above limitation this is an additional strong argument for the density $n_0$ in this type of flare being actually higher than the derived lower limit.

In conclusion we surmise that, for the impulsive bursts of August 2 and May 19:

1. they have one or more small ($\ll 10^9$ cm) acceleration regions.
2. their hard X-ray source region should have at least a density of $n_0 \sim 10^{10}$ cm$^{-3}$ and, in the case of event No. 4 on August 2, a size $\ll 10^9$ cm.

5.3. PERIODICITIES

Some of the hard X-ray time profiles have a quasi-periodic appearance, notably the flare of August 4 (Figure 16) and also that of March 21 (Figure 2). A real periodicity in the time profile has potentially important implications because if it is inferred that they are due to real physical oscillations of some kind in the flare region, then the choice of possible flare configurations is restricted. In addition, the period is expected to have a simple relationship to other parameters of the acceleration region like density, dimension, etc.

In this section we therefore inquire into the reality of such periodicities, using Fourier power spectra. The Fourier transform of a time series (e.g. the count rates $c_0 \cdots c_{N-1}$ of one channel) is constructed, resulting in the series $a_0 \cdots a_{N-1}$. The power spectral density (power spectrum) is defined as $|a_p|^2$ for $p = 0, \cdots, N-1$; $p$ is a dimensionless frequency, $p = 1$ corresponding to (series length)$^{-1}$.

In all figures the quantity $10^6 \log |a_p|^2$ is plotted. Periodicities in the time series will show up as peaks in the power spectrum.

Figure 24 gives the result for the August 4 event, showing power spectra for the count rate in channel 2, for $F_{25}$ and for $\gamma$. In all three power spectra large peaks show up at $p = 10, 20$ and $35$, corresponding to periods of 124 s, 52 s and 35 s and suggesting that these periods are really present in the data.
Since the data are contaminated with Poisson noise, we must investigate the reality of the peaks.

When Poisson statistics apply to the series \( c_0 \cdots c_{N-1} \), the relative uncertainty in the power \(|a_p|^2\) at any \( p \) due to fluctuations is given by (Hoyng, 1976):

\[
(2x - x^2)^{1/2}; \quad x = N^{-2} (\Sigma c_k) / |a_p|^2.
\] (14)

\( N^{-2} \Sigma c_k \) is the expected average high frequency power level, drawn in as a horizontal line in Figure 24. This line coincides with the average high frequency power as computed from the power spectrum, showing once again that the noise in the signal is purely instrumental Poisson noise (the ratio \( x \) can be read easily from the Figure as a difference). For the peaks at \( p = 10, 20 \) and 35 in Figure 24, top, the relative error ranges from 5% to 15% and these peaks are therefore quite

Fig. 24. Power spectra of the count rate in channel 2, \( F_{25} \) and \( \gamma \) for the August 4, 1972 event. The usual technicalities were applied before the actual Fourier transformation (Brault and White, 1971). Due to this the power spectra are distorted for \( p \leq 5 \). Frequency \( p \) and period \( T \) are related by \( p \times T = 1024 \times 1.2 \) s. In the top figure, the straight line represents the average high frequency power expected from Poisson noise (see text).
real (no such analysis for $F_{25}$ and $\gamma$ is immediate as their statistical properties are unknown). Note that this establishes spectral periodicities for the first time.

Power spectra of other events have been obtained. In the March 21 event, Figure 25 (cf. Figure 2), there is an indication of a periodicity in the time profile of channel 2 at $p \sim 17$, or 9 s [width $\Delta p/p \sim 0.1$; relative peak accuracy 40%, from (14)]. The other events of Table II do not show anything.

A dynamic Fourier spectrum was constructed for the August 4 event, at the suggestion of Dr. G. K. Parks, Figure 26.* In brief outline, the method applies a gaussian window of constant relative width $\Delta p/p$ to the Fourier series $a_0 \cdots a_{N-1}$, after which it is transformed back to the time domain. In this way a wave packet is constructed with central frequency $p$ whose power at any time is known (Dzierzowski and Hales, 1972). Time and frequency resolution $\Delta t$ and $\Delta p$ satisfy the relation $(\Delta p/p) \times (\Delta t/T) = \pi^{-1}$ ($\Delta p$, $\Delta t =$ half width at $1/e$ of maximum: $T =$ period; $p \times T = 1024 \times 1.2$ s), and in Figure 26 $\Delta p/p = 0.2$ and $\Delta t/T = 2$, the best possible combination attainable with the method used.

In view of the time resolution obtained, the local maxima at $p \sim 10$, $\sim 20$ and $\sim 50$ cannot be said to occur at different times; the $p \sim 50$ maximum is rather weak.

![Power Spectra Raw Data Channel 2](image)

* Fig. 25. Power spectrum of the count rate in channel 2 for the March 21, 1972 event. The straight line represents the average high frequency power expected from Poisson noise.

* The help of Dr. G. Nolet, Vening Meinesz Laboratory, Utrecht, is gratefully acknowledged.
Fig. 26. Dynamic spectrum of the count rate of channel 2 of the August 4, 1972 event. The lineprinter resolution is much better than the actual resolution (see text).

Our conclusions are:

(1) In the August 4 event significant and well separated periodicities of \(~120\) s, \(~60\) s and \(~30\) s are indicated at the onset of the hard X-ray emission. The two higher frequencies die out and the lowest one seems to shift towards lower frequencies as time progressess. The vibrating coronal trap model for this flare
(Brown and Hoyng, 1975) admits a possible interpretation in terms of trap-eigenmodes, but this needs further theoretical substantiation.

(2) With the possible exception of event No. 1, on March 21, no other flare shows a trace of a periodic X-ray emission, within the statistical limitations of the present data.

5.4. Timing Differences in Spikes

The hard X-ray time profiles of impulsive X-ray bursts sometimes exhibit very narrow spikes in all four channels at almost the same time. We analyze here to what extent apparent small timing differences are real and indicate possible implications. The purpose of this section is mainly to establish the phenomenon, as it is not easily derivable from the figures.

Seven such narrow spikes were identified, in the events No. 4 (August 2), No. 6 (May 19, 1973) and No. 11 (August 23), indicated by arrows in Figure 8, 12 and 21. For purposes of intercomparison, a median time $t_m$ is defined for each spike:

$$t_m = \frac{\sum_{i=1}^{n} t_i c_i}{\sum_{i=1}^{n} c_i},$$

where $c_i$, $t_i$ are the total count within, and the time halfway through the integration period $i$. In all four channels the summation runs over the same time interval, and in practice we used various $n$, from $n = 3$ up to $n = 9$.

The uncertainty in $t_m$ due to Poisson noise follows by computation of the (expectation value and) standard deviation of the random variable $t_m = \Sigma t_i c_i / \Sigma c_i$:

$$E t_m = \frac{\Sigma t_i \lambda_i / \Sigma \lambda_i}{\lambda_i} \approx \frac{\Sigma t_i c_i / \Sigma c_i}{c_i} = t_m$$

$D t_m = \left[ \Sigma \lambda_i (t_i - t_m)^2 / \Sigma \lambda_i \right]^{1/2} = \left[ \Sigma c_i (t_i - t_m)^2 / c_i \right]^{1/2}$

$(t_m = \Sigma t_i \lambda_i / \Sigma \lambda_i)$. $D t_m$ turns out to be typically of the order of 0.05 s. Because a spike could be due to a change in only a part of the hard X-ray flare region, $t_m$ is also calculated subtracting the background flare emission by using (15) with $c_i = c_i - c_1 - (c_n - c_1)(i-1)/(n-1)$ instead of $c_i$. Errors in these $t_m$ have not been evaluated; they will be substantially greater than (16).

Results are shown in Figure 27. Channel number is converted to mean channel photon energy using the relevant power law weighting function. $t_m$ is seen to either increase or decrease systematically with energy and subtraction of the background flare emission does not alter the energy dependence. The three spikes from the May 19 event are not shown; in each case, all $t_m$ are equal within the error limits.

A detailed analysis of possible origins of these timing differences is outside the scope of this paper. One must, however, think in terms of factors such as the acceleration as a function of time and the magnetic field geometry.

One simple explanation could be the following. Suppose a bunch of electrons is accelerated 'instantly', with downward velocities. If the acceleration region is
Fig. 27. Median times $t_m$ as a function of photon energy for the events of August 2 and 23, 1972, with and without subtraction of background flare emission (crosses and dots, respectively). The total length of the error bar is $2D t_m$. The number of integration periods, $n$, used in (15) is indicated, as is the UT of $t_m$ of channel 1. Because $n = 3$ in the top figure, the crosses are all based on one integration period and therefore coincide in time.

...high up in the atmosphere, then due to velocity dispersion, the fastest electrons are stopped first. In this case $t_m$ is expected to increase with decreasing energy, as it does in two spikes (Figure 27 middle). If the travel distance $l$ is about the same for all energies (i.e. the scale height in the hard X-ray source is much smaller than $l$), then $t_m \sim lE^{-1/2}$. We estimate $l$ from the timing difference $\Delta t$ between channel 1 and 4 ($\beta = v/c$):

$$l = (\beta_1^{-1} - \beta_4^{-1})^{-1} c \Delta t \approx 1.1 c \Delta t.$$  

Taking $\Delta t = 0.2$ s from Figure 26, $l \sim 7\times10^9$ cm is obtained.

The reverse situation, $t_m$ increasing with energy, could perhaps arise if the acceleration takes place in a high density region, such that low energy electrons virtually do not propagate, but the high energies need some time to slow down.
6. Correlations

We have searched for the existence of correlations between the thick target parameters, time scales and count rates.

6.1. Correlations between the thick target parameters $\gamma$ and $F_{25}$

A marked correlation was found to exist between the instantaneous electron flux $F_{25}(t)$ and $\gamma(t)$ in the case of the extended burst of August 4, see Figure 28. During the brief rising phase of the event – cf. Figure 17, top – the ‘dog-leg’ part AB is swept out, and all later ($F_{25}$, $\gamma$) are seen to exhibit a correlation.

Fig. 28. Correlation diagrams for the August 4, 1972 event, obtained by eliminating time between $F_{25}(t)$ and $\gamma(t)$ in Figure 17. Only every fifth data point is shown for clarity. The solid line (drawn on the basis of all data points) indicates the smoothed path of the event development (see text). The letters A–E are time markings and they refer to those in Figure 17. Note that point C is passed several times and so does not refer to a definite time in Figure 17, but to an extremum flux.
Following the actual behaviour as a function of time, the point \((F_{25}, \gamma)\) moves on the average along the drawn-in line, that is neglecting scatter due to statistical fluctuations and to small digressions. Each of the three counter-clockwise 'ellipses' corresponds to one of the three large oscillations visible in the time profile, Figure 16. After the oscillations have died out there is a monotonic decay down the line DE. The velocity along the curve is inversely proportional to the density of points.

The existence of this correlation has stimulated the coronal electron trap interpretation for this flare (Brown and Hoyng, 1975). The reasoning is the following. In the first place, the occurrence of 'ellipses' is imputed to secondary, deviating effects and the existence of a strict correlation line is assumed along which the point \((F_{25}, \gamma)\) moves up and down and finally down along DE. Then it would follow that the hard X-ray source region must be describable in terms of one parameter that determines the position on the correlation line. Specifically, the system must be able to return to the same physical state at later times, implying that there is no continuous injection (but only a brief initial acceleration) and negligible collisional losses. This points already towards a large, low density trap; the (relative) magnetic field changes, associated with trap oscillations, serves as the free parameter. Betatron acceleration is continuously acting on the fast electron population, causing the X-ray flux modulations (Figure 16); the trap eventually expands and oscillations die out (part DE); see further Brown and Hoyng (1975). Of course it is always possible to explain any event, also this one, with the thick target model. The attractiveness of the present model is however, that for the first time it has been possible to give a simple analytic treatment of a (re)acceleration mechanism.

The corresponding diagram for the other large flare of August 7 is given in Figure 29. A correlation between \(F_{25}\) and \(\gamma\) is seen not to exist in this case (in the sense that knowledge of either parameter enables one to do a probability statement on the other). The existence of a well-defined path in the \((F_{25}, \gamma)\)-plane need not be a surprise: Any well-behaved photon flux implies continuous functions \(F_{25}(t)\) and \(\gamma(t)\) – in other words, a path in the \((F_{25}, \gamma)\)-plane. The flux must be strong enough to eliminate the influence of statistical fluctuations.

In this respect it is mentioned that the 'correlation diagrams' for the other events Nos. 1–7 are virtually structureless (scatter diagrams). No path exists because fluctuations dominate.

Therefore, Figure 29, if suggestive, does not necessarily contain something that has to be explained. There are, however, a few similarities between Figure 28 and 29. Both events begin in the same way, although the August 7 event more slowly, and there is also a tendency towards counter-clockwise movements in Figure 29. It is a pity that the end phase of this flare was not observed. A coronal trap interpretation for this event seems therefore impossible, perhaps until an explanation is given for the minor counter-clockwise movements in the August 4 event, which seem more dominant in the August 7 event.
6.2. Correlation between $\gamma$ and count rate in the August 4 and 7 events*

In Figure 16 and 17 a correlation is seen to exist between a time profile of any channel and $-\gamma$, provided the latter is shifted backwards in time some 15 s at most. This point is closely related to Section 6.1.

The time profiles of the August 4 event, Figure 16, show that a 'temporal feature' in general occurs progressively later at higher energy, totalling up to a shift of $\sim$15 s; and this holds until point $D$ in Figure 17. As a consequence,

* We are grateful to Dr A. O. Benz for drawing our attention to these points.
channel 1 and $\gamma$ will oscillate, with a phase shift. As variation in $\gamma$ are small, $F_{25}$ is just proportional to the count rate in channel 1. Counter-clockwise 'ellipses' in Figure 28 appear as a result and their axes are approximately aligned with the co-ordinate axes because of the relative size of $\Delta F_{25}$ and $\Delta \gamma$, and because the phase shift is about a quarter period.

The relation between $\gamma$ and a time profile manifests itself as an anticorrelation because that requires the smallest time shift. For the August 7 event also this anticorrelation between channel 1 and $\gamma$ is seen to exist, Figure 19. There is no indication for a time shift (possibly because the features are much less pronounced).

6.3. Correlation between $\gamma$ and the time scale $\tau$

In no event of Table II we have been able to establish any correlation between $\gamma$ and the time scale $\tau$ (cf. Vorpahl and Takakura, 1974, who compared rise times and $\gamma$ for 36 different events).

7. Implications for X-Ray Flare Models

Despite all the effort invested, theoretical flare models are still in a poorly developed state and a reasonable prediction of rates of electron acceleration and hence of their hard X-ray emission is out of the question for some time to come. This is true even for the best elaborated flare model, Petschek's wave assisted diffusion mode (Petschek, 1964) and follow-ups (Syrovat-ski, 1966; Green and Sweet, 1967; Petschek and Thorne, 1967; Sonnerup, 1970; Yeh and Axford, 1970; Coppi and Friedland, 1971; Anzer, 1973; Priest, 1973).

The present observations therefore do not have discriminating power against existing flare models (this situation could change considerably when hard X-ray observations with good spatial resolution become available). However they do so for the existing X-ray source models such as thick and thin target model, and the electron trap model (Brown, 1971; Datlowe and Lin, 1973; Brown and Hoyng, 1975).

The aim of this paper so far has been to present the data in usable form as a basis for future physical analysis. Here we consider a few possible topics: (1) The question of possible physical differences between impulsive and extended bursts. (2) The possibility of a thermal contribution to $\gamma$ in impulsive bursts. (3) Evidence for in situ power law electron spectra in flares. (4) Implications of the present observations for thick and thin target models.

7.1. Physical differences between impulsive and extended X-ray bursts?

It was suggested that the observations can be divided, phenomenologically, in two groups, impulsive and extended X-ray bursts:

Extended bursts have a much longer duration than impulsive bursts and are
usually associated with a type II radio burst. Only a few observations of this type exist.

The X-ray time profile of extended bursts exhibits a much more 'gradual' development than those of impulsive bursts, and especially during later stages of the X-ray emission a prolonged, featureless decay is observed. Impulsive bursts on the other hand consist of one or a few brief peaks [this is e.g. confirmed by the discussion on time scales, Section 5.2.].

One the average, $\gamma$ decreases with time in extended bursts, but increases in impulsive bursts.

It is possible that these different characteristics reflect physical differences in the X-ray source region of these flares.

The characteristics of extended bursts indicate a large, low density coronal X-ray source region. This can be substantiated for the flare of March 30, 1969 (Frost and Dennis, 1971); this event occurred far behind the limb and therefore the coronal interpretation is rather cogent, as pointed out by Hudson (1973). In addition to this, it has recently been possible to account for the major characteristics of the flare of August 4, 1972 (Brown and Hoyng, 1975; see also Section 6), in terms of a large ($\sim 10^{32}$ cm$^3$), low density ($\sim 10^7$ cm$^{-3}$) coronal volume.

On the other hand, impulsive bursts would originate in a relatively small, high density region in the chromosphere or low corona. The present observations are consistent with this for the flares of August 2 and May 19 (cf. Section 5.2.).

In further support it is pointed out that existing observations of smaller soft X-ray flares suggest that they are dense and small in extent (Walker and Rugge, 1970; Takakura et al., 1971; Purcell and Widing, 1972; Catalano and Van Allen, 1973; Neupert et al., 1974; Phillips et al., 1974; Widing and Cheng, 1974; Alissandrakis and Kundu, 1975). See also beginning of next Section 7.2.

7.2. Possibility of a Thermal Contribution to $\gamma$ in Impulsive Bursts

The fact that $\langle \gamma \rangle \geq 0$ in impulsive bursts could be due to an increasing contribution of thermal bremsstrahlung in the lower energy channel(s), cf. Takakura, 1969. In fact, preliminary analysis showed that the 'tails' in channel 1 occurring in events Nos. 2 and 3 on May 18 (Figure 4 and 6) can be of thermal origin, for reasonable source parameters. It is emphasized that firm settlement of this question requires further investigations, as well as very precise, concurrent measurements of both soft and hard X-ray flare emission. These measurements must be really free of any instrumental defect.

7.3. Evidence for In Situ Power-Law Electron Spectra in Flares

Models for hard X-ray flare emission often assume power-law electron spectra in the flare region. It is pointed out that there is no direct and very little indirect evidence for this assumption.

The spectrum of fast electrons is best (but still very poorly) determined by hard X-ray observations: An integral equation must be solved in which the brems-
strahlung cross section serves as the kernel. The condition of this equation is such that small variations in the photon spectrum induce very large distortions in the electron spectrum (Brown, 1975). Given the present status of hard X-ray flare observations, a great variety of electron spectra—among which sometimes a single power law—is compatible with observations. In addition, the deduced electron spectra are space-averaged. Direct observations of power law electron spectra in e.g. the cosmic ray spectrum and in solar cosmic rays (Lin, 1974a) may give some comfort, but at best single power-law electron spectra in solar flares provide reasonable working models only. Model calculations assuming a single power law must be considered with great caution insofar conclusions are drawn from deviations from power laws. In situ fast electron spectra may well be no power law at all, or only so on very limited energy intervals.

Recently, Petrosian (1973) has explained the break that is often observed in the hard X-ray photon spectrum (Section 4.3.2.) by the effect of relativistic beaming of the emitted bremsstrahlung from a beam of fast electrons shot downwards into the photosphere. Although the effect is undeniable, it is doubtful whether this explanation is satisfactory, because an exact power law electron spectrum is assumed at the onset of the beam. This is essentially an arbitrary assumption.

In this context it is mentioned also than a thermal explanation of hard X-ray bursts up to higher energies that usual (say e.g. ~100 keV) by a suitable temperature distribution cannot be excluded on the basis of the present observations (Chubb et al., 1966; Chubb, 1970; Milkey, 1971; Brown, 1974). Such a thermal explanation has not yet been worked out fully, but it is very attractive in that the actual electron flux is much smaller than the values given for $F_{25}$ in Table III, thereby relieving some of the difficulties formulated in the next section.

Further investigations are badly needed to settle this matter as well as the above mentioned high quality concurrent observations of soft and hard X-ray flare emission.

7.4. ELECTRON BEAMS, THICK AND THIN TARGET

Thick and thin target model invoke the existence of streams of fast electrons propagating downward or upward in the solar atmosphere. The following discussion is largely based on the values of $\langle F_{25}\rangle$ (Table III). As it will be argued that these are very large, it is relevant first to review the interpretation of $F_{25}$ (cf. also Section 4.2.).

If an X-ray burst is due to an influx of fast electrons into a target, then $F_{25}$ usually represents a lower limit to the electron flux, for the following reasons:

(a) If the target is thin, the actual flux should be higher to reproduce the observed X-ray bursts.

(b) 25 keV was taken as low energy cut-off, just for instrumental reasons, but the true figure could easily be lower, say ~5–20 keV (Kahler and Kreplin, 1971; McKenzie et al., 1973, and Peterson et al., 1973; for event No. 4, on August 2, see Datlowe and Peterson, 1973). On the other hand, there is the above mentioned
possibility of a power law resulting from spatial distribution of maxwellians, in which case the actual nonthermal electron flux is much smaller.

(c) Expression (2) for \( F_{25} \) is based on the expression for collisional energy loss of a single electron (Brown, 1971). To obtain the actual flux one needs an expression for the total energy loss of an average beam electron, which is always larger than or equal to single electron collisional losses (e.g. due to collective losses); hence causing the required flux to be larger than our \( F_{25} \).

As the values of \( F_{25} \) (Table III) are very high (see below) it is noted here that they would go down if somehow the nett electron energy losses were smaller than the collisional losses. This seems to be almost impossible; resistive instabilities (Bekefi, 1966, Furth et al., 1963) provide the only theoretical possibility for this, but it is unclear if any efficiency can be reached.

(d) Direct electron effects. The distribution of fast electrons is supposed to be isotropic in deriving (2), but is of course anisotropic when a beam is considered. This can increase as well as decrease the actual flux, but only little; less than a factor 10 (Elwert and Haug, 1971; Brown, 1972).

We now inquire into the physical nature of the electron streams. First, if the electrons are supposed to have about parallel velocities, then \( F_{25} \sim 10^{36} \text{s}^{-1} \) implies an enormous current \( I = eF_{25} \sim 10^{17} \text{A} \) (corresponding to \( \geq 100 \text{Am}^{-2} \), or, or to \( \geq 2 \times 10^{-2} \text{G cm}^{-1} \) assuming an area \( \leq 10^{19} \text{cm}^{2} \)); and essentially because of this large value, the following can be concluded:

(a) It seems unlikely that the fast electrons form (part of) a current system, with all plasma electrons flowing in the same direction. This follows by considering the time evolution of a current \( \mathbf{J} \), given by

\[
\frac{\partial}{\partial t} \mathbf{J} = \frac{c^2}{4\pi} \Delta (\mathbf{J}/\sigma) - \frac{c}{4\pi} \Delta (\mathbf{v} \times \mathbf{H}),
\]  

where \( \sigma \) is the conductivity. Consider now the time scales involved in this equation, and to this end the current is taken to have a perpendicular area \( A = l d \), with \( l > d \).

The first term in (18) describes the evolution of a current when little or no mass motion occurs. The associated time scale is given by \( \tau \sim \sigma d^2/c^2 \), which is very difficult to evaluate since \( \sigma \) and \( d \) are essentially unknown and since each of them can vary by orders of magnitude.

The second term in (18) has a time scale \( \tau \sim d/v \geq d/v_A \) (\( v_A \) = Alfvén velocity) which is more easily evaluable.

Estimating the self field of the current by

\[
H \sim 4I/c \quad \text{(with} \ I \geq eF_{25} \text{)}
\]

one obtains

\[
\tau \sim 7 \times 10^{-6} A_{18} n_{10}^{1/2} / F_{25}
\]

(20)
(\(\tau\) in sec.; \(A_{18}\) in \(10^{18}\) cm\(^2\); \(n_{10}\) = ambient density in \(10^{10}\) cm\(^{-3}\); \(F_{25}\) in \(10^{36}\) s\(^{-1}\)). Therefore, for this second time scale one has \(\tau \approx 10^{-5}\) s.\(^*\) Now the characteristic evolution time of the current as described by (18) cannot be larger than the value given by (20); physically this means that a current of the required strength pinches within \(\sim 10^{-5}\) s after being switched on. As observations, on the other hand, require \(\tau \approx\) few sec. at least (Section 4.3.), a quasistationary solution of (18) is indicated, with the observed time profile governed by slowly varying boundary conditions. However, this is impossible because if one requires e.g. \(H \leq 10^4\) G, then (19) implies \(l \geq 7 \times 10^{12}\) cm! Only by allowing extreme field strengths of \(\sim 10^7\) G one can bring \(l\) down to \(\sim 10^{10}\) cm. Although one is dealing here with chromospheric or coronal magnetic fields during flares, about which nothing is known, such large fields must be excluded, because the energy in the current-associated magnetic field is estimated to be many orders of magnitude larger than the total flare energy.

This admits the interesting conclusion that observations exclude the possibility that the fast electrons are (part of) a runaway current e.g. set up by the appearance of a large scale transient electric field in the local restframe of the fluid. The total current density \(J\) must satisfy the inequality \(J \leq e f d^4 v f_1(v)\), where \(f_1(v)\) is the fast electron velocity contribution, and \(\lambda = 10^{-3}-10^{-4}\).

(b) Under these circumstances it is conceivable that the fast electrons constitute a beam. A reverse current is set up in the background plasma, neutralizing the beam current (Benford and Book, 1971). An electric field is supposed to be absent in this approach. The reverse current is excited upon passage of the beam front. The beam density is given by

\[
n_b \approx \frac{2\gamma}{2\gamma + 1} \frac{F_{25}}{Av_{25}}
\]

\((A\) is the beam area; \(v_{25} = 0.3\) c is the velocity of a 25 keV electron), from which, for \(A \sim 10^{18}-10^{19}\) cm\(^2\), beam densities \(n_b \approx 10^7-10^8\) cm\(^{-3}\) are obtained. The beam could therefore be dilute.

For this reverse current to be stable against electrostatic wave excitation, the ratio of the reverse current velocity \(v_r(\approx F_{25}/n_0 A)\) to the sound velocity \(v_s\) must satisfy the inequality

\[
3.5F_{25}/n_{10}A_{18}T_7^{1/2} \approx v_r/v_s \approx 1
\]

\((F_{25}\) in \(10^{36}\) s\(^{-1}\); the background plasma density \(n_{10}\) in \(10^{10}\) cm\(^{-3}\), etc.). Beam instabilities will keep \(v_r/v_s \leq 1\) [in any case \(\leq (M/m)^{1/2} = 43\), but in solar flare conditions it seems likely that \(T_e \gg T_i\) holds]. In this context it should be recalled that the above cited observations indicate a small flare area, \(A_{18} \approx 1\).

* \(\tau^{-1}\) is much larger than both the electron cyclotron- and plasma frequency, justifying \textit{a posteriori} the use of the single-fluid Ohm's law \(\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{H}/c)\) in deriving (18), as well as the neglect of the displacement current in (19).
The occurrence of beams during flares is not ruled out by (22), though it is rather strongly restricted; as soon as a beam enters a region violating (22), it can be largely destroyed.

In particular, it is possible that beams of fast electrons exist in flares, propagating in any direction, but travelling over a limited distance determined by:

- either: violation of (22). As (21) and (22) imply $n_d/n_0 < 3 \times 10^{-3} T_i^{1/2}$ (or $<10^{-1} T_i^{1/2}$ in case $T_e \sim T_i$), this happens before the beam density surpasses the ambient density.
- or: indefinitely increasing collisional losses (thick target case).

Note that the former certainly implies that beams, as strong as they would have to be in flares, could not propagate upward through the corona. Rather, they would produce a shock wave separating the quiet coronal plasma from the flare plasma.

Finally, two remarks:

- The required beams are much stronger than those in type III bursts, $\sim 10^{36} \text{ s}^{-1}$, compared to $\sim 10^{33} \text{ s}^{-1}$ at the very most in type III bursts. The stabilization of such strong beams is not considered here, and this could easily impose further constraints.
- It is recalled that, apart from the question whether beams can exist, it follows from the discussion under (a) that beams of the required strength cannot be accelerated by a ‘static’ electric field.

(c) In view of the above points it seems attractive to suppose that the fast electron distribution function $f_1(v)$ -- in addition to isotropy around the magnetic field -- also roughly possesses mirror symmetry with respect to the field direction. It is in principle also conceivable that the fast electrons are contained in some volume and constantly reaccelerated in there for the duration of the hard X-ray flare ($\sim 10^3 \text{ s}$). Such a containment as well as reacceleration can in principle be achieved in a weakly turbulent plasma.

It is not possible at this stage to reach any conclusion on the degree of anisotropy of $f_1(v)$; this question is closely related to the question of microscopic stability, which is outside the scope of the present paper. $f_1(v)$ could well be nearly isotropic, although some anisotropy may be demanded by observations such as X-ray burst polarisation (Tindo et al., 1972).

If a current is invoked to sustain the above mentioned turbulence (which is very likely) then the required high magnetic field gradient $(\nabla H \gg 4 \pi n_0 e v_s/c = 6 \times 10^{-2} n_10 T_i^{1/2} \text{ G cm}^{-1})$ allows only a sheet-like structure with a very small thickness $d \sim H_m \nabla H$, where $H_m$ is some maximum field strength. If such a sheet, having a lateral ‘active’ area $A$, is required to contain all fast electrons, then either $A$ or the sheet density $n_0$ (or both) must be very large: From Table IV, $\langle n_0 N_{23} \rangle = a n_0^2 \times A d = 5 \times 10^{45}$. Estimating the fractional fast electron density $\alpha$ to be as high as $10^{-1}$ and taking $H_m \sim 500 \text{ G}$, one obtains $n_0 A \cong 6 \times 10^{32} T_i^{1/2}$: Even for a very large area $A \sim 10^{20} \text{ cm}^2 (2' \times 2'$; cf. above cited observations on X-ray source size) a density $n_0 \cong 10^{13} \text{ cm}^{-3}$ is required!
In summarizing (a), (b) and (c) it is concluded that the observed values of \( \langle F_{25} \rangle \), when translated in terms of a current, a beam or an isotropic distribution of fast electrons, are very large and put quite stringent requirements on flare parameters. In principle two possible ways out suggest themselves:

- a more efficient mechanism for generation of X-rays than bremsstrahlung, i.e. synchrotron or compton processes (Korchak, 1971; Brown, 1976), or even a coherent mechanism such as an X-ray laser.
- the effective electron energy losses are smaller than the collisional losses.

However, supposing the present interpretation to be correct, it is clear that further work on flare theory has to explain how—loosely speaking—so many electrons can be accelerated, as expounded above.

(d). The values of \( \int P_{25} \, dt \) as given in Figures 2–19 and summarized in Table IV represent the total energy dumped in the flare region by collisions of fast electrons in any bremsstrahlung model, cf. Section 4.2. These values are also large, from \( \sim 2 \times 10^{30} \) erg in impulsive X-ray bursts up to \( \sim 10^{32} \) erg in extended X-ray bursts, and in addition they are lower limits because of the cut-off at 25 keV. It follows that the energy put into fast electrons is about equal to the total energy released (Brown, 1974; Lin, 1974a, Petrosian, 1973). It is therefore possible that all flare energy is channelled through fast electrons into other forms such as mass motion, heating, optical and EUV emission.

Acknowledgements

The results described in this paper have been obtained thanks to the contributions of many individuals, whose efforts the authors gratefully recognize. For the development and manufacturing of the instrument our thanks are due to Messrs P. J. de Groene, J. J. van der Laan, W. A. Mels, W. Zar-dee and many other members of the Physics, Electronics and Mechanics Department of the Space Research Laboratory at Utrecht. The software development for data handling was mainly the responsibility of Mr Ch. Lapoutre. Messrs W. J. van Iersel and G. W. Geytenbeek have assisted with the data analysis. We are indebted to Drs H. S. Hudson, D. W. Datlowe and M. J. Elcan (UCSD) for providing us with their OSO-7 data. We thank Professor C. de Jager and Dr G. A. Stevens for their critical remarks.

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