A DETERMINATION OF $R$ FROM OPTICAL AND RADIO OBSERVATIONS OF PLANETARY NEBULAE*

STEVEN A. HAWLEY AND DOUGLAS K. DUNCAN
Lick Observatory, Board of Studies in Astronomy and Astrophysics
University of California, Santa Cruz

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The value of the ratio of total to selective extinction ($R = A_V/E_{B-V}$) is determined from existing optical and radio observations. The results are free from many of the systematic effects encountered in purely optical methods of determining $R$; however, uncertainties in the individual determinations are large, making conclusions concerning possible variations of $R$ in the galaxy impossible. Rigorous treatment of all sources of error yields a mean value of $R = 3.4 \pm 0.2$.

Key words: interstellar absorption — reddening — planetary nebulae

I. Introduction

Photometric studies have determined the value of $R = A_V/E_{B-V}$ for many different regions in the Galaxy. The most widely used methods for determining the ratio of total to selective extinction have been the variable-extinction method and the color-difference method. These methods have been discussed by Johnson (1968); however, in many cases his results are vitiated by the effects of infrared excesses in stars.

The bulk of the evidence from a variety of different techniques favors a value of $R$ slightly greater than 3 (Aannestad and Purcell 1973). Schalén (1975) has determined $R$ from star counts for a number of dark nebulae, finding $R = 3.1$. Harris (1973) has utilized the cluster-diameter method and obtained $R = 3.15 \pm 0.20$, independent of the cluster's position in space. Burstein and McDonald (1975) incorporated reddening from globular clusters and total absorption from galaxy counts. They conclude that their most reliable data are consistent with $R \approx 3$. Recently Sandage (1975) has discussed determining $R$ from photometry of groups of galaxies. He obtains $R = 3.35 \pm 0.29$ from the 3C 129 group.

However, several values of $R$ significantly different from 3 have been reported. Sharpless (1952) found a value of $R = 6$ using the variable-extinction method for the Orion Sword region. Herbst (1974) used the same method to study seven stars in the emission nebula NGC 6193. Schmidt telescope photographs were used to select stars definitely associated with the nebulosity, and a value of $R = 5.6 \pm 0.3$ (s.e.) was determined. In a similar study, Racine (1974) studied 14 stars in the nebulous cluster vB 130 in Cygnus. He found $R = 8.1 \pm 1.2$ (m.e.).

A variety of uncertainties are evident in these photometric methods. The variable-extinction method can only measure that component of the extinction which is variable across the cluster. There is uncertainty in the spectral classification and resulting absolute magnitude, as well as in the determination of cluster membership. In the color-difference method infrared excesses can cause an erroneously large value of $R$ to be determined.

It is therefore desirable to have an accurately determined value of $R$ independent of stellar photometry. Gebel (1968) has used H II regions to determine $R$ as a function of galactic longitude. He confirms the classical reddening law for the range $l = 5^\circ - 130^\circ$. In the region from $l = 130^\circ - 210^\circ$ the calculated $R$ values are higher although the uncertainties preclude a confident statement concerning any variation of $R$. He finds a total range in $R$ from 2.6 to 6.5.

Planetary nebulae are prime candidates for determinations of $R$. Subtending smaller solid angles than H II regions, they are less susceptible to the effects of variable extinction. They represent "standard candles" of well-known intrinsic fluxes. The physical processes giving rise to the recombination line spectrum are well understood and the theoretical Balmer line intensities can be calculated quite accurately (Miller 1974). Miller and Mathews (1972) have shown that the spectrum of NGC 7027 agrees remarkably well with theoretical predictions. They derive a value for $R$ of $3.47 \pm 0.19$. The purpose of this study was, therefore, to compile the best available observational data for a large number of planetaries and to look for variations in $R$ from place to place.

II. Procedure

The amount of reddening is found by determining the value of the intrinsic H$\beta$ flux from observations

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of the thermal radio continuum and comparing it with the observed Hβ flux. The ratio of \( I(\text{H} \beta) \), the intrinsic flux in Hβ, to \( I(\nu) \), the flux in the radio bremsstrahlung, has been computed from the free electron emissivity of Oster (1961) and the Hβ emissivity of Brocklehurst (1971)

\[
\frac{I(\text{H} \beta)}{I(\nu)} = \left( \frac{N_\beta N_e}{c \sum_z N_z c_1 Z^2 T_e^{-1/2} \ln(c_2 T_e^{1/2}/Z) \nu} \right),
\]

where \( \nu \) is the Hβ emission coefficient for Case B. The numerical coefficients in equation (1) are

\[
c_1 = \frac{32e^6}{3m^2c^3} (2\pi m/k)^{1/2} = 3.771 \times 10^{-38}
\]

and

\[
c_2 = \frac{m}{\Gamma e^2 \pi} (2k/T m)^{3/2} = 4.954 \times 10^7
\]

(in cgs units) where \( \Gamma = 1.78107 \). It is necessary that \( \nu \) be a radio frequency at which the nebula is optically thin.

The reddening may be discussed in terms of a Balmer line color excess

\[
E_{\beta-\alpha} = 2.5 \log \left[ \frac{I(\text{H} \beta)/I(\text{H} \alpha)}{F(\text{H} \beta)/F(\text{H} \alpha)} \right],
\]

where \( I(\text{H} \beta)/I(\text{H} \alpha) \) is the known theoretical ratio of Hβ to Ha (Brocklehurst 1971) and \( F(\text{H} \beta)/F(\text{H} \alpha) \) is the observed decrement; the extinction in magnitudes at Hβ is given by

\[
A_\beta = 2.5 \log \left[ \frac{I(\text{H} \beta)/F(\text{H} \beta)}{I(\text{H} \alpha)/F(\text{H} \alpha)} \right].
\]

With an assumed reddening curve, \( A_\beta \) and \( E_{\beta-\alpha} \) may be converted to \( A_V \) and \( E_{V-\alpha} \). We assume the Whitford reddening curve (Whitford 1958) which is a good fit to the extinction in the region 3200 Å - 11,000 Å (Miller and Mathews 1972). Then it can be shown (Miller and Mathews 1972) that

\[
R = \frac{A_\beta - 0.425 E_{\beta-\alpha}}{0.863 E_{\beta-\alpha}}.
\]

The coefficients differ somewhat from those determined by Gebel (1968).

III. Data

In most cases optical fluxes were obtained from photoelectric scanner observations made at Lick Observatory. Many observations made with the prime-focus scanner (Wampler 1966) on the 36-inch (91-cm) Crossley reflector were kindly provided by Tim Barker prior to publication. These were combined with observations by Miller (1973) with the prime-focus scanner on the 120-inch (3.05-m) telescope. A list of the nebulae studied is given in Table I, with reference to the data shown in parentheses. The uncertainties in the quantities in Table I derive from the errors quoted by the various authors. An exception is the case of NGC 7008 for which values for the helium abundance and the electron temperature were adopted as shown in Table I.

Code's (1960) calibration for the flux of \( \alpha \) Lyrae at \( \lambda \lambda 4861-5007 \) was compared to that of Oke and Schild (1970) and was found to be systematically 8% higher; we therefore adopt values for the Hβ flux 8% lower than O'Dell's (1962) measurements. The same correction has been applied to the measurements of O'Dell and Terzian (1970), and Collins, Daub, and O'Dell (1961).

IV. Discussion of Results

Values for \( R \) for each nebula are given in the final column of Table I. Clearly the errors in the individual values of \( R \) are large. Nevertheless, they are the rigorous standard deviations and should accurately represent the uncertainties in the available data. The errors are smallest for the few most heavily reddened nebulae. Changing the rather poorly known electron temperature alters the value of \( R \) weakly. Since the abundance of neutral helium is expected to be low in planetary nebulae (see Osterbrock 1974a), the total helium abundance, given by \( N(\text{He}^+) + N(\text{He}^{++}) \), was assumed to be 0.125 relative to hydrogen for NGC 7008. Yet, even for the case with unknown abundances of helium, the errors introduced due to the uncertainties in the assumed abundances are much smaller than the uncertainties caused by errors in the optical and radio observations.

Care was taken to examine the radio frequency spectra of the nebulae used in the study in order to include only those frequencies for which the nebulae were optically thin in the radio bremsstrahlung. Planetaries with confused radio spectra were not included.

A possible systematic source of error is the effect of partial optical obscuration. If part of a nebula is unobscured, the value of \( R \) we determine will be higher than that which characterizes the obscuring material. We investigate the likelihood that a nebula partially obscured by material with one value of \( R \) may masquerade as a nebula completely obscured by material with a different value of \( R \).

We consider the simple-minded case of a fraction \( f \) of a nebula optically obscured by material characterized by a value of \( R = R_p \). The rest of the nebula is assumed to be unobscured. For this situation it is possible to derive \( R_{ob} \), the value of \( R \) observed given uniform obscuration over the fraction \( f \) of the nebula, as a function of \( f \), \( A_p \), and \( R_p \). The "true" \( R \) of the material obscuring the nebula. Figure 1 is a plot of \( R_{ob} \) versus \( f \) for six values of \( R_p \) assuming \( A_p = 1 \) magnitude. There is a minimum \( f \) for each value of \( A_p \) when the Hβ flux from the unobscured fraction be-
### TABLE I

**DATA FOR INDIVIDUAL NEBULAE**

<table>
<thead>
<tr>
<th>Name</th>
<th>Hα/Hβ</th>
<th>T_e (°K)</th>
<th>ν(GHz)</th>
<th>F_ν (f. u.)</th>
<th>He⁺/H</th>
<th>He++/H</th>
<th>A_β</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 7008</td>
<td>4.40±.44 (2)</td>
<td>(1.38±.14)x10⁻¹¹ (3)</td>
<td>10,000±2,000*</td>
<td>15.5 (1)</td>
<td>.27±.07 (1)</td>
<td>.1±.05*</td>
<td>.025±.05*</td>
<td>2.0±0.4</td>
</tr>
<tr>
<td>IC 3568</td>
<td>3.30±.09 (5)</td>
<td>(1.31±.15)x10⁻¹¹ (3)</td>
<td>10,800±500 (5)</td>
<td>15.5 (1)</td>
<td>.15±.06 (1)</td>
<td>.096±.0098 (5)</td>
<td>.001±.005 (5)</td>
<td>1.5±0.5</td>
</tr>
<tr>
<td>IC 5217</td>
<td>3.70±.045 (5)</td>
<td>(6.92±.48)x10⁻¹² (5)</td>
<td>11,300±500 (5)</td>
<td>10 (6)</td>
<td>.13±.08 (6)</td>
<td>.104±.006 (5)</td>
<td>.0064±.005 (5)</td>
<td>1.9±0.7</td>
</tr>
<tr>
<td>IC 351</td>
<td>3.51±.06 (5)</td>
<td>(3.80±.09)x10⁻¹² (5)</td>
<td>12,200±600 (5)</td>
<td>6.63 (6)</td>
<td>.04±.02 (6)</td>
<td>.0824±.0093 (5)</td>
<td>.0287±.005 (5)</td>
<td>1.2±0.5</td>
</tr>
<tr>
<td>IC 2003</td>
<td>3.61±.13 (5)</td>
<td>(6.49±.15)x10⁻¹² (5)</td>
<td>14,200±1,000 (5)</td>
<td>6.63 (6)</td>
<td>.043±.02 (6)</td>
<td>.0852±.006 (5)</td>
<td>.048±.005 (5)</td>
<td>0.5±0.5</td>
</tr>
<tr>
<td>PK 38+12*1</td>
<td>3.76±.12 (5)</td>
<td>(1.17±.03)x10⁻¹² (5)</td>
<td>6,500±800 (5)</td>
<td>10.63 (6)</td>
<td>.118±.047 (6)</td>
<td>.0368±.006 (5)</td>
<td>.0012±.005 (5)</td>
<td>1.7±0.4</td>
</tr>
<tr>
<td>51+9*1</td>
<td>4.15±.05 (5)</td>
<td>(1.51±.07)x10⁻¹² (5)</td>
<td>10,500±1,500 (5)</td>
<td>7.8 (6)</td>
<td>.12±.05 (6)</td>
<td>.1040±.006 (5)</td>
<td>.0008±.005 (5)</td>
<td>1.0±0.5</td>
</tr>
<tr>
<td>NGC 7027</td>
<td>7.47±.31 (5)</td>
<td>(6.25±.31)x10⁻¹¹ (7)</td>
<td>15,000±1,000 (7)</td>
<td>15.5 (1)</td>
<td>.595±.50 (1)</td>
<td>.085±.005 (7)</td>
<td>.042±.005 (7)</td>
<td>3.5±0.1</td>
</tr>
<tr>
<td>IC 418</td>
<td>3.35±.16 (2)</td>
<td>(2.95±.32)x10⁻¹⁰ (3)</td>
<td>9,000±500 (5)</td>
<td>15.5 (1)</td>
<td>1.24±.1 (1)</td>
<td>.079±.008 (5)</td>
<td>.0001±.005 (5)</td>
<td>0.6±0.2</td>
</tr>
<tr>
<td>NGC 6567</td>
<td>5.10±.14 (5)</td>
<td>(1.17±.12)x10⁻¹¹ (6)</td>
<td>12,000±1,000 (5)</td>
<td>10 (6)</td>
<td>.12±.04 (6)</td>
<td>.110±.006 (5)</td>
<td>.0016±.005 (5)</td>
<td>1.4±0.4</td>
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<tr>
<td>IC 2149</td>
<td>3.75±.2 (2)</td>
<td>(3.16±.3)x10⁻¹¹ (5)</td>
<td>10,200±600 (5)</td>
<td>15.5 (1)</td>
<td>.43±.24 (1)</td>
<td>.0987±.006 (5)</td>
<td>.0000±.005 (5)</td>
<td>1.8±0.6</td>
</tr>
<tr>
<td>+30°3639</td>
<td>3.98±.4 (4)</td>
<td>(9.77±.1)x10⁻¹¹ (10)</td>
<td>9,500±1,000 (5)</td>
<td>8.085 (9)</td>
<td>.78±.10 (9)</td>
<td>.042±.01 (5)</td>
<td>.0000±.005 (5)</td>
<td>1.3±0.2</td>
</tr>
<tr>
<td>NGC 7662</td>
<td>3.13±.15 (2)</td>
<td>(1.05±.1)x10⁻¹⁰ (5)</td>
<td>13,500±800 (5)</td>
<td>8.085 (9)</td>
<td>.53±.10 (9)</td>
<td>.0582±.006 (5)</td>
<td>.0393±.006 (5)</td>
<td>0.4±0.2</td>
</tr>
</tbody>
</table>

* Value is uncertain and was estimated.

### REFERENCES TO TABLE I

1. Sistla et al. (1974)
2. Miller (1973)
3. O'Dell (1962)
4. Peimbert and Torres-Peimbert (1971)
6. Higgs (1971)
7. Miller and Mathews (1972)
8. Collins et al. (1961)
10. O'Dell and Terzian (1970)
Fig. 1 — The observed value of $R$, $R_{ob}$, as a function of the fraction of the nebula obscured, $f$, and the “true $R$” of the material; for $A_p$ equal to one magnitude.

comes equal to the observed flux. This limit is 60% for $A_p = 1$. The plots for $A_p = 2$ and $A_p = 3$ are similar to the one for $A_p = 1$ with limiting fractions of 84% and 94%, respectively.

Figure 1 illustrates that the range of $R$ values in Table I can be explained by a “true $R$” of roughly 2-4 with differing amounts of obscuration and that they tend to be upper limits. These results show the advantage of using small-angular-diameter planetary nebulae, as opposed to H II regions, to minimize the effects of obscuration which varies across the source.

Another possible source of systematic error would be internal reddening due to dust mixed in with the gas. Osterbrock (1974a) has observed the Balmer lines in NGC 7027 at high dispersion and found that the red and blue sides of the profiles show no difference which can be attributed to dust within the nebula.

On the whole, information concerning variations of $R$ in the galaxy is lost due to the errors in the flux measurements. Nevertheless, there are a few objects of potential interest that might warrant further, more-accurate observations: IC 3508, IC 5217, and NGC 6567. Unfortunately, in all but the most reddened nebulae, optical fluxes to accuracies of a few percent are required to keep the uncertainties in the computed values of $R$ small. In the more heavily reddened nebulae the error arises primarily from uncertainties in the radio fluxes which in many cases are 30%-50%. Therefore, more accurate radio and optical fluxes are necessary to determine quantitatively the possible variations in $R$.

A useful result of this study is the mean value of the calculated $R$s weighted by their individual uncertainties. We obtain

$$R = 3.4 \pm 0.2.$$

This result includes all nebulae in Table I.

To examine the sensitivity of our result to the particular choice of nebulae, the value of $R$ was computed from several subsets of the data. Without the most accurately known $R$ value, that obtained for NGC 7027, we compute

$$R = 3.3 \pm 0.4.$$

The average $R$ obtained exclusive of the two largest individual values (NGC 5217 and IC 3508) is

$$R = 3.3 \pm 0.2,$$

and excluding the lowest individual value (NGC 6567) yields

$$R = 3.5 \pm 0.2.$$

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— ibid. pp. 185-212.
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