ARE THERE SPOTS ON MAGNETIC WHITE DWARFS?

EGIDIO LANDI DEGL'INNOCENTI
Astrophysical Observatory of Arcetri, Florence, Italy
Received 1975 December 29; revised 1976 March 4

ABSTRACT

The temporal variation of the circular polarization observed in the magnetic white dwarf G195–19 is interpreted in terms of an oblique rotator model. The peculiar behavior observed in red light strongly suggests the presence of a spot on the surface of this star. A model based on this assumption is found to be consistent with observations.

Subject headings: polarization — stars: magnetic — stars: white dwarfs

Since the first observations of the polarization of magnetic white dwarfs (Kemp et al. 1970b) several attempts have been made to build up a convenient theory to interpret the spectral dependence of both circularly and linearly polarized light.

It is nowadays well established that the physical mechanisms able to introduce continuous circular polarization in the optical spectra of magnetic white dwarfs are essentially (a) bound-free opacities and (b) cyclotron absorption or emission (Lamb and Sutherland 1974). The magnetic fields required to obtain a fractional circular polarization of few percent in the optical range are essentially $10^6$–$10^7$ gauss for the first mechanism and $1-3 \times 10^8$ gauss for the second. On the contrary, for the linear polarization, only the second mechanism would be efficient. This fact has led Sazonov and Chernomordik (1974) to the subdivision of the magnetic white dwarfs in two categories according to the presence, or not, of linearly polarized light in their spectra.

Despite our knowledge of the basic physics underlying the formation of the continuous polarized spectrum, the theoretical interpretation of the wavelength dependence of polarization in magnetic white dwarfs is often hampered by the lack of information regarding the detailed structure of the atmosphere (chemical abundances, $T$ versus $\tau$ relation, etc.), and the geometry of the magnetic field. Even the effective temperature, which should be one of the directly measurable parameters, is sometimes unknown to such an extent that different authors (Sazonov and Chernomordik 1974; Angel and Landstreet 1974) referring to the same magnetic white dwarf, G99-37, give estimates of this quantity differing by more than 30 percent. For these reasons, every attempt to reproduce the observed wavelength dependence of the circular and linear polarization in the spectra of magnetic white dwarfs can give, at the moment, no information other than a rough estimate of an ill-defined “mean” longitudinal or transverse magnetic field. A better possibility of getting some insight on the geometrical distribution of the magnetic field is offered by the (polarized) light curve of the variable magnetic white dwarfs. Only two stars of this type are presently known: G195–19, a DC white dwarf (Angel et al. 1974) and GD 229, a white dwarf suspect, which shows irregular variations (Swedlund et al. 1974); the discovery of a third variable magnetic white dwarf, OX +25°6725 (Shulov and Belokon 1972), has not been confirmed by Landstreet, Angel, and Illing (1975). In this paper we will concentrate mainly on G195–19.

In LTE the equations of radiative transfer for polarized light can be written in the form (Landi Degl'Innocenti and Landi Degl'Innocenti 1975, hereafter referred to as Paper I):

$$\frac{d}{ds} \begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = - \begin{pmatrix} \eta_I & \eta_Q & \eta_U & \eta_V \\ \eta_Q & \eta_I & \rho_V & -\rho_U \\ \eta_U & -\rho_V & \eta_I & \rho_Q \\ \eta_V & \rho_U & -\rho_Q & \eta_I \end{pmatrix} \begin{pmatrix} I - B \\ Q \\ U \\ V \end{pmatrix}. \tag{1}$$

Assuming the magnetic field sufficiently low on the surface of the star ($H \ll 10^7$ gauss), the principle of the “rigidity” of the initial and final state wave functions can be applied to compute the bound-free opacities within the hydrogenic approximation (Lamb and Sutherland 1974).

Using the formalism developed in Paper I we find that only a few terms of the Mueller matrix above are different from zero to first order in the magnetic field. These can be related to the continuous absorption coefficient $\sigma(\omega)$ in the absence of a magnetic field as follows:

$$\eta_I(\omega) = \sigma(\omega),$$

$$\eta_Q(\omega) = \cos \psi(\omega) \left\{ -1 - \frac{1}{\sigma(\omega)} \frac{d\sigma}{d\omega} \right\} - \frac{1}{\exp (x) - 1} \left[ 1 + \frac{1}{2} \int (2 + ll'(l + 1) - l' l' + 1) \right], \tag{2}$$
where $\psi$ is the angle between the direction of the magnetic field vector and the line of sight, $z = \omega_l/\omega$, $y = \hbar \omega_l/kT$, $x = \hbar \omega_l/kT$, with the Larmor frequency $\omega_l = eB/2mc$. Finally $l$ and $l'$ are the quantum numbers of the orbital angular momentum of the photoionized electron in the bound and free states, respectively. Details on the derivation of equation (2) will be given elsewhere. Here we simply give the physical meaning of the various terms. The first term in braces is the magnetoemission term (Kemp 1970); the second, or "spectroscopic" term arises from the spectral dependence of the opacity (Kemp, Swedlund, and Evans 1970a); finally, the last two terms, which have been neglected in previous papers but are nevertheless important, arise from stimulated emission and from the differences in the thermal populations of the magnetic sublevels of the electronic bound state, respectively. For a magnetic field of $10^7$ gauss and a temperature of $10^4$ K at the wavelength $\lambda 5000$, we have, for instance, $z = 0.023$, $y = 0.067$, both quantities being multiplied by factors of the order of unity. Irrespective of the presence of Faraday rotation, we can solve the equations of radiative transfer for the two Stokes parameters $I$ and $V$ (Sazonov and Chernomordik 1974). Introducing the frequency-dependent optical depth $d\tau/\mu = -\sigma(\omega)ds$, and the quantity $k_\nu(\tau)$ (generally $\ll 1$) by the expression

$$H \cos \psi k_\nu(\tau) = \eta_\nu(\omega)/\sigma(\omega),$$

where $H$ is the intensity of the magnetic field, the emerging Stokes parameters at a particular point $P$ on the surface of the star are given by the expression (to first order in $k_\nu$)

$$I_\rho = \int_0^\infty B(\tau) \exp(-\tau/\mu)d\tau/\mu,$$

$$V_\rho = H \cos \psi \int_0^\infty (k_\nu(\tau) - \bar{k}_\nu(\tau)/\mu)B(\tau) \exp(-\tau/\mu)d\tau/\mu,$$

where $B$ is the local Planck function and $\bar{k}_\nu = \tau^{-1} \int_0^\infty k_\nu(\tau')d\tau'$; to obtain expressions (3), $H \cos \psi$ has been assumed constant with depth at the particular point $P$.

We now integrate on the visible hemisphere of the star assuming for the magnetic field geometry that due to a centered magnetic dipole (Babcock 1947); introducing the polar magnetic field intensity $H_p$ and denoting by $\gamma$ the angle between the line of sight and the magnetic axis (see Fig. 1), we obtain for the degree of circular polarization $q = V/I$:

$$q = q_0(\omega) \cos \gamma = \frac{1}{2}H_p \cos \gamma \int_0^\infty d\tau B(\tau)[k_\nu(\tau)G_1(\tau) - \bar{k}_\nu(\tau)G_2(\tau)]\left[\int_0^\infty d\tau B(\tau)E_n(\tau)\right]^{-1},$$

where

$$G_1(\tau) = 3E_4(\tau) - E_2(\tau), \quad G_2(\tau) = \tau(3E_4(\tau) - E_2(\tau)),$$

$E_n(\tau)$ being the $n$th exponential integral.

For a rotating star whose polar axis is inclined with respect to the magnetic axis, the quantity $q$ will change in time following the law:

$$q(\omega, t) = q_0(\omega)[\cos \delta \cos \epsilon + \sin \delta \sin \epsilon \cos (2\Tilde{\sigma}t)],$$

where $f = t/T$ is the phase and the meaning of the angles $\delta$ and $\epsilon$ is explained in Figure 1.

A time-varying $q$ of this form is just observed in the blue-green band for the magnetic white dwarf G195–19 (Angel, Illing, and Landstreet 1972); taking into account a difference in sign in the definitions of $q$, the light curve is reproduced in our model if $\tan \delta \tan \epsilon = 1.04$. For the reasons explained above, we cannot give, for the magnetic field $H_p$, more than a rough estimate; if we retain only the term proportional to $z$ in equation (2) to determine the circular polarization and assume a depth dependence of the temperature of the form $T = T_0(\frac{1}{2} + \frac{3}{2}T_0)^{1/4}$, where $T_0$ is the optical depth measured at $\lambda 4600$ (center of the blue-green filter), for $T_0 = 8000$ K (Greenstein, Gunn, and Kristian 1971), we obtain

$$H_p \cos \delta \cos \epsilon = (1 + \alpha)^{-1} \times 6.1 \times 10^6 \text{ gauss},$$

where $\alpha$ is the spectral index for the continuous absorption coefficient in that spectral range:

$$\sigma(\omega) \sim \omega^{-\alpha}.$$

The explanation just given for the time variation of the circular polarization in the blue-green band in terms of a purely thermal origin agrees with the absence of observed linear polarization (Angel and Landstreet 1971). In fact, as pointed out by Sazonov and Chernomordik (1974), radiation produced by the synchrotron mechanism in an intense coronal magnetic field ($\sim 3 \times 10^6$ gauss) would show linear polarization of the same order as the circular polarization.
Considering now the observations by Angel, Illing, and Landstreet (1972) in the red wave-bands ($\lambda\lambda 6000-7000$ and $\lambda\lambda 6000-8800$), we see that they cannot be reproduced in our simple model which predicts a time dependence of the circular polarization of the form given by equation (4) at all wavelengths, independently of the atmospheric model. A striking feature of these observations is that the scatter of measurements repeated over several periods is greater than that expected from counting statistics. This fact suggests in a very natural way the idea of a sort of "activity" going on in the atmosphere of G195-19. If we assume the presence on the surface of the star of a maximum magnetic field of the order of $2 \times 10^8$ gauss, the corresponding cyclotron emission would largely influence the measured circular polarization only at wavelengths longer than 6000 Å. On the other hand, the existence of localized high-field regions on magnetic white dwarfs has already been suggested by Landstreet and Angel (1975) for the interpretation of the polarization spectrum of Grw +70°8247 and by Angel et al. (1974) to account for the unusual polarization of G240–72.

We now develop the spot model assuming a circular spot whose radius $r$ is much smaller than the star radius $R$. The polarization properties of the light emerging from a particular point of the spot will be mainly determined by electronic free-free transitions in the intense magnetic field. For magnetic fields of the order of $10^8$ gauss, applying the methods of Paper I (see also Lamb and Sutherland 1974), we find

$$\eta_{1} = k_{e} + \eta_{0}(\omega - \Omega)(1 + \cos^{2}\psi), \quad \eta_{0} = -\eta_{0}(\omega - \Omega)\sin^{2}\psi\cos 2\psi,$$
$$\eta_{u} = -\eta_{0}(\omega - \Omega)\sin^{2}\psi\sin 2\psi, \quad \eta_{v} = -2\eta_{0}(\omega - \Omega)\cos\psi,$$
$$\rho_{0} = -\rho_{0}(\omega - \Omega)\sin^{2}\psi\cos 2\psi, \quad \rho_{u} = -\rho_{0}(\omega - \Omega)\sin^{2}\psi\sin 2\psi,$$
$$\rho_{v} = -2\rho_{0}(\omega - \Omega)\cos\psi,$$

where $k_{e}$ is the continuous absorption coefficient due to other opacity sources, $\eta_{0} = n_{e}v_{e}^{2}e^{2}/mc$, $n_{e}$ being the electronic number density, $\eta(\omega - \Omega)$ is the normalized function describing the absorption line profile, centered around the cyclotron frequency $\Omega$, due to electronic transitions between successive Landau levels $\int [\eta(\omega - \Omega)ds = 1]$, $\rho(\omega - \Omega)$ is the corresponding anomalous dispersion profile, and, finally, $\psi$ and $\varphi$ are the angles describing the direction of the spot magnetic field vector (see Fig. 1); if this direction can be considered constant with depth, the equations of radiative transfer can be solved exactly and the emerging Stokes parameters can be expressed by

$$I = J_{1} + J_{2}, \quad Q = -\Lambda_{1}(J_{1} - J_{2}),$$
$$U = -\Lambda_{2}(J_{1} - J_{2}), \quad V = -\Lambda_{3}(J_{1} - J_{2}),$$

where

$$\Lambda_{1} = \sin^{2}\psi\cos 2\varphi(1 + \cos^{2}\psi), \quad \Lambda_{2} = \sin^{2}\psi\sin 2\varphi(1 + \cos^{2}\psi), \quad \Lambda_{3} = 2\cos\psi(1 + \cos^{2}\psi),$$

and

$$J_{1} = \frac{1}{2}\int_{-\infty}^{0} B(s)[2\eta(s) - k_{e}(s)]\exp\left\{ -\int_{s}^{0} [2\eta(s') - k_{e}(s')]ds'\right\}ds,$$
$$J_{2} = \frac{1}{2}\int_{-\infty}^{0} B(s)k_{e}(s)\exp\left\{ -\int_{s}^{0} k_{e}(s')ds'\right\}ds,$$

where $s$ is the coordinate measured along the ray path, $s = 0$, corresponding to the outermost layers of the star.

As $\eta_{1} \gg k_{e}$ (Lamb and Sutherland 1974), the main contribution to the integral $J_{1}$ comes from the exterior layers of the star; assuming then the existence of a corona having an electronic temperature $T_{e}$, we can write

$$J_{1} \sim B(T_{e})N_{e}\frac{\pi^{2}e^{2}}{\mu mc} \eta(\omega - \Omega), \quad J_{2} \ll J_{1},$$

where $N_{e}$ is the coronal column density measured at the center of the star while the factor $1/\mu$ (limb brightening) is due to geometrical effects. Incidentally, we want to remark here that the possibility of the existence of a corona for white dwarfs has already been suggested on the basis of independent considerations by Böh and Cassinelli (1971), by Strittmatter, Brecher, and Burbidge (1972), and by Sazonov and Chernomordik (1974).

We now integrate on the spot assuming a radial magnetic field and a continuous intensity distribution ranging from a maximum value, corresponding to a cyclotron frequency $\Omega_{0}$, to a much lower value on the borders of the spot. The contribution to $V$ arising from the spot will then be

$$V_{\text{spot}} \sim -B(T_{e})N_{e}\frac{\pi^{2}e^{2}}{mc\Omega_{0}}\pi r^{2}\Lambda_{g}(\Lambda_{g})$$

© American Astronomical Society • Provided by the NASA Astrophysics Data System
Fig. 1.—Geometrical relations among the line of sight, the magnetic field vector and the couple of unit vectors \((e_a, e_b)\) defining the Stokes parameters. \(P\) is the pole, \(M\) the magnetic pole, and the dotted area represents the spot.

where \(\theta\) is the step-function; and the degree of circular polarization at a frequency \(\omega' < \Omega_0\) will be expressed by

\[
q(\omega', t) = q_0(\omega') \cos \gamma + q_{\text{spot}} \cos \psi \theta(\cos \psi)/(1 + \cos^2 \psi),
\]

where the term \(q_0(\omega')\) is the photospheric term analogous to the one introduced in equation (4) and

\[
q_{\text{spot}} \approx \frac{B(T_c)}{B(T' = 2/3)} \frac{N_e \pi^2 e_0^2}{mc \Omega_0} \left(\frac{r'}{R}\right)^2,
\]

with \(r'\) the optical depth in the continuum at the frequency \(\omega'\). For a temperature \(T\), at \(\tau' = \frac{2}{3}\), still equal to 8000 K and a spot field equal to \(2 \times 10^8\) gauss we obtain numerically at \(\lambda 7000\):

\[
q_{\text{spot}} = -1.38 \times 10^{-6} N_e T_c (r/R)^3,
\]

with \(N_e\) expressed in cm\(^{-2}\) and \(T_c\) in K. To obtain equation (5), we have neglected the contribution to \(q\) arising from the corona outside the spot column. This is justified as the magnetic field \((\sim 10^6\) gauss) is sufficiently low to exclude synchrotron radiation at optical wavelengths unless relativistic electrons are invoked in the corona (Bekefi 1966). The temporal behavior of \(q(\omega', t)\) deduced from equation (5) is of the form

\[
q(\omega', t) = a(1 + 1.04 \cos (2\pi f)) + b \cos \psi \theta(\cos \psi)/(1 + \cos^2 \psi),
\]

where

\[
a = -q_0(\omega') \cos \delta \cos \epsilon,
\]

\[
b = -q_{\text{spot}},
\]

\[
\cos \psi = \cos \delta \cos \epsilon + \sin \delta \sin \epsilon \cos (2\pi f - \varphi_0),
\]

and the meaning of the angles \(\epsilon\) and \(\varphi_0\) is explained in Figure 1. The best fit to the observations is reproduced in Figure 2 and is obtained for the following values of the single parameters: \(a = 0.0041, b = 0.0174, \delta = 35^\circ, \epsilon = 75^\circ, \varphi_0 = 205^\circ\). From these values we can further obtain: \(\epsilon = 56^\circ, H_p \sim 1.3 \times 10^7(1 + \omega)^{-1}\) gauss, \(q_0(\omega')/q_0(\omega) = 1.77, N_e T_c (r/R)^3 = 1.3 \times 10^{18}\) cm\(^{-2}\) K. The increase of the circular polarization due to thermal radiation with decreasing wavelength, expressed by the ratio \(q_0(\omega')/q_0(\omega)\), is not surprising as it is observed also in other magnetic white dwarfs like G99-37 (Landstreet and Angel 1971); on the other hand, the value for the product \(N_e T_c (r/R)^3\) gives some extra information on the physical properties of the corona of G195-19. Concerning the linear polarization, the values expected from our model at wavelengths longer than 6000 \(\AA\) are reproduced in Figure 3. The maximum value \((0.87\%_o)\) for \(P_L = (Q^2 + U^2)^{1/2}/I\) are obtained at phases 0.26 and 0.88, corresponding to the times of appearance and disappearance of the spot.

Observations of this kind would be crucial in proving the correctness of our model, but as far as we know, they have not been performed in this spectral range. On the other hand, the observations by Angel and Landstreet (1971) in the band \(\lambda\lambda 3800-6000\) are consistent with our model which predicts a value of the linear polarization of the order of 0.01 percent. Before accepting the spot model for G195-19 one should examine alternative explanations. A possibility would be the presence of a centered-dipole field of the order of \(2 \times 10^8\) gauss that, according
Fig. 2.—The degree of circular polarization at $\lambda > 6000$ Å, computed according to eq. (5), is plotted versus the phase for G195–19; the vertical bars are observational data (Angel et al. 1972).

to Sazonov and Chernomordik (1974), would give rise to a circular polarization, mainly of coronal origin, proportional to $\cos \gamma$. Since in this case the temporal variation of the circular polarization would still be given by equation (4), in disagreement with what is observed, this first possibility can be ruled out. One could then wonder whether an appropriate combination of photospheric (thermal) and coronal radiation could be able to reproduce the observations. In this case the circular polarization would be

$$q = \frac{V_{\text{cor}} + V_{\text{phot}}}{I_{\text{cor}} + I_{\text{phot}}} \sim \frac{V_{\text{cor}} + V_{\text{phot}}}{I_{\text{phot}}}.$$

On the other hand, the thermal radiation originating in a magnetic field of the order of $10^8$ gauss would necessarily be strongly polarized, as can be argued on general physical grounds, being $\omega \sim \omega_L$. Since $V_{\text{phot}}$ would then be a relevant function of $I_{\text{phot}}$, a rather high value of $q$ would be expected, again in disagreement with the observations which give values of $q$ of the order of 1 percent.

In conclusion, the argument presented in this paper can be considered, in our opinion, as a piece of evidence as to the existence of activity on some magnetic white dwarfs. From this point of view the irregular variability of GD 229 could be easily interpreted as the result of a large number of spots appearing and disappearing on the surface of the star itself. We think that further observations on G195–19 and GD 229, taken at time intervals of the order of a few months or years, may perhaps bring to the discovery of a cycle of activity on magnetic white dwarfs.
Fig. 3.—The degree of linear polarization expected for G195-19 at $\lambda > 6000$ Å is plotted with the phase as parameter (numbers along the curve). $Q$ is defined as the linearly polarized light in the plane containing the line of sight and the stellar polar axis, minus the linearly polarized light in the orthogonal direction. $U$ is defined accordingly.

The author is indebted to Dr. C. Chiuderi and Dr. M. Landi Degl’Innocenti for helpful and stimulating discussions.

REFERENCES


E. LANDI DEGL’INNOCENTI: Osservatorio Astrofisico di Arcetri, Largo Enrico Fermi, 5, 50125 Firenze, Italy