THE Mg II h AND k LINES. II. COMPARISON WITH SYNTHESIZED PROFILES AND Ca II K

T. R. Ayres and J. L. Linsky*

Joint Institute for Laboratory Astrophysics, University of Colorado and National Bureau of Standards

Received 1975 August 11; revised 1975 October 15

ABSTRACT

We compare the Mg II h and k resonance line data of Kohl and Parkinson and profiles of the Ca II K line with synthetic spectra computed using a partial redistribution formalism and several single-component solar upper photosphere and lower chromosphere models. We find that the HSRA and Vernazza et al. (VAL) models predict systematically lower intensities in the h, k, and K inner wings than are observed, but that models with a somewhat larger minimum temperature \( T_{\text{min}} \approx 4450 \) K, such as proposed previously by Shine et al., can reproduce the measured inner wing intensities and limb darkening of these resonance lines. Upper photosphere temperature distributions with \( T_{\text{min}} \approx 4450 \) K are also more consistent with the radiative equilibrium models of Athay (non-LTE) and Kurucz (LTE), when nonradiative energy dissipation is taken into account. We propose a "hot" \( T_{\text{min}} \) solar model, which is reasonably consistent with the empirical emission cores and wing intensities of the Ca II and Mg II resonance lines, to serve as an alternative to the class of models, such as the HSRA and VAL, based on continuum observations.

Subject headings: line profiles — Sun: atmosphere

I. INTRODUCTION

Recently, Kohl and Parkinson (1976, hereafter Paper I) have described the center and limb measurements and absolute calibration of high-dispersion profiles of the solar Mg II h (2803 Å) and k (2796 Å) resonance lines obtained by a rocket-borne photoelectric spectrometer. In this paper we compare these measured center and limb profiles of h and k with synthetic spectra based on several solar upper photosphere and lower chromosphere temperature distributions. In addition, we synthesize profiles of the analogously formed Ca II K (3934 Å) resonance line for the same atmospheric models. In both cases we adopt the so-called "partial redistribution" line formation formalism of Milkey and Mihalas (1973a, b) and Milkey, Shine, and Mihalas (1975), although we reformulate their approach for the sake of numerical tractability.

a) Previous Studies of the Mg II and Ca II Resonance Lines

The importance of the Mg II and Ca II resonance lines for inferring chromospheric structure in the Sun and other late-type stars is well established (Dumont 1967, 1969; Athay and Skumanich 1968; Linsky and Avrett 1970; Lemaire and Skumanich 1973; Kondo et al. 1972; Ayres, Linsky, and Shine 1974; Ayres and Linsky 1975a). In short: (1) The doubly reversed emission cores of Ca II H and K and Mg II h and k are sensitive to the temperature, density, and microvelocity gradients of the lower chromosphere. (2) The Mg II resonance lines can be better indicators of these quantities than H and K, owing to the larger opacities of the former, and hence smaller geometrical thermalization lengths in the chromosphere. For this reason, the solar h and k lines usually show broader emission profiles and larger \( k_\text{s}/k_\text{e} \) emission contrasts than the Ca II resonance lines. (3) The extensive damping wings of the Ca II and Mg II lines are formed throughout the solar photosphere, and can be good indicators of upper photospheric structure, especially in the vicinity of the temperature minimum.

Unfortunately, previous studies of the solar h and k emission features have either been based on the physically inappropriate "complete redistribution" (CRD) approximation (Dumont 1967; Lemaire and Skumanich 1973) or have been based on the more realistic "partial redistribution" (PRD) approach (Milkey and Mihalas 1974) but mainly in an illustrative sense. In fact, part of the motivation in the Milkey and Mihalas analysis for not deriving a semiempirical model, per se, can be traced to uncertainties in the absolute calibration of the earlier available profiles of h and k (Bates et al. 1969; Lemaire and Skumanich 1973) and uncertainties in fundamental atomic parameters for Mg II, in particular the van der Waals broadening coefficients. The latter play a central role in determining the extent to which partial coherence effects are important in the h and k inner wings (see discussion by Milkey and Mihalas 1974). However, such objections to deriving semiempirical solar models based on the Mg II resonance lines have now largely been resolved owing to the availability of high-quality, accurately calibrated

* Staff Member, Laboratory Astrophysics Division, National Bureau of Standards.
profiles of \( h \) and \( k \) (e.g., Paper I) and the use of semiempirical techniques to estimate van der Waals parameters for resonance lines (Shine 1973; Ayres 1975a).

b) Semiempirical Solar Models: The Upper Photosphere, Temperature Minimum, and Lower Chromosphere

Recently, Vernazza, Avrett, and Loeser (1976) have proposed an upper photosphere and lower chromosphere temperature model (hereafter VAL) to fit a wide variety of ultraviolet, visible, and infrared solar continuum data. Their model is intended to replace previous reference solar models, such as the BCA (Gingerich and de Jager 1968) and HSRA (Gingerich et al. 1971), which were constructed similarly, but were based on a smaller sample of continuum data and a more restrictive set of physical assumptions (Vernazza, Avrett, and Loeser 1973, 1976). Both the HSRA and VAL models are characterized by “cool” temperature minima (\( T_{\text{min}} \approx 4170 \, \text{K} \) and \( 4150 \, \text{K} \), respectively), compared with the value suggested by Athay’s (1970) non–LTE radiative equilibrium (RE) model (\( T_{\text{min}} \approx 4330 \, \text{K} \pm 150 \, \text{K} \)). Significantly, the pioneering partial redistribution synthesis of the solar Mg \( \Pi \) \( h \) and \( k \) and Ca \( \Pi \) \( H \) and K lines (Milkey and Mihalas 1974; Shine, Milkey, and Mihalas 1975) implied that a minimum temperature larger than \( T_{\text{min}} \approx 4150 \, \text{K} \) is required to fit the measured absolute intensities of the \( k \) and \( K \) minimum features, which are formed near \( T_{\text{min}} \). In fact, Shine, Milkey, and Mihalas (1975) have proposed a minimum temperature in the solar atmosphere of \( 4450 \, \text{K} \) based on the observed disk center \( K \) intensity, a five-level-plus-continuum representation of Ca \( \Pi \) with partial redistribution, and a somewhat schematic modification to the HSRA semiempirical model.

The major intent of this paper is to compare synthetic spectra with observed profiles of the Mg \( \Pi \) (Paper I) and Ca \( \Pi \) (Brault and Testerman 1972) resonance lines using a variety of possible solar temperature models in an attempt to clarify this apparent \( T_{\text{min}} \) controversy. We synthesize both disk center and limb profiles in order to more completely model the atmosphere, and to help identify possible multicomponent effects, if present.

This paper is sectioned as follows: In § II we formulate the spectrum synthesis approach for Ca \( \Pi \) K and Mg \( \Pi \) \( h \) and \( k \) and discuss the collisional and fixed radiative rates appropriate to the adopted model atoms and the solar atmosphere. In § III we describe the atmospheric models upon which we base the spectrum synthesis of § IV. Finally, in § V, we comment on the implications of the comparison between the measured and theoretical intensities of the Mg \( \Pi \) and Ca \( \Pi \) resonance lines for deriving models of the solar upper photosphere and lower chromosphere.

II. Atomic Models, Spectrum Synthesis, Fixed Rates

In this section we describe the spectrum synthesis formalism for the Ca \( \Pi \) and Mg \( \Pi \) resonance lines. Our approach to the non–LTE partial redistribution problem is similar to that proposed recently by Heasley and Kneer (1975) and is a straightforward generalization of the LTE partial coherent scattering approximation appropriate to the Mg \( \Pi \) and Ca \( \Pi \) damping wings described by Ayres (1975a).

a) Ca \( \Pi \)

i) Atomic Model: Three Levels plus Continuum

The Ca \( \Pi \) model we consider here consists of a ground state (level 1; \( 4s^{2}S_{1/2} \)), a radiating state (level 3; \( 4p^{2}P_{3/2} \)), and an intermediate metastable state (level 2; \( 3d^{2}D_{5/2} \)). In addition, we include the Ca \( \Pi \) continuum (Ca \( \Pi \)), and neglect all other stages of ionization. The permitted radiative transitions in this model atom are \( 3 \rightarrow 1 \) (the 3934 Å \( K \) line) and \( 3 \rightarrow 2 \) (the 8542 Å subordinate infrared triplet line), while collisions couple all three levels and the continuum.

The three-level-plus-continuum, two-transition approach adopted here is less accurate than the five-level-plus-continuum, five-transition approach used by Shine, Milkey, and Mihalas (1975). However, the difference in computed profiles is significant only at the \( H_{\alpha} \) and \( K_{\alpha} \) features, while the computing time required for the five-level representation of Ca \( \Pi \) is increased by an order of magnitude over that required for the three-level solutions.

ii) The PRD Formalism for Ca \( \Pi \)

Following Milkey, Shine, and Mihalas (1975), we divide the radiating state 3 of the Ca \( \Pi \) model atom into sublevels \( 3_{1} \) and \( 3_{2} \). The two distinct sublevels represent a partition of the radiating state according to the fraction of excited atoms that spontaneously decay by emitting a photon into line transition \( 3 \rightarrow 1 \) or \( 3 \rightarrow 2 \), respectively.

Each of the sublevels is further divided into frequency substates having a population distribution \( n_{3_{1}}(v) \) where

\[
\int_{-\infty}^{\infty} n_{3_{1}}(v) dv = n_{3_{1}}.
\]

Here \( n_{3_{1}} \) is the total population of the sublevel \( 3_{1} \).

\( ^{1} \) In what follows, we neglect stimulated emissions.
The coarse partition of the radiating state into sublevels is made in order to properly account for "cross" (i.e., fluorescent) scatterings of the form \(1 \rightarrow 3 \rightarrow 2\) and \(2 \rightarrow 3 \rightarrow 1\) (the "direct" scatterings are \(1 \rightarrow 3 \rightarrow 1\) and \(2 \rightarrow 3 \rightarrow 2\)). The fine division of each sublevel into substate bands is made in order to account for the effects of the redistributed radiation on the population distribution within the particular sublevel.

A. The emission profile. For a given frequency \(\nu\) in transition \(3 \rightarrow l\), the monochromatic absorptivity is

\[
\chi_{\nu}^{(i)} = \sigma_{13} n_{13} + X_{\nu},
\]

where \(\sigma_{13} \equiv (4\pi e^2/mc) f_{13}\) is the total line absorption cross section (\(f_{13}\) is the oscillator strength); \(\phi_{\nu}\) is the normalized absorption profile (taken to be the ordinary Voigt function); and \(X_{\nu}\) is the background opacity.

The emissivity is

\[
\eta_{\nu}^{(i)} = \frac{2h\nu^3}{c^2} \frac{g_3}{g_\nu} \sigma_{13} n_{13}(\nu) + E_{\nu},
\]

where \(g_{13}\) is the statistical weight of sublevel \(3\), and \(E_{\nu} = X_{\nu} B_{\nu}\) is the LTE background emissivity.

We rewrite the substate population distribution \(n_{13}(\nu)\) in terms of a normalized emission profile \(\psi_{\nu}\):

\[
n_{13}(\nu) = n_{13} \psi_{\nu},
\]

with

\[
\int_{-\infty}^{\infty} \psi_{\nu} d\nu = 1.
\]

In complete redistribution, the emission profile \(\psi_{\nu}\) is identical to the absorption profile \(\phi_{\nu}\) (Mihalas 1970). Because the actual departure of the PRD emission profile from the CRD emission profile is likely to be small compared with the strong frequency dependence of \(\phi_{\nu}\), we introduce an emission factor \(\beta_{\nu}\) which measures the departure of the true emission profile from what otherwise would obtain in complete redistribution. In other words

\[
\beta_{\nu} \equiv \psi_{\nu}/\phi_{\nu}.
\]

B. Solution of the transfer equation. The total emissivity \(\eta_{\nu}^{\text{total}}\) for transition \(3 \rightarrow l\) is given by the first term on the right-hand side of equation (3) with \(n_{13}(\nu)/g_{13}\) replaced by \(n_{3}/g_{3}\), where \(n_{3}\) is the total population of level \(3\). The monochromatic emissivity can be written in terms of the total emissivity and the emission profile

\[
\eta_{\nu} = \beta_{\nu} \phi_{\nu} \eta_{\nu}^{\text{total}} + E_{\nu}.
\]

We write the second-order differential form of the transfer equation

\[
\frac{d^2}{dr_{\nu}^2} (f J_{\nu}) = J_{\nu} - \eta_{\nu}/X_{\nu}
\]

in terms of the mean intensities \(J_{\nu}\) and variable Eddington factors \(f_{\nu}\) (Auer and Mihalas 1969). We solve the linearized differenced representation of equation (8) by the scheme described by Auer, Heasley, and Milkey (1972), except now we introduce the emission factors \(\beta_{\nu}\) into the monochromatic line emissivities.

C. Explicit representation of \(\beta_{\nu}\). By manipulating the statistical equilibrium equation satisfied by the substate band \(n_{13}(\nu) d \nu\) (see Milkey, Shine, and Mihalas 1975, their eq. [7]), we obtain an explicit representation of the emission factor \(\beta_{\nu}\) for transition \(3 \rightarrow l\) in terms of the line radiation fields and integrated level populations

\[
\beta_{\nu}^{(i)} = \left\{ \sum_{l < 3} n_{i} \left[ C_{l3} + (B_{l3}/p_{l3}) \int_{-\infty}^{\infty} R^{(l')(\nu, \nu')} J_{l'}(\nu') d\nu' \right] + n_{3}(R_{e3} + C_{3k}) \right\} /
\]

\[
\left\{ \sum_{l < 3} n_{i} \left[ C_{l3} + B_{l3} \int_{-\infty}^{\infty} \phi_{\nu} J_{l'}(\nu') d\nu' \right] + n_{3}(R_{e3} + C_{3k}) \right\}.
\]

Here \(B_{l3} = (4\pi h/\nu_{l3}) p_{l3}\) is the induced absorption rate coefficient; the \(C_{il}\) are inelastic collision rates; \(R_{e3}\) is the radiative recombination rate; and \(R^{(l')(\nu, \nu')}\) is a redistribution function describing the joint probability in the observer's frame of an atom absorbing a photon at frequency \(\nu'\) in transition \(l' \rightarrow 3\) and then reemitting at frequency \(\nu\). (Note: the ordering convention of \(\nu\) and \(\nu'\) here is reversed from that adopted by Hummer 1962.) For transitions between a sharp lower level and a broadened upper level which are coherent in the atom's frame, the appropriate redistribution function is (Milkey, Shine, and Mihalas 1975)

\[
R^{(l')(\nu, \nu')} = (1 - \Lambda) \left( \begin{array}{c} \frac{R_{p3}^{(l')(\nu, \nu')} (l' = l)}{R_{p3}^{(l')(\nu, \nu')} (l' = l)} \\ \frac{R_{e3}^{(l')(\nu, \nu')} (l' \neq l)}{R_{e3}^{(l')(\nu, \nu')} (l' \neq l)} \end{array} \right)
\]

\[
+ \Lambda R^{(l)(\nu, \nu')}.
\]
where \( R_D^{II} \) and \( R_D^{III} \) are the redistribution functions described by Hummer (1962); \( R_{x}^{II} \) is the "cross" redistribution function of Milkey, Shine, and Mihalas (1975); and \( \Lambda \) is the incoherence fraction

\[
\Lambda = \frac{\Gamma_E}{\Gamma_r + \Gamma_E + \Gamma_{rad}},
\]

as described by Milkey and Mihalas (1973a). Here \( \Gamma_{rad} \) is the radiation damping width, which is depth-independent; \( \Gamma_E \) is the elastic damping parameter, which has a depth dependence of the form (van der Waals broadening)

\[
\Gamma_E = \gamma_{vw}(T_e/5000)^{0.3} n_{HI},
\]

where \( n_{HI} \) is the neutral hydrogen density; and \( \Gamma_r \) is the the inelastic collision width, which we ignore because it is usually negligible compared with \( \Gamma_{rad} \) or \( \Gamma_E \) (Ayres 1975a).

In addition, following the usual practice, we approximate \( R^{III} \) with complete redistribution

\[
R^{III}(v, v') = \phi v v'.
\]

The normalization condition for each of the redistribution functions is

\[
\int_{-\infty}^{\infty} R(v, v') dv' = \phi v v'.
\]

Notice from equation (9) that \( \beta_v = 1 \) in the limit of complete redistribution [i.e., \( R^{III}(v, v') \equiv \phi v v' \)] independent of the frequency dependence of the radiation field. This would not have been the case had we included stimulated emissions owing to the residual coherence introduced by the induced emission process.

In practice, we replace the integrals appearing in equation (9) with quadrature sums. The redistribution weights entering into these quadratures are interpolated from tables representing the appropriate redistribution functions (i.e., "cross" or "direct" \( R^{II} \)). We use the Adams, Hummer, and Rybicki (1971) \( R^{II} \) generator as modified by Shine, Milkey, and Mihalas (1975) to construct the redistribution weights tables.

In order to assure numerical accuracy, we also interpolate the complete redistribution components of the total redistribution function, as well as the CRD weights appearing in the denominator of equation (9), from tables constructed identically to the PRD weights tables.

**iii) Fixed Rates for Ca II**

We adopt the atomic parameters, collision rates, and photoionization cross sections for Ca II given by Shine and Linsky (1974), except for the van der Waals broadening coefficient \( \gamma_{vw}(K) \) (see below). The Shine and Linsky (1974) value for the important resonance collisional excitation \( 1 \rightarrow 3 \) was taken from the experimental work of Crandall et al. (1974). In addition, we assume a solar calcium abundance of \( A_{Ca}^{(0)} = 2.14 \times 10^{-6} \pm 25 \) percent (\( A_{HI} = 1.0 \) based on the work of Lambert and Warner (1968), Lambert, Mallia, and Warner (1969), Withbroe (1971), and Holweger (1972).

Because the photoionization rates from the \( 4s, 3d, \) and \( 4p \) levels are dominated by ultraviolet background radiation fields, we can estimate these rates (actually, the "surface" values) by numerically integrating the adopted photo cross sections and measured intensities of the solar EUV spectrum (Dupree and Reeves 1971). In the two cases for which \( \Lambda \) lies within the photoionization bandpass (Ca II \( 4p \) and \( 3d \)), \( \Lambda \) accounts for approximately 80 percent of the total rate. In the remaining case, the \( 4s \) ground state, the \( H i \) Lyman continuum accounts for nearly half of the total rate. In order to simulate the actual thermalization behavior of the background ultraviolet radiation fields, we set the photoionization rates equal to their empirical surface values in the chromosphere and equal to the local thermal values (\( J_e = B_e \)) in the upper photosphere. Our results are relatively insensitive to the location at which the ultraviolet radiation fields are assumed to be thermal. Although the integrated intensities and profiles of the synthesized Ca II lines are somewhat sensitive to the adopted photo-rates, the effect of even a factor of 2 error in the solar EUV background radiation fields is considerably less than that attributable to other sources, especially the solar calcium abundance.

In order to derive \( \gamma_{vw}(K) \) we adopt the Vernazza, Avrett, and Loeser (1976) and Kurucz (1974) lower photosphere models to establish the temperature structure below \( \tau_{5000} = 0.1 \), where the wing shapes of \( K \) and \( k \) are no longer reliable atmospheric probes, but provide important constraints on possible values of the abundances and elastic damping parameters. In particular, we require that for a given value of \( A_{Ca} \), \( \gamma_{vw}(K) \) be chosen to force agreement between synthesized and measured disk center intensities at \( +6.1 \) Å redward of K line-center (\( f_{K} = 1.06 \times 10^{-6} \) ergs cm\(^{-2}\) s\(^{-1}\) sr\(^{-1}\) Hz\(^{-1}\)). This particular interval in the K line damping wing is formed only about 20 km above \( \tau_{5000} = 1 \) (Liu and Skumanich 1974) and does not appear to be strongly affected by nearby absorption lines.
For the adopted solar calcium abundance, \( A_{\text{Ca}}^{(0)} = 2.14 \times 10^{-6} \), we infer \( \gamma_{\text{Ca}}^{(0)}(K) = 1.8 \times 10^{-8} \) for the VAL and \( 1.2 \times 10^{-8} \) for Kurucz's model. In addition, we find that the ±0.1 dex uncertainty in \( A_{\text{Ca}} \) corresponds to a ±0.1 dex uncertainty in \( \gamma_{\text{Ca}}^{(0)}(K) \) for both “basis” temperature distributions. The difference between the “VAL” and “Kurucz” \( \gamma \)'s is attributable to the differences in temperature structure between the VAL and Kurucz models in that portion of the lower photosphere where \( f_{\text{b}2} \) is formed.

\section*{b) Mg II}

\subsection*{i) Atomic Model: Three Levels plus Continuum}

We adopt a representation for Mg II consisting of three bound levels and the continuum (Mg iii). As with Ca ii, we ignore all other stages of ionization. The Mg ii model includes a ground state (level 1; \( 3s^2S_{1/2} \)) and two radiating states (levels 2 and 3; \( 3p^2P_{1/2} \) and \( 3p^2P_{3/2} \), respectively). The permitted transitions are \( 3 \rightarrow 1 \) (the 2796 Å \( k \) line) and \( 2 \rightarrow 1 \) (the 2803 Å \( h \) line). As in the Ca ii model, collisions couple all three levels and the continuum.

\subsection*{ii) The Mg II Emission Factors}

The Mg ii system described here is much simpler than the Ca ii system described above owing to the lack of intermediate metastable states analogous to Ca ii \( 3d^2D_{5/2} \) and \( 3d^2D_{3/2} \). For the Mg ii case, the sublevel partition introduced above is unnecessary because there is only one permitted radiative channel for each radiating state. The emission factor \( \beta_{\text{Mg}}(v) \) for transition \( u \rightarrow 1 \) reduces to

\[
\beta_{\text{Mg}}(v) = \sum_{i=1}^{3} n_i C_i + n_1 (B_{1W}/\phi_i) \int_{-\infty}^{\infty} R^{0\nu}(v', v') J_{\nu}(v') dv' + n_u^*(v_R + C_{\text{Mg}}),
\]

where \( u = 2, 3 \). (Note: the Mg ii resonance lines are denoted by the upper level \( u \) of the particular transition, in contrast to the Ca ii \( K \) and \( \lambda 8542 \) case where the upper levels are shared and the lower levels are distinct.)

\subsection*{iii) Fixed Rates for Mg II}

We adopt the atomic parameters, collisional rates, and photoionization cross sections given by Shane (1973), with the exception of \( \gamma_{\text{Mg}}(h, k) \). In addition, we assume a solar magnesium abundance of \( A_{\text{Mg}}^{(0)} = 3.89 \times 10^{-5} \pm 15 \) percent \( (A_p = 1.0) \). This abundance is based on the same sources cited for the calcium abundance in § II a(iii) above with the addition of Holweger (1971). As with Ca ii, we estimate semiempirical values for the Mg ii photoionization rates by numerically integrating the adopted photo cross sections and calibrated solar EUV intensities. Mg ii \( h \) and \( k \) are much less sensitive to photoionization than the Ca ii lines, because the Mg ii \( 3 \) and \( 3p \) edges lie shortward of the strong solar \( L \alpha \) emission.

Unfortunately, we cannot determine \( \gamma_{\text{Mg}}^{(0)}(h, k) \) reliably by the approach cited above for \( \gamma_{\text{Ca}}^{(0)}(K) \) owing to the severe line blanketing in the far wings of the Mg ii lines. Therefore, we scale \( \gamma_{\text{Mg}}^{(0)}(K) \) of the previous section to Mg ii following Unsöld (1955; see Shane 1973). We find \( \gamma_{\text{Mg}}^{(0)}(h, k) = 1.0 \times 10^{-8} \) and \( 1.6 \times 10^{-8} \) for the Kurucz and VAL models, respectively. Both \( \gamma \)'s are consistent with the values necessary to reproduce the estimated disk center specific intensity at 9.5 Å shortward of \( k \) line-center, \( I_{k=9.5} \approx 1.3 \times 10^{-6} \) ergs cm\(^{-2}\) s\(^{-1}\) sr\(^{-1}\) Hz\(^{-1}\), for the adopted Mg abundance \( A_{\text{Mg}}^{(0)} = 3.89 \times 10^{-5} \), and for the particular basis models.

\section*{III. MODEL ATMOSPHERES}

The solar models we construct below are based on the assumptions of plane-parallel geometry, hydrostatic equilibrium, lateral homogeneity, and LTE ionization equilibrium for all elements except hydrogen. These assumptions are made so that we might readily obtain solutions for what otherwise would be an intractable numerical problem. Our hope is that simple, semiempirical models constructed to fit “average” profiles, but properly taking partial redistribution effects into account, can provide some insight into global properties of the solar atmosphere, for instance the average heating and cooling rates in the lower chromosphere. As a justification for the analysis of spatially unresolved Mg ii data, we note that the average network and cell profiles of Lemaire and Skumanich (1973) are very similar to their quiet Sun average profiles. These “average Sun” models should also provide a foundation for the next generation of multicomponent solar models, when the necessary observations and interpretive techniques become available.

\subsection*{a) Ionization Balance}

We adopt a straightforward method to construct self-consistent density runs of neutral hydrogen, protons, and electrons from a given distribution of temperature (K) and microvelocity (km s\(^{-1}\)) versus mass column density (g cm\(^{-2}\)). The calculation is based on the complete linearization method of Auer, Heasley, and Milkey (1972) specialized to a three-level-plus-continuum representation of hydrogen. In this model atom, the Lyman continuum and Hα are solved explicitly: \( L_\alpha \) and \( L_\beta \) are assumed to be in radiative detailed balance; and the Balmer and Paschen continua are treated in an optically thin approximation by means of prespecified fixed rates. For simplicity, the equations of ionization balance in the model atmosphere and statistical equilibrium of the model
atoms are solved iteratively rather than linearized. Following Vernazza, Avrett, and Loeser (1973), we allow for turbulent pressure support by including the appropriate $P_t = \frac{1}{2} \alpha \varphi^2$ term in the hydrostatic equilibrium constraint.

Near the base of the chromosphere (4500–6500 K) where the majority of the Ca II and Mg II resonance line emission is formed, Lα and Lβ have line-center optical depths of $10^7-10^{10}$ and $10^6-10^9$, respectively. Although these optical depths are not quite sufficient for detailed balance to hold precisely, the assumption of detailed balance is adequate for determining the total electron densities. The errors in the derived electron densities in the lower chromosphere attributable to errors in the detailed balance assumption are probably less than the errors attributable to the assumption of LTE ionization equilibrium for the metals (see Vernazza, Avrett, and Loeser 1973).

The collisional excitation and ionization rates and fixed photoionization rates appropriate to this model hydrogen atom and the solar chromosphere are described by Ayres and Linsky (1975b). Incidentally, our approach is very similar to that adopted by Noyes and Kalkofen (1970) in their analysis of the solar Lyman continuum.

**b) Adopted Solar Models**

The semiempirical solar temperature-mass distributions adopted for the spectrum synthesis of § IV below are depicted in Figure 1a, and the associated microvelocity models are illustrated in Figure 1b. In addition to the HSRA and VAL models previously described, we have included a small grid of schematic models from Ayres's (1975b) study of stellar chromospheres. These latter models were constructed relative to Kurucz's radiative equilibrium (RE) model photosphere and flux averaged profiles of the solar K line. We include this grid of schematic chromosphere temperature distributions because it is characteristic of the “hot” temperature minimum class of solar models. In § IVd we will propose variants on this grid of models, but constructed relative to the VAL lower photosphere, to optimally match the Ca II and Mg II specific intensitic data.

The individual models in the solar-type “stellar” grid are characterized by (1) constant gradients of temperature and microvelocity ($dT/\log m = \text{constant}$) in the lower chromosphere; (2) constant $\xi_t$ in the upper photosphere, and (3) an upper photosphere temperature structure determined by trial and error synthesis of the Ca II K damping wings. In practice, we fix the microvelocity, temperature, and mass column density at the temperature minimum ($\xi_t = 2 \text{ km s}^{-1}$; $T_{\text{min}} = 4350 \text{ K}$; $\log m_r = -1.50$ for the Sun), and fix the temperature at $T_{\text{Ly0}} = 1$ at $T_0 = 8000 \text{ K}$ as described by Ayres and Linsky (1975a, b). Therefore, each model in the grid is uniquely specified by...
the value of the "top mass" \( n_0 = m(T_{\text{top}} = 1) \) and the surface value of microvelocity \( \xi_l^{(0)} = \xi_l(T_{\text{top}} = 1) \). Owing to the sharp temperature rise in the solar atmosphere above \( T_{\text{top}} = 1 \) to coronal temperatures in much less than a pressure scale height, the "surface" pressure \( P_s = g n_0 \) should be roughly equal to the pressure in the transition region and at the base of the corona (Shine and Linsky 1974).

In addition to the grid models characterized by \( T_{\text{min}} = 4350 \) K, we have also included models in which the entire photospheric temperature distribution is raised or lowered in increments of 100 K.

IV. SYNTHETIC SPECTRA

The resonance line profile calculations presented below fall into two categories: (1) the damping wings, which are synthesized according to the partial coherent scattering formalism described by Ayres (1975a) (this approximation applies to portions of the line profile many Doppler widths \([ > 10 \Delta v_p \text{max} ] \) from line center); (2) the emission cores, which are synthesized according to the method described in § II above.

a) The Solar Ca II K line

We adopt the high-dispersion (\( \lambda/\Delta \lambda \approx 4 \times 10^5 \)), disk center (\( \mu = 1 \)) and limb (\( \mu = 0.2 \)) profiles of the solar Ca II K line from the KPNO Atlas (Braught and Testerman 1972). We calibrate these profiles by means of Houtgast's (1970) absolute photometry and the limb darkening measurements in the near wings of K (\( \Delta \lambda \approx \pm 4 \) \( \AA \)) by White and Suemoto (1968). These absolute calibrations are probably accurate to better than \( \pm 10 \) percent. Because the KPNO observations were obtained in the double-pass mode, we assume that the scattered light level in these data is negligible.

i) Synthesized K Wing Profiles: The Upper Photosphere Model

Figure 2a illustrates synthesized K wing profiles for the models of Figure 1a based on the "Kurucz" \( \gamma_{vw}^{(0)}(K) \). The agreement in the extreme inner wings (\( \Delta \lambda < 1 \) \( \AA \)) between the \( \mu = 1.0 \) and 0.2 data and the synthetic profiles for the 4350 model indicates that \( T_{\text{min}} \) is near 4350 K. Discrepancies at larger \( \Delta \lambda \) suggest slight modifications to the model below the temperature minimum. Several conclusions can be drawn from the comparison of the synthesized and measured wing profiles of Figure 2a: (1) Both the HSRA and VAL models appear to be too cool in the upper photosphere. In fact, the inferred value (from the extreme inner wings) of \( T_{\text{min}} \approx 4350 \pm 70 \) K, including the estimated uncertainty in the absolute calibration of K, is much closer to the "hot" \( T_{\text{min}} \approx 4450 \) K proposed by Shine, Milkey, and Mihalas (1975). (2) As Shine, Milkey, and Mihalas (1975) have shown, we find it possible to reproduce quantitatively the measured limb darkening of the K inner wings using partial coherency and single-component photospheric models. This is not the case in complete redistribution calculations, however, which typically predict a large increase in \( \Delta K_L \) toward the limb and constant \( K_L \) brightness temperature (Shine, Milkey, and Mihalas 1975), contrary to the empirical limb darkening behavior (e.g., Linsky and Avrett 1970). (3) The 4350 grid model does not fully match the shape of the inner wings. In § IVd we will attempt to optimally fit this data and more accurately derive \( T_{\text{min}} \).

ii) Synthesized Core Profiles: The Lower Chromosphere Model

Figure 2b depicts the core and inner wings of the K line synthesized with the VAL, HSRA, and the 4350 K grid model characterized by \( \log m_0 = -5.5 \) ga cm\(^{-2}\) and \( \xi_l^{(0)} = 7.5 \) km s\(^{-1}\). We adopt the "Kurucz" \( \gamma_{vw}^{(0)}(K) \). Notice that both the cores and inner wings of K synthesized with the VAL and HSRA models are too low compared with the observed profiles. Notice also that the widely different microvelocity distributions for these three models all reproduce the observed wavelength displacements of the \( K_2 \) peaks—a sensitive velocity indicator—at disk center, presumably because each of the turbulence models has similar velocities in the region of \( K_2 \) formation. However, all of the models predict profiles which are too narrow at the limb. This center-to-limb difference suggests the possibility of anisotropic turbulence, although atmospheric inhomogeneities could conceivably be responsible. Indeed, a much improved fit to the emission cores, especially at the \( K_2 \) features, could be obtained by using the more accurate five-level, five-transition representation of Ca ii, which typically predicts a brighter \( K_3 \) core than the three-level, two-transition solution (Shine, Milkey, and Mihalas 1975). Both approaches, however, yield the same \( K_1 \) intensities. Alternatively, convolution of the calculated profile with a \( \sim 10 \) km s\(^{-1}\) Gaussian to simulate macroturbulent broadening would substantially improve the agreement of the synthetic and measured core intensities, particularly for Mg ii h and k (see below) where the three-level, two-transition representation is expected to be more accurate.

iii) The Effects of Temperature and Microvelocity Gradients on the K Line Core

Figure 2c illustrates synthesized profiles for the grid models \([-5.5, 7.5] \), \([-5.5, 10.0] \), and \([-5.0, 7.5] \). (The first quantity within each set of square brackets refers to \( m_0 \) and the second to \( \xi_l^{(0)} \).) Increasing \( m_0 \)—and hence the temperature gradient—increases the integrated K emission for fixed \( \xi_l^{(0)} \), while increasing \( \xi_l^{(0)} \)—and hence the velocity gradient—for fixed \( m_0 \) has little effect on the total emission (\( K_{1\alpha} \) and \( K_{1\beta} \) but shifts the \( K_2 \) peaks somewhat.
Fig. 2.—(a) Comparison of measured (mean profile, filled circles) and synthesized inner wing intensities of Ca II K for the upper photosphere temperature models (same line symbols) of Fig. 1a (cf. § IVa[i]). The grid model labels designate $T_{\text{min}}$. The error bars refer to the estimated $\pm 10\%$ accuracy of the absolute calibration. (b) Synthesized core and inner wing profiles of Ca II K for models of Fig. 1a. The grid model is $[-5.5, 7.5]$. (c) Synthesized core and inner wing profiles of Ca II K for grid models $[-5.5, 7.5]$ (---); $[-5.5, 10.0]$ (-----); and $[5.0, 7.5]$ (----).
further from line center. Notice also the effect of an increased microvelocity gradient on the $K_1$ position and intensity, owing to the "spilling over" of emission from the Doppler core into the line wings. The same effect can be produced by raising the total intensity of the $K$ emission core (i.e., by increasing $m_0$), but in both cases only the extreme inner wings ($\Delta \lambda \lesssim 0.5 \, \text{Å}$) are affected. The empirical limb darkening of $K_1$ can be reproduced reasonably well by a single-component model having a temperature minimum of 4350 K and a horizontal microvelocity gradient 30 percent larger than that in the vertical direction.

### b) The Solar Mg II $h$ and $k$ Lines

The calculations of the previous section were presented primarily to establish classes of solar reference models as a basis for comparing synthesized Mg II $h$ and $k$ profiles with the absolute intensities measured by Kohl and Parkinson (Paper I).

#### i) Synthesized $h$ and $k$ Wing Profiles

In Figure 3a we plot synthesized $h$ and $k$ wing intensities over a 25 Å bandpass centered near 2800 Å for the VAL model and a variety of van der Waals parameters and Mg abundances. Notice the twofold effect of $\gamma_{vw}$ described...
by Milkey and Mihalas (1974): increasing $\gamma_{\text{vw}}$ reduces the far wing intensities by increasing the damping width, but raises the intensities in the radiation damping dominated inner wings of $h$ and $k$ by increasing the incoherence fractions, thereby forcing the monochromatic wing source functions closer to $B_{\nu}$. Significantly, Figure 3a shows that even large departures of $\gamma_{\text{vw}}$ and $A_{\text{mg}}$ from their nominal values have little effect on the synthesized inner wing intensities ($\Delta \lambda \leq 3 \text{ Å}$), compared to the effects produced by the different atmospheric models (see below).

In fact, the VAL synthesized intensities for even the extreme limits on $\gamma_{\text{vw}}$ and $A_{\text{mg}}$ fall roughly a factor of 2 below the measured profiles in the inner wings, much in excess of the probable uncertainty of ($+12$, $-20$) percent (Paper I) in the absolute calibration.

Figure 3b illustrates the synthesized intensities for the photospheric models of Figure 1a and the scaled "Kurucz" $\gamma_{\text{vw}}(h, k)$. Again, we find that the VAL and HSRA models fail to match the inner wing intensities, shapes, and limb darkening of $h$ and $k$. The 4350 grid model, on the other hand, reproduces these properties much better. The comparison in Figures 4a and 4b between measured inner wing and $k_1$ intensities and synthesized grid model profiles suggests that a 4500 grid model would produce the best, but not optimum, agreement. As with Ca II, the measured inner wing shapes are not accurately fit by the grid models. As stated in Paper I the random errors in this data are ±12 percent. In addition there is a systematic error due to scattered light which is estimated to be $-8$ percent at the $k_1$ feature and correspondingly less at higher intensities. Thus from the comparison with the grid models we derive a minimum temperature of 4500 ($+60$, $-100$) K including only errors associated with the data and not the modeling procedure.
Fig. 4.—Same as Fig. 2a, but for Mg II h and k. The error bar refers to the estimated (+12, −20) percent accuracy of the absolute calibration at $k_1$.
Fig. 4.—Continued

© American Astronomical Society • Provided by the NASA Astrophysics Data System
Fig. 5.—(a) Same as Fig. 2b but for Mg II h and k. The measured core profiles are represented by +'s. The profiles synthesized for the VAL and HSRA models overlap in the line cores: For clarity, only the VAL profile is plotted. (b) Same as Fig. 2c but for Mg II h and k.
The h and k Emission Cores

Figure 5a depicts the core and inner wings of h and k synthesized with the same models used for Ca II K in Figure 2b above and the scaled “Kurucz” γW(h, k). The synthesized profiles have been smeared with the instrumental profile (~28 mA FWHM) described in Paper I. As with the K line, we find that the HSRA and VAL models predict intensities which are systematically too low in the core and inner wings of h and k, and that this discrepancy appears at both disk center and the limb. In addition, the empirical h and k wavelength displacements are matched at disk center by the velocity models of Figure 1b, but not at the limb.

The Effects of Temperature and Microvelocity Gradients on the h and k Emission Cores

In Figure 5b we plot k profiles synthesized for the same grid models applied to Ca II K in Figure 2c above. We find essentially the same behavior for k as we did for K. Again, convolution of the calculated profiles with a ~10 km s⁻¹ Gaussian would improve the general agreement between the synthetic and measured intensities, while a slight anisotropy in the turbulence model would improve the limb fits.

c) h, k, and K Indices for a Grid of Solar Chromospheric Temperature Models

In addition to the profiles illustrated in the various figures above, we have calculated “emission indices” for Mg II h and k and Ca II K. By “emission index” we mean the total specific intensity (ergs cm⁻² s⁻¹ sr⁻¹) emitted in the particular line core between the minimum features (i.e., h, k, or K). This definition differs somewhat from the usually adopted fixed-frequency bandpass, but has the advantage that the synthesized indices are roughly independent of the microvelocity model (Ayres 1975b). Because the computed indices depend mainly on the lower chromosphere temperature structure, a comparison of measured and synthesized emission indices for lines such as k and K allows us to estimate the temperature gradient semiempirically.

Figure 6 illustrates the run of calculated emission indices with m₀ (which is a measure of the lower-chromosphere temperature gradient) for h, k, and K. These values, including the measured quantities and those appropriate to the VAL and HSRA models, are listed in Table 1. Both the disk center and limb indices of all three lines suggest log m₀ ≈ −5.25 ± 0.20, which is similar to the top masses of both the HSRA and VAL (see Fig. 1a). Furthermore, the corresponding surface pressure P₀ ≈ 0.15 ± 60 percent dyn cm⁻² is similar to the value derived by Dupree (1972) from an analysis of solar transition region emission lines.
From the experience gained in comparing synthesized profiles from the grid models and measured line profiles we are now in a position to derive models which optimally fit the data. We are specifically interested in the value and uncertainty of $T_{\text{min}}$ and whether one temperature distribution can adequately explain the inner wing profiles of both the Ca $\Pi$ and the Mg $\Pi$ lines.

i) Ca $\Pi$

We assume the lower photosphere of the VAL model and the VAL derived $\gamma_{\text{mix}}$'s, because their model is based on a wider variety of data than Kurucz's analysis, and because their semiempirical method is not subject to the restrictive assumptions of LTE and RE adopted by Kurucz. We modify the surface layers of the VAL model according to the following scheme: First, we determine the mass point in the model which corresponds to monochromatic optical depth unity at the wavelength where the observed disk center K wing profile begins to depart

<table>
<thead>
<tr>
<th>TABLE 1</th>
<th>Measured and Computed Emission Indices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>K Index</td>
</tr>
<tr>
<td></td>
<td>$\mu = 1.0$</td>
</tr>
<tr>
<td>VAL (VAL $\xi$)</td>
<td>1.08(5)</td>
</tr>
<tr>
<td>HSRA (LA $\xi$)</td>
<td>9.33(4)</td>
</tr>
<tr>
<td>$-5.5$ (2, 7.5)</td>
<td>1.32(5)</td>
</tr>
<tr>
<td>$-5.5$ (2, 10)</td>
<td>1.40(5)</td>
</tr>
<tr>
<td>$-5.0$ (2, 7.5)</td>
<td>2.04(5)</td>
</tr>
<tr>
<td>$-5.0$ (2, 10)</td>
<td>2.06(5)</td>
</tr>
<tr>
<td>Paper I</td>
<td>1.56(5)</td>
</tr>
<tr>
<td>Bandpass</td>
<td>$[\pm 0.275 \text{ Å}]$</td>
</tr>
</tbody>
</table>
noticeably from the synthesized intensity distribution. This mass point, \(m_1\), becomes the first "pivot." Next, we construct a series of linear (in \(\log m\)) temperature models connecting the pivot point \([m_1, T_{e_1}]\) and \([m_{\text{min}}, T_{\text{min}}]\).

In order to simplify the parametrization of the resulting models, we fix \(m_{\text{min}}\) at the value \(10^{-5.2}\) \(\text{g cm}^{-2}\) suggested by the detailed line core and \(K_1\) synthesis of § IV. Above \(m_{\text{min}}\), we adopt a "linear" temperature rise into the lower chromosphere with \(m_0 = 10^{5-25}\). We then adjust \(T_{\text{min}}\) to force agreement between the synthesized and measured profiles of the \(K\) wing immediately shortward of the first pivot point wavelength \(\Delta \lambda^{(1)}\). For instance, we choose the first pivot point in the VAL model to occur at \(T_{e_1} = 5040\) K and find that \(T_{\text{min}} = 4200\) K produces an excellent fit between synthesized wing intensities and the calibrated KPNO \(\mu = 1\) K wing profile between the pivot-point wavelength \(\Delta \lambda^{(1)} = 3\) \(\text{Å}\) and \(\Delta \lambda = 1\) \(\text{Å}\). However, the synthetic profile for \(\Delta \lambda < 1\) \(\text{Å}\) falls somewhat below the measured intensities (see Fig. 8). We therefore choose \(\Delta \lambda^{(2)} = 1\) \(\text{Å}\) as the second pivot point and again vary \(T_{\text{min}}\) for the now shorter piecewise linear temperature segment to force agreement between the calculated and measured inner wing profiles shortward of \(\Delta \lambda^{(2)}\). For \(\Delta \lambda < 1\) \(\text{Å}\) we find it necessary to account for the differences between profiles synthesized with the LTE coherent scattering formalism and those synthesized with the PRD core method. In particular, we find that the LTE partial coherent scattering (LTE-PCS) solution overestimates the PRD synthesized intensities by an average 2 percent between \(\Delta \lambda = 1\) \(\text{Å}\) and \(\Delta \lambda = 0.33\) \(\text{Å}\), and underestimates the PRD intensities by about 6 percent at \(\Delta \lambda = 0.28\) \(\text{Å}\), 10 percent at \(\Delta \lambda = 0.26\) \(\text{Å}\), and 20 percent at \(\Delta \lambda = 0.24\) \(\text{Å}\). The rapid increase in error of the LTE-PCS approximation compared with the non-LTE PRD solution is attributable primarily to the increasing dominance of the Doppler core relative to the damping wings as \(\Delta \lambda \rightarrow 0\). We use the partial coherent scattering approximation to construct upper-photosphere models here, because the minor increase in accuracy of the PRD approach is more than offset by the massive amounts of computing time required by that method.

We find that the \(\Delta \lambda^{(2)} = 1\) \(\text{Å}\) pivot point corresponds to \(T^{(2)} = 4450\) K, and requires \(T_{\text{min}} = 4400-4500\) K to fit the extreme inner wing intensities near \(K_1\), taking the small systematic errors of the LTE–PCS method into account. The range of models derived here is similar to the schematic modification to the HSRA adopted by Shine, Milkey, and Mihalas (1975). We find essentially the same \(T_{\text{min}}\) despite the slightly large value of \(I_{K_1}^{\text{obs}}(\mu = 1) = 1.46 \times 10^{-6}\) \(\text{ergs cm}^{-2} \text{ s}^{-1} \text{ sr}^{-1} \text{ Hz}^{-1}\) (compared with their \(I_{K_1}^{\text{obs}}(\mu = 1) = 1.41 \times 10^{-6}\) adopted here, based on the KPNO K profiles and Houtgast’s (1970) absolute calibration. The small differences between our derived model and that of Shine et al. are attributable to the (1) \(\sim 10\) percent larger \(\gamma_{\text{ew}}^{(0)}(K)\) adopted here and (2) the fact that the model of this work has generally larger upper photosphere temperatures than the "flat" \(T_{\text{min}}\) modification to the HSRA adopted by Shine et al. It is important to recognize that the derived \(T_{\text{min}}\) depends on the shape of the upper photospheric temperature distribution because the frequency-dependent source function at \(K_1\) closely approximates the radiation field. Consequently to match \(I_{K_1}^{\text{obs}}\), \(T_{\text{min}}\) must increase as the temperature gradient in the upper photosphere becomes shallower. This is why we derive \(T_{\text{min}} = 100\) K hotter than in § IVa when we accurately match the entire inner wing profile.

The inferred "Ca ii" model \((T_{\text{min}} = 4450\) K\) is illustrated in Figure 7; the inner wing profile fits, in Figure 8.
Fig. 8.—Comparison of synthesized and measured Ca II K wings for single-pivot model with $T_{\text{min}} = 4200$ K (---), adopted "Ca II" model with $T_{\text{min}} = 4450$ K, and adopted "Mg II" model.

Fig. 9.—Comparison of synthesized and measured Mg II k short-wavelength wing for adopted "Ca II" and "Mg II" models.
ii) Mg II

We can match the measured \( k \) inner wing intensities with a single-pivot model systematically 100–200 K hotter than the inferred Ca II model (the single-pivot is at \( T_e = 5200 \) K and requires \( T_{\text{min}} = 4500 \) K). We cannot, however, strictly reproduce the Mg II intensities using the inferred Ca II model, or, conversely, the Ca II intensities using the model inferred from Mg II, despite varying the \( \text{FWHM} \) and abundances over a fairly wide range. The lower range of "Mg II" temperatures based on the estimated (\( +12, -20 \)) percent uncertainty at \( k_e \) is barely consistent with the upper range of "Ca II" temperatures based on the estimated \( \pm 14 \) percent error of the K line analysis (see below).

The inferred model for Mg II is illustrated in Figure 7; the inner-wing profile fits, in Figure 9.

iii) The Adopted Model

Because the models of this work differ significantly from those proposed in previous investigations (i.e., HSRA and VAL), we present a tabulated version of a "Ca II" model to serve as an example of an alternative class of solar upper photosphere models constructed to fit a different variety of data than the HSRA or VAL. For this purpose, we adopt the "Ca II" model with \( T_{\text{min}} = 4450 \) K, \( \log m_p = -5.25 \) g cm\(^{-2} \), and \( \xi(0) = 10 \) km s\(^{-1} \). We choose the "Ca II" model because the observed data and absolute calibrations are probably more reliable than the Mg II

### TABLE 2
"Ca II" SOLAR MODEL
\( (T_{\text{eff}} = 5770 \) K, \( \log g = 4.44 \text{ cm s}^{-1} \)\)

<table>
<thead>
<tr>
<th>( m(g \text{ cm}^{-2}) )</th>
<th>( T_e (K) )</th>
<th>( n_t (\text{cm}^{-3}) )</th>
<th>( n_e (\text{cm}^{-3}) )</th>
<th>( b_1(\text{H I}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.80(−3)</td>
<td>5321</td>
<td>1.05(14)</td>
<td>1.06(11)</td>
<td>1.54</td>
</tr>
<tr>
<td>3.94 (−3)</td>
<td>5307</td>
<td>1.09(13)</td>
<td>1.05(11)</td>
<td>1.51</td>
</tr>
<tr>
<td>4.47(−3)</td>
<td>5255</td>
<td>1.27(14)</td>
<td>1.02(11)</td>
<td>1.38</td>
</tr>
<tr>
<td>5.32 (−3)</td>
<td>5183</td>
<td>1.55(14)</td>
<td>9.90(10)</td>
<td>1.23</td>
</tr>
<tr>
<td>6.31 (−3)</td>
<td>5113</td>
<td>1.88(14)</td>
<td>9.63(10)</td>
<td>1.09</td>
</tr>
<tr>
<td>7.51 (−3)</td>
<td>5041</td>
<td>2.31(14)</td>
<td>9.27(10)</td>
<td>0.97</td>
</tr>
<tr>
<td>8.91 (−3)</td>
<td>4971</td>
<td>2.80(14)</td>
<td>9.24(10)</td>
<td>0.85</td>
</tr>
<tr>
<td>1.06(−2)</td>
<td>4899</td>
<td>3.42(14)</td>
<td>9.18(10)</td>
<td>0.75</td>
</tr>
<tr>
<td>1.26(−2)</td>
<td>4829</td>
<td>4.17(14)</td>
<td>9.32(10)</td>
<td>0.66</td>
</tr>
<tr>
<td>1.50(−2)</td>
<td>4757</td>
<td>5.10(14)</td>
<td>9.68(10)</td>
<td>0.57</td>
</tr>
<tr>
<td>1.78(−2)</td>
<td>4687</td>
<td>6.21(14)</td>
<td>1.03(11)</td>
<td>0.50</td>
</tr>
<tr>
<td>2.12(−2)</td>
<td>4615</td>
<td>7.60(14)</td>
<td>1.12(11)</td>
<td>0.43</td>
</tr>
<tr>
<td>2.51(−2)</td>
<td>4545</td>
<td>9.22(14)</td>
<td>1.25(11)</td>
<td>0.37</td>
</tr>
<tr>
<td>2.99(−2)</td>
<td>4473</td>
<td>1.13(15)</td>
<td>1.41(11)</td>
<td>0.32</td>
</tr>
<tr>
<td>3.16(−2)</td>
<td>4450</td>
<td>1.20(15)</td>
<td>1.47(11)</td>
<td>0.30</td>
</tr>
<tr>
<td>3.55(−2)</td>
<td>4450</td>
<td>1.35(15)</td>
<td>1.63(11)</td>
<td>0.30</td>
</tr>
<tr>
<td>4.22(−2)</td>
<td>4450</td>
<td>1.60(15)</td>
<td>1.90(11)</td>
<td>0.30</td>
</tr>
<tr>
<td>5.01(−2)</td>
<td>4450</td>
<td>1.91(15)</td>
<td>2.20(11)</td>
<td>0.30</td>
</tr>
<tr>
<td>5.97(−2)</td>
<td>4450</td>
<td>2.27(15)</td>
<td>2.56(11)</td>
<td>0.30</td>
</tr>
<tr>
<td>7.08(−2)</td>
<td>4450</td>
<td>2.69(15)</td>
<td>2.97(11)</td>
<td>0.30</td>
</tr>
<tr>
<td>8.43(−2)</td>
<td>4450</td>
<td>3.21(15)</td>
<td>3.44(11)</td>
<td>0.30</td>
</tr>
<tr>
<td>1.00(−1)</td>
<td>4462</td>
<td>3.79(15)</td>
<td>4.01(11)</td>
<td>0.31</td>
</tr>
<tr>
<td>1.19(−1)</td>
<td>4502</td>
<td>4.48(15)</td>
<td>4.76(11)</td>
<td>0.34</td>
</tr>
<tr>
<td>1.41(−1)</td>
<td>4541</td>
<td>5.26(15)</td>
<td>5.62(11)</td>
<td>0.37</td>
</tr>
<tr>
<td>1.68(−1)</td>
<td>4581</td>
<td>6.22(15)</td>
<td>6.68(11)</td>
<td>0.40</td>
</tr>
<tr>
<td>2.18(−1)</td>
<td>4620</td>
<td>7.34(15)</td>
<td>7.92(11)</td>
<td>0.43</td>
</tr>
<tr>
<td>2.38(−1)</td>
<td>4660</td>
<td>8.67(15)</td>
<td>9.39(11)</td>
<td>0.47</td>
</tr>
<tr>
<td>2.82(−1)</td>
<td>4699</td>
<td>1.02(16)</td>
<td>1.11(12)</td>
<td>0.51</td>
</tr>
<tr>
<td>3.36(−1)</td>
<td>4739</td>
<td>1.20(16)</td>
<td>1.31(12)</td>
<td>0.55</td>
</tr>
<tr>
<td>3.98(−1)</td>
<td>4778</td>
<td>1.42(16)</td>
<td>1.55(12)</td>
<td>0.60</td>
</tr>
<tr>
<td>4.74(−1)</td>
<td>4817</td>
<td>1.67(16)</td>
<td>1.83(12)</td>
<td>0.64</td>
</tr>
<tr>
<td>5.62(−1)</td>
<td>4856</td>
<td>1.97(16)</td>
<td>2.16(12)</td>
<td>0.69</td>
</tr>
<tr>
<td>6.69(−1)</td>
<td>4896</td>
<td>2.33(16)</td>
<td>2.56(12)</td>
<td>0.75</td>
</tr>
<tr>
<td>7.94(−1)</td>
<td>4935</td>
<td>2.74(16)</td>
<td>3.02(12)</td>
<td>0.80</td>
</tr>
<tr>
<td>9.46(−1)</td>
<td>4975</td>
<td>3.24(16)</td>
<td>3.57(12)</td>
<td>0.86</td>
</tr>
<tr>
<td>1.12</td>
<td>5014</td>
<td>3.81(16)</td>
<td>4.20(12)</td>
<td>0.92</td>
</tr>
<tr>
<td>1.34</td>
<td>5072</td>
<td>4.51(16)</td>
<td>5.04(12)</td>
<td>1.00</td>
</tr>
<tr>
<td>1.58</td>
<td>5190</td>
<td>5.20(16)</td>
<td>6.21(12)</td>
<td>1.00</td>
</tr>
<tr>
<td>1.89</td>
<td>5322</td>
<td>6.08(16)</td>
<td>7.90(12)</td>
<td>1.00</td>
</tr>
<tr>
<td>2.24</td>
<td>5470</td>
<td>7.02(16)</td>
<td>1.03(13)</td>
<td>1.00</td>
</tr>
<tr>
<td>2.68</td>
<td>5640</td>
<td>8.12(16)</td>
<td>1.41(13)</td>
<td>1.00</td>
</tr>
<tr>
<td>3.16</td>
<td>5840</td>
<td>9.30(16)</td>
<td>2.07(13)</td>
<td>1.00</td>
</tr>
<tr>
<td>3.76</td>
<td>6130</td>
<td>1.06(17)</td>
<td>3.64(13)</td>
<td>1.00</td>
</tr>
<tr>
<td>4.40</td>
<td>6423</td>
<td>1.18(17)</td>
<td>6.51(13)</td>
<td>1.00</td>
</tr>
<tr>
<td>5.11</td>
<td>6865</td>
<td>1.29(17)</td>
<td>1.46(14)</td>
<td>1.00</td>
</tr>
<tr>
<td>5.81</td>
<td>7460</td>
<td>1.35(17)</td>
<td>3.82(14)</td>
<td>1.00</td>
</tr>
<tr>
<td>6.54</td>
<td>8100</td>
<td>1.40(17)</td>
<td>9.37(14)</td>
<td>1.00</td>
</tr>
<tr>
<td>6.81</td>
<td>8333</td>
<td>1.42(17)</td>
<td>1.25(15)</td>
<td>1.00</td>
</tr>
</tbody>
</table>

© American Astronomical Society • Provided by the NASA Astrophysics Data System
observations described in Paper I, owing to the (1) less severe line blanketing in the Ca II K data, and (2) smaller probable error of radiometric calibrations in the visible compared with the vacuum ultraviolet.

This model is tabulated in Table 2.

V. DISCUSSION

a) An Error Analysis for $T_{\text{min}}$

The errors associated with deriving a value of $T_{\text{min}}$ by matching the extreme inner wings of the Ca II and Mg II lines using a PRD analysis arise from (1) errors in the absolute calibrations of the data, (2) errors in the abundances and van der Waals parameters, (3) errors in the shape of $T_m(m)$ in the upper photosphere, and (4) errors in the PRD technique itself.

For Ca II we estimate a $\pm 10$ percent rms uncertainty in the absolute calibration compared of a $\pm 8$ percent probable error cited by Houtgast (1970) and an additional $\pm 5-6$ percent estimated for the scaling of Houtgast's absolute intensities and White and Suemoto's (1968) limb darkening measurements to the KPNO center-limb K profiles.

We estimate a $\pm 5$ percent uncertainty owing to the effect of the $\pm 0.1$ dex accuracy of $A_{\text{Ca}}$ on $\gamma_{\text{vw}}$ as "calibrated" against the $I_{\text{Ca}}$ absolute intensity. This uncertainty arises from the fact that the $I_{\text{Ca}}$ calibration forces a roughly $\gamma_{\text{vw}} \propto A_{\text{Ca}}^{-1}$ relationship on the two parameters. Therefore, lowering $A_{\text{Ca}}$ below the nominal value $A_{\text{Ca}}(0)$ has a twofold effect on $I_{\text{Ca}}$, through (a) the narrowing of the radiation damping dominated inner wings of K, thereby increasing the $K_1$ intensity, and (b) requiring a larger $\gamma_{\text{vw}}$ to fit $I_{\text{Ca}}$, thereby also raising the $K_1$ intensity owing to the effect of the larger $\gamma_{\text{vw}}$ on increasing the upper photosphere K line incoherence fractions. Conversely, raising $A_{\text{Ca}}$ above $A_{\text{Ca}}(0)$ has a twofold lowering effect on $I_{\text{Ca}}$ by broadening out the inner-wing profiles and reducing the incoherence fractions. Errors in the modeling process include $\pm 5$ percent owing to the uncertainty in the microvelocity distribution and an additional $\pm 7$ percent ($\pm 50$ K) owing to errors in $T_m(m)$. This estimate is derived by assuming half the difference between $T_{\text{min}}$ derived from the grid model and "optimal" fit model.

The total rms uncertainty is on the order of $\pm 14$ percent, corresponding to roughly $\pm 130$ K. Hence, $T_{\text{min}} = 4450 \pm 130$ K derived from the Ca II K line. (The uncertainty cited here is exclusive of possible errors attributable to the PRD approach itself.)

For Mg II the uncertainty in absolute calibration is $\pm 12$ percent and an additional $\pm 8$ percent at k1 to account for possible scattered light. We estimate $\pm 15$ percent uncertainty attributable to the scaling of $\gamma_{\text{vw}}(h, k)$ from $\gamma_{\text{Ca}}$ and $\pm 5$ percent uncertainty owing to the $\pm 0.05$ dex accuracy of $A_{\text{Mg}}$. The error attributable to $\gamma_{\text{vw}}(h, k)$ and $A_{\text{Mg}}$ is larger than for Ca II since we cannot "calibrate" $\gamma_{\text{vw}}(h, k)$ and $A_{\text{Mg}}$ against the $h$ and $k$ far wings as reliably as for Ca II owing to severe line blanketing in the vicinity of the Mg II lines. We assume similar errors to Ca II arising from the model building process. The total rms uncertainty, exclusive of that attributable to the PRD approach, is of order $(+20, -28)$ percent at k1 and, hence $T_{\text{min}} = 4500 (+80, -110)$ K derived from the Mg II k line.

b) The Value of the Solar Temperature Minimum

In this analysis we have "derived" a solar temperature minimum value of 4450 $\pm$ 130 K based on the Ca II K line and 4500 ($+80, -110$) K based on the Mg II k line. These values are internally consistent but differ substantially from the values of 4170 K for the HSRA and 4150 K for the VAL model, which are both based on fitting ultraviolet and far-infrared continuum data. We wish to make several comments on this apparent discrepancy.

1. Because the values of $T_{\text{min}}$ derived from the Ca II and Mg II data are consistent, possible errors in the absolute calibration and scattered-light estimate of the Mg II data in excess of those present in Paper I cannot account for the discrepancy.

2. The physical basis for the PRD theory has been discussed in detail by Milkey and Mihalas (1973a,b, 1974), Milkey, Shire, and Mihalas (1975), and Shire, Milkey, and Mihalas (1975). However, until the present work this theory has not been adequately tested against high-quality solar data. We have found that the solar upper photosphere temperature distributions derived using the theory disagree with the models based on continuum data, but also that the models derived to explain the Ca II and Mg II data differ somewhat and may be inconsistent with each other, even though the values of $T_{\text{min}}$ agree. This inconsistency suggests that the physical basis for and manner of application of the PRD theory could be at fault. We point out that the essential difference between the CRD and PRD approaches is in the presence of coherence effects in the scattering process. As a result, the inferred value of $T_{\text{min}}$ is a sensitive function of the degree of coherence in the inner wings of the Ca II and Mg II lines. If the degree of coherence is increased, a hotter temperature minimum is required to produce the observed $K_1$ or $K_2$ intensities. Conversely, if the scattering process is less coherent, then a cooler $T_{\text{min}}$ will suffice. However, a nonnegligible degree of coherence is almost certainly necessary to explain the observed limb darkening of the K and k inner wings (see, e.g., Milkey and Mihalas 1974; Shire, Milkey, and Mihalas 1975). Therefore, a CRD-derived value for $T_{\text{min}}$ is almost certainly a lower limit. ($T_{\text{min}}^{\text{CRD}} \approx 4200$ K for both Ca II K and Mg II k [see, e.g., Linsky and Avrett 1970; Shire and Avrett 1973].)

Unfortunately, we cannot reliably estimate the possible error attributable to the application of the PRD technique itself. Therefore, we feel that the PRD approach should be studied in even greater detail.
3. The continuum synthesis method is inherently more difficult to apply than the PRD method because it requires many individual, independent, and therefore possibly inconsistent absolute calibrations over large frequency bandpasses covering much of the solar spectral energy distribution. In addition, the continuum method requires accurate atomic data for the many important continuous absorbers in the far ultraviolet and knowledge of the line blanketing distribution function.

By way of comparison, Vernazza, Avrett, and Loeser (1976) have computed the ultraviolet and infrared continuum flux distributions for a "hot" $T_{\text{min}} = 4450$ K model (their model "S") similar to the "Ca II" model proposed here. They find flux enhancements of $\leq 50$ percent for $2500 \leq \lambda \leq 2100$ Å and factors of $\sim 2-4$ below 2100 Å, as compared with their "best fit" model ("M"). In addition, Vernazza et al. find enhanced brightness temperatures in the far-infrared, free-free spectrum corresponding to the hotter $T_{\text{min}}$ region of model S. They suggest that model S is therefore inconsistent with the measured solar continuum fluxes. However, the computed fluxes shortward of $\lambda \sim 4000$ Å are very sensitive to the adopted line blanketing distribution function (see Fig. 27 in Vernazza et al.), which is not well determined. Furthermore, the wide separation of the ultraviolet and infrared intensity measurements and possible systematic errors in the absolute calibrations allow enough uncertainty that model S and hence also our Ca II model are not inconsistent with the continuum data. We therefore feel that the existing solar continuum data do not rule out a "hot" value for $T_{\text{min}}$.

4. Athay (1970) has derived $T_{\text{min}} = 4330 \pm 150$ K assuming radiative equilibrium and non-LTE line blanketing, and Kurucz (1974) finds $T_{\text{r}} = 4300$ at $T_{\text{r}} = 5000 \approx 10^{-4}$, where $T_{\text{r}}$ is located, for an LTE radiative equilibrium model including the blanketing of $1.7 \times 10^9$ lines. Short-period acoustic waves are very likely damped in the upper photosphere (Ulmschneider and Kalkofen 1973; Ayres 1975a) and could produce temperature enhancements over the nominal temperature profile otherwise expected in radiative equilibrium of $\Delta T \approx 50-100$ K. The $\Delta T$'s are small owing to the effectiveness of H $^+$ cooling at the relatively high densities of the upper photosphere.

It is difficult to reconcile the "cool" $T_{\text{min}}$ HSRA and VAL models with the RE temperature structures of Athay (1970) and Kurucz (1974), because the apparent temperature deficiencies of the semiempirical models would require net nonradiative energy production (mechanical cooling) rather than dissipation (mechanical heating) in the vicinity of $T_{\text{r}}$.

5. It could be argued that departures from a strictly homogeneous atmosphere might be responsible for the apparent disagreement between the resonance line models and those based on continuum intensities, especially because the analysis of the shorter-wavelength Mg II h and k lines suggests a slightly hotter $T_{\text{min}}$ than Ca II K. However, ultraviolet continuum data do not appear to imply a hotter $T_{\text{min}}$ than infrared continuum data (e.g., Vernazza et al. 1976). Because the Planck function is an exponential function of temperature in the ultraviolet but linear in the infrared, we conclude from the apparent agreement of far-ultraviolet and far-infrared $T_{\text{min}}$'s that temperature fluctuations near the temperature minimum are likely to be small (cf. Bonnet and Blamont 1968). On the other hand, the role played by atmospheric inhomogeneities on the Sun is not well understood, and it merits further research including high spatial resolution studies of the Ca II and Mg II lines with the OSO-8 experiment.

We thank the National Center for Atmospheric Research, which is operated by the University Corporation for Atmospheric Research under contract with the National Science Foundation, for providing computing time. We acknowledge support of the National Aeronautics and Space Administration through grants NGL-06-003-057 and NAS 5-23274 to the University of Colorado. We also acknowledge useful discussions with J. Kohl and W. Parkinson, and thank an anonymous referee for helpful comments and suggestions.

REFERENCES


———. 1975b, ibid., 201, 212.


Lemaire, P. 1975, private communication.


Thomas R. Ayres: Center for Astrophysics, 60 Garden St., Cambridge, MA 02138.

Jeffrey L. Linsky: Joint Institute for Laboratory Astrophysics, University of Colorado, Boulder, CO 80302.