RESPONSE OF A BOUNDED ATMOSPHERE TO A
NON-RESONANT EXCITATION

I: Isothermal Case

JANINE PROVOST
Observatoire de Nice, Le Mont-Gros 06300, Nice, France

(Received September, 1974)

Abstract. The response of a bounded atmosphere to a non-resonant excitation applied at its basis is studied. It is shown that the essential feature related to this kind of excitation is that the distribution of the energy of the velocity field relatively to the frequency and horizontal wavelength is a function of height and merely depends on the structure of the atmosphere above the level at which it is considered. The preliminary results concerning an isothermal atmosphere are presented and their relevance to the solar case is discussed.

1. Introduction

Many attempts to interpret the five-minute solar oscillation discovered by Leighton (1960) have been proposed in the literature: the theoretical studies have been reviewed by Schatzman and Souffrin (1967) and by Stein and Leibacher (1974). Essentially there exist two different approaches: either the involved mechanism is an instability of the subphotospheric layers occurring for some frequencies, or the oscillation corresponds to the response of the photospheric and chromospheric layers to an underlying excitation which is supposed related to the convective instability. In this paper some aspects of this latter approach will be developed.

In some works (Souffrin, 1966, 1970; Zhugzda, 1973; Moore, 1974; Chen, 1974) the model considered is a semi-infinite isothermal atmosphere with a radiation condition at infinity, so that reflection of the waves does not take place. However, as far as the Sun is concerned, the reflections cannot be reasonably neglected: Bahng and Schwarzschild (1963) have shown that for the propagation of vertical waves in an isothermal atmosphere the presence of a very hot corona can be taken into account by using a free surface upper boundary condition. This result can be extended to oblique waves in a more general atmosphere (Provost, 1974). The model considered here will take into account this reflection property due to the steep increase of the temperature in the transition zone chromosphere – corona.

The structural properties of the atmosphere play the predominant role in various theoretical models in the following way: it is considered that if the atmosphere has undamped eigenmodes, these modes will be resonant for any excitation of this atmosphere. Following this conception many resonant models have been constructed (Aure et al., 1971) but the failure of these models is that the results are very dependent of the choice of the parameters of the models and of the boundary conditions: the number of parameters is such that arbitrary resonant frequencies can be obtained. In
other models the observed motions are considered as forced motions induced by convection and the excitation is represented in different explicit ways. As the process of generation of the motions is not well known, the assumption is made, that the convective zone acts as an ‘oscillating lower boundary’, i.e. the dependence of a physical quantity on space and time \((X(x, t))\) is given at the basis of the atmosphere. In the literature there exist different choices for the assumed quantity \(X\): vertical velocity (Whitney, 1958; Meyer and Schmidt, 1967; Stix, 1970; Zhugzhda, 1973); Eulerian pressure perturbation (Worrall, 1972; Moore, 1974). This choice is not discussed in most of these works. Nevertheless it is fundamental because it determines the modes in which the energy of the response is distributed: in an atmosphere where the value of a quantity \(X(x, t)\) is imposed at the basis, most of the energy is contained in the eigenmodes of the atmosphere corresponding to the lower boundary condition \(X=0\). We note that if the excitation is stationary in time the amplitude of such eigenmodes is infinite. The above argument is valid as long as a non trivial motion may exist with a zero value of \(X\).

The aim of the present paper is to study the response of an atmosphere, bounded above by a hot corona, to a statistically stationary excitation specified in such a way as to avoid resonances: the quantity imposed at the oscillating lower boundary is the energy, quadratic quantity which cannot be zero. It is understood that this is the best we can do as long as the power spectra of the oscillatory modes relevant to the external solar layers remain unknown. This form of the excitation will permit us to determine the evolution with the altitude of the motions in a model of atmosphere, which satisfies two important properties of the solar atmosphere: the existence of reflections due to the hot corona and the statistically stationary character of the motions suggested by the observations.

In a first part the model is presented and the calculations of the spatio-temporal power spectra of the velocity field are given when the energy of the motions is imposed at the basis of the atmosphere. In a second part the general characteristics of these spectra are discussed and illustrated by the results corresponding to an isothermal atmosphere. In the conclusion the solar case is considered and we try to determine if the specific properties of the response to a non-resonant excitation are supported by the observational data about the photospheric and chromospheric oscillation.

2. Definition and Formal Solution of the Problem

We consider the following simplified model of the solar atmosphere, an isothermal atmosphere \(1\) of perfect gas horizontally stratified by an uniform gravity field directed along the vertical axis \(Oz\), is bounded above by a very hot atmosphere \(2\) representing the corona. This hot corona behaves like a perfectly reflecting layer, for the perturbations of the atmosphere \(1\), at the limit of infinite temperature and will be replaced by a free surface upper boundary condition situated at the level \(z=L\). This approximation is discussed by Bahng and Schwarzschild (1963) in the case of waves with vertical propagation in an isothermal atmosphere; many authors used this condition a priori
(Whitney (1958) who was the first to consider it as relevant to the solar atmosphere, Meyer and Schmidt (1967), Stein and Leibacher (1969), Jones (1969)). This condition can be justified using a model in which the corona is approximated by a hot isothermal atmosphere in the limit of an infinite temperature (Provost, 1974).

At the basis of the atmosphere 1, which we assume to be situated at the top of the convective zone, the underlying layers generate small random perturbations statistically stationary in time and in horizontal space. Our purpose is to calculate the response of the atmosphere to this excitation, assuming that the mean statistical energy of these perturbations is given at the basis of the atmosphere. The quantities which we derive are the spatio-temporal power spectrum \( F_{v_z} \) and \( F_{v_{\perp}} \) of the field of the vertical velocity \( v_z \) and horizontal velocity \( v_{\perp} \), which are the Fourier transforms of the autocorrelation function of the velocity field relatively to time and horizontal space,

\[
F_{v_z, \perp}(k_\perp, \omega, z) = \int_{-\infty}^{\infty} d\tau \int_{\mathbb{R}_2} d\mathbf{r} <v_{z, \perp}(t, \mathbf{r}) v_{z, \perp}(t + \tau, \mathbf{r} + \mathbf{R})> e^{-i(\omega \tau + \mathbf{k}_\perp \cdot \mathbf{R})},
\]

(1)

\( k_\perp \) is the horizontal wave number and \( \omega \) the frequency. The properties of stationarity of the statistical velocity field allows to relate the power spectra to the Fourier components of the velocity field in the following way:

\[
F_{v_z, \perp}(k_\perp, \omega, z) = v_{z, \perp}(k_\perp, \omega, z) v_{z, \perp}^*(k_\perp, \omega, z).
\]

(2)

Henceforth we assume that the velocity field is statistically isotropic in horizontal space and \( |k_\perp| \) will be denoted \( k \).

For the atmosphere 1, the linearized equations of conservation of momentum, matter and energy are

\[
\begin{align*}
\frac{d\mathbf{v}}{dt} &= -\nabla p + \frac{\rho}{\rho_0} \mathbf{g}, \\
\frac{d\rho}{dt} + \rho \nabla \cdot \mathbf{v} &= 0, \\
\frac{dp}{dt} &= c_0^2 \frac{d\rho}{dt}.
\end{align*}
\]

(3)

Here \( \rho_0, \mathbf{v}, c_0 \) are respectively the density of the unperturbed atmosphere, the velocity and the velocity of sound, \( p \) and \( \rho \) are Euler's perturbations of pressure and density, \( d/dt \) is the Lagrangian derivative; \( \mathbf{g} \) is the gravity field. The motion is supposed to be adiabatic.

These perturbations satisfy two boundary conditions. The upper condition, or free surface condition, is expressed by

\[
\left( \frac{\partial p}{\partial t} + \rho_0 \mathbf{g} \cdot \mathbf{v} \right)_{z=L} = 0.
\]

(4)
As a lower boundary condition we impose at the basis of the atmosphere the mean statistical energy of the perturbations. We use the concept of external energy, defined by Eckart (1960), which can be constructed with the amplitude assigned to the perturbations in the linearized theory. This definition takes into account simultaneously the kinetic, elastic and thermodynamic properties of the fluid. Following Eckart’s definition, this external energy $E$ is given by

$$2c_0E = \varrho_0c_0 v^2 + \frac{1}{\varrho_0 c_0} p^2 + N^2 \varrho_0 c_0 \left( \frac{\eta}{\eta_0} \right)^2,$$

where $N$ is the Brunt-Väissälä frequency, $\eta$ the eulerian first order perturbation of entropy, $\eta_0'$ is the entropy gradient of the unperturbed atmosphere. If the perturbations propagate adiabatically, the entropy perturbation depends in a simple way on the vertical velocity and the mean energy of the perturbations can be expressed as a function of the power spectra of the vertical and horizontal velocities,

$$\frac{2\langle E \rangle(z)}{\varrho_0(z)} = \sum_{k, \omega} \left\{ \left( 1 + \frac{N^2(z)}{\omega^2} \right) F_{v_z}(k, \omega, z) + \left( 1 + \frac{\omega^2}{k^2 c_0^2(z)} \right) F_{v_\perp}(k, \omega, z) \right\} = \sum_{k, \omega} \varepsilon(k, \omega, z);$$

$\varepsilon(k, \omega, z)$ represents the contribution of each mode $(k, \omega)$ to the mean energy. We are able now to formulate more precisely the lower boundary condition by assuming that the value of $\varepsilon(k, \omega, 0)$ is known at the basis of the atmosphere. In the case of convective stability, $N^2$ is a positive quantity and $\varepsilon(k, \omega, 0)$ cannot be zero. Consequently this excitation cannot generate resonance, contrary to the case where a quantity as the vertical velocity or the eulerian pressure perturbation is applied to the basis of the atmosphere.

The vertical and horizontal Fourier components of the velocity are derived from the system (3), for an isothermal atmosphere:

$$v_z(k, \omega, z) = I e^{iz} + J e^{iz},$$

$$v_{\perp}(k, \omega, z) = I \left( \frac{r_1 - k^2 g/\omega^2}{k^2 - \omega^2 / c^2} + \frac{g}{\omega} \right) \varepsilon^{iz} + J \left( \frac{r_2 - k^2 g/\omega^2}{k^2 - \omega^2 / c^2} + \frac{g}{\omega} \right) \varepsilon^{iz},$$

where

$$r_{1,2} = \frac{1}{2H} \left( 1 \pm \sqrt{1 - \frac{\omega^2}{\omega_c^2} + k^2 4H^2 \left( 1 - \frac{N^2}{\omega_0^2} \right)} \right),$$

$\omega_c$ is the acoustic cut-off frequency, $H$ the density scale height. $I$ and $J$ are two amplitude factors, to which the reflexion condition at the top of the atmosphere (Equation 4) imposes the following relation:

$$I(r_1 - k^2 g/\omega^2) e^{iL} + J(r_2 - k^2 g/\omega^2) e^{i2L} = 0.$$ 

Thus the variables $v_z(k, \omega, z)$ and $v_{\perp}(k, \omega, z)$ depend on a single parameter ($I$ for example). Consequently the power spectra will depend on $II^*$. This quantity can be
calculated from Equations (6, 7, 8) as a function of \( \varepsilon(k, \omega, 0) \) which is given by the lower boundary condition.

In order to compare the analytic results with the informations given by the observations, let us introduce the spatial spectrum \( \psi(k_{\perp}) \) and the temporal spectrum \( \Phi(\omega) \) defined by

\[
\psi_{v_x, \perp}(k_{\perp}) = \int F_{v_x, \perp}(k_{\perp}, \omega) \, d\omega, \\
\Phi_{v_x}(\omega) = \int F_{v_x, \perp}(k_{\perp}, \omega) \, dk_{\perp}.
\]

(9)

3. Discussion of the Results

3.1. General Characteristics of the Response

Before we present the results relative to the response of the isothermal atmosphere, we shall show that the non resonant character of the excitation applied at the basis of any atmosphere allows to predict some properties of the response. An excitation is non resonant if the generated perturbations are finite for any physical variable and for any horizontal wave number and frequency.

Let us consider an atmosphere with a given upper boundary condition excited in a non-resonant way. For each pair \( (k, \omega) \) the generated wave will have a vertical velocity equal to zero at all level such that the pair \( (k, \omega) \) is an eigen mode of the slab of the atmosphere above this level \( z \), with a rigid lower boundary condition; similarly the horizontal velocity will be zero at all level \( z \) such that the pair \( (k, \omega) \) is an eigen mode of the slab of the atmosphere above the level \( z \), with a zero eulerian pressure perturbation lower boundary condition. In the following we design for each level \( z \) by modes I and II the sets of the pairs \( (k, \omega) \) which satisfy respectively \( F_{v_x}(k, \omega, z) = 0 \) and \( F_{v_{\perp}}(k, \omega, z) = 0 \). We emphasise the fact that the zeros of the horizontal and vertical power spectra at the level \( z \) depend on the layers above this level and not on the exact way of the generation of the motions, as long as the excitation is non-resonant.

The perturbations generated by the excitation have an Eckart’s external mean energy given by the Equation (6), where \( F_{v_x}(k, \omega, z) \), \( F_{v_{\perp}}(k, \omega, z) \) and \( \varepsilon(k, \omega, z) \) are finite positive quantities, for a non-resonant excitation. Considering first the vertical velocity power spectrum we derive from the Equation (6) the following inequality

\[
F_{v_x}(k, \omega, z) \leq \varepsilon(k, \omega, z) \left( 1 + \frac{N^2(z)}{\omega^2} \right).
\]

(10)

The equality occurs for the modes II. If we represent graphically in a three dimensional space \( F_{v_x}(k, \omega, z) \) as a function of \( k \) and \( \omega \) at a given level \( z \), the corresponding surface \( S_{v_x} \) is anywhere situated below a surface named ‘envelope surface’ and which is defined by the right member of the inequality (10). The coordinates of the relative maxima of \( S_{v_x} \) are very close to the pair \( (k, \omega) \) corresponding to the modes II, where the surface \( S_{v_x} \) and the envelope surface are in contact. Consequently we obtain the following
interesting result which justifies this kind of approach: the positions of these maxima in the $k$-$\omega$ diagram merely depend on the structure of the layers above the level $z$. Their amplitudes are governed by the properties of the excitation and by the structure of the atmosphere. For a small frequency $\omega$, the equation of the envelope surface is proportional to $\omega^2 \varepsilon(k, \omega, z)$ and this quantity tends to zero as long as $\varepsilon(k, \omega, z)$ is bounded. To illustrate the above remarks, we have drawn in Figure 1 the section of $S_{v_z}$ and of the envelope surface by a plane $k = \text{const}$.

Fig. 1. Spatio-temporal power spectrum of the vertical velocity field as a function of $\omega$, $k$ and $z$ being given. The dashed curve is the section of the envelope surface by a plane $k = \text{const}$. The symbols * (or +) represent modes I (or II).

Considering now the horizontal velocity power spectrum, we can discuss similar properties from the following inequality, which results from the Equation (6):

$$F_{v_\perp}(k, \omega, z) \leq \varepsilon(k, \omega, z)\left(1 + \frac{\omega^2}{k^2c^2(z)}\right).$$

The situation is now reversed as compared with the case considered above: the zeros of $F_{v_\perp}(k, \omega, z)$ occur for the modes II and the maxima are located very close to the modes I, for which there is equality in relation (11). In the graphical representation the equation of the envelope surface is proportional to $\omega^{-2} \varepsilon(k, \omega, z)$ for large $\omega$ and this quantity tends to zero as long as $\varepsilon(k, \omega, z)$ is bounded. In Figure 2 we have plotted as a function of $\omega$ the values of $F_{v_\perp}(k, \omega, z)$ and the section of the envelope surface for a given $k$ and at a given level.

The evolution of the response with the altitude will depend on the structure of the atmosphere in particular through the corresponding displacement of the modes I and II in the $k$-$\omega$ plane. This displacement is illustrated in Figure 3 where we have plotted
the modes I and II corresponding to two altitudes \(z_1\) and \(z_2\) in the isothermal case. As it is well known in an isothermal atmosphere (Eckart, 1960) modes I and II can be classified in acoustic, gravity and evanescent modes. We see that for a given \(k\), when the altitude increases, the acoustic modes I and II have increasing frequencies, while the gravity ones have decreasing frequencies. The only mode which is weakly dependent on the altitude is the evanescent mode.

This property is interesting in two ways: first the properties of the response related to this mode are weakly dependent on the dimensional specification of the model representing the solar atmosphere; second the observed quantities are partly integrated in the \(z\) direction. We remark moreover that for large \(k\) around the maximum relative to the evanescent mode, as the positions of the zero and of the maximum are very near, the variation of the vertical velocity power spectrum is very steep around this maximum.

Before studying the particular case of the isothermal atmosphere, we will now discuss the form of the excitation which is defined by the value of \(\varepsilon(k, \omega, 0)\) which is the contribution of each oscillation mode \((k, \omega)\) to the mean energy of the perturbations at the basis of the atmosphere. A minimal mathematical condition for \(\varepsilon(k, \omega, 0)\) in order to have a finite value of the mean energy, is that the integral of \(\varepsilon(k, \omega, 0)\) in the \((k, \omega)\)-plane converges. Physically, we suppose that the excitation is generated by the turbulent motions at the top of the convective zone, but we do not have any information about the physical characteristics of this excitation. Stein (1967) gives the power spectrum of the acoustic energy generated by the turbulence, derived from Lighthill's method, but the results depend strongly of the adopted model of turbulence, particu-
larly of its characteristic dimension and velocity. Due to the lack of knowledge concerning the excitation, we will choose the function $\varepsilon(k, \omega, 0)$ as smooth as possible. The above general study has shown that for the spatio-temporal power spectra the $\omega-k$ dependence of $\varepsilon(k, \omega, 0)$ influences their amplitude but does not influence the $k-\omega$ location of the zeros and of the relative maxima: henceforth the general shape of these spatio-temporal spectra is weakly modified by the form of the excitation and in the following we have calculated these quantities with $\varepsilon(k, \omega, 0) = 1$. In Figures 1 and
2 the envelope curves correspond to this value of \( \varepsilon(k, \omega, 0) \). The temporal and spatial spectra, derived by integration, are calculated with 
\[
\varepsilon(k, \omega, 0) \sim e^{-k^2\lambda^2} e^{-\omega^2\theta^2}
\]
where \( \lambda^{-1} \) and \( \theta^{-1} \) are the widths in \( k \) and \( \omega \) of the excitation spectrum \( \varepsilon(k, \omega, 0) \). This choice implies that the shape of the envelope curve plotted in Figure 1 will have a maximum for
\[
\omega = \frac{N}{2} \left( -1 + \sqrt{1 + \frac{4}{N^2 \theta^2}} \right)^{1/2}
\]
and will tend to zero for large \( \omega \).

From the preceding discussion we retain in particular first that the shape of the spatio-temporal spectra at a given level, namely the location of the zeros and of the relative maxima, is determined by the structure of the atmosphere above this level, second that the behaviour of the vertical and horizontal power spectra are opposite, which can provide an observational test.

3.2. Response of the Isothermal Atmosphere

The response of an isothermal atmosphere with a thickness \( L = 5H \) bounded above by a free surface has been calculated in the following range of frequency and horizontal wave number: \( \theta = 1/\omega_c \) and \( \lambda = 2H \). This domain has been chosen in relation to the observations. The observations (White and Cha, 1973; Frazier, 1968) are limited to \( 6.5 \times 10^{-3} \) Hz which corresponds approximately to twice the acoustic cut-off frequency in the solar atmosphere at the temperature minimum. For the horizontal wave number the observations (Mein, 1966; Frazier, 1968) concern values of \( k \) below \( 5 \times 10^{-3} \) km\(^{-1} \) which corresponds to the inverse of the solar density scale height at the temperature minimum. In this \( k-\omega \) range we calculate the spatio-temporal power spectra of the velocity field \( F_{v_x}(k, \omega, z) \) \( F_{v_y}(k, \omega, z) \). By integrating and \( \omega \) and \( k \) we obtain respectively the spatial power spectra and the temporal power spectra. All these quantities are calculated at three levels: \( z = 0, z = L/2, z = L \).

3.2.1. Spatio‐Temporal Power Spectra

The results are given in Figures 4 and 4’ for \( \varepsilon(k, \omega, 0) = 1; F_{v_x}(k, \omega, z) \) and \( F_{v_y}(k, \omega, z) \) are plotted as functions of the frequency for various given wave numbers and at a given level.

These results illustrate the preceding general considerations (Section 3.1.) for the response of an atmosphere to a non-resonant excitation: comparing Figures 4a and 4’a and Figure 3a, we verify that at the level \( z = 0 \) the zeros of \( F_{v_x}(k, \omega, z) \) correspond to the modes I and those of \( F_{v_y}(k, \omega, z) \) to the modes II. The same property is verified at the level \( z = L/2 \) comparing Figures 4b and 4b’ and Figure 3b. If we take \( \varepsilon(k, \omega, 0) \sim e^{-\omega^2\theta^2} e^{-k^2\lambda^2} \) the results are similar, but the envelope curve exhibits then a maximum (cf. Section 3.1.). The horizontal and vertical initial kinetic energy is distributed in the frequencies of the modes I and II at the level of the excitation. The filtering caused by the propagation through the atmosphere shifts these frequencies towards the high
frequencies for the acoustical modes and towards the low frequencies for the gravity ones; the frequency is relatively stable for the evanescent mode.

3.2.2. *Temporal and Spatial Power Spectra.*

The temporal and spatial power spectra are derived from the spatiotemporal power spectra by integration respectively on the horizontal wave number and the frequency. Due to the rather complicated variation of the spatio-temporal power spectra, no general properties can be predicted for the integrated quantities. In Figure 5, repre-
Fig. 4c. Spatio-temporal power spectrum of the vertical velocity field as a function of ω, k and z being given. (--- 2Hk = 1.28; ---- 2Hk = 1.66; ······ 2Hk = 2.04).

Fig 4a.

senting the temporal power spectrum of the vertical velocity field, there appear two peaks in the power spectrum which correspond to the contribution of the gravity and acoustic modes II; the relative minimum between the two peaks is due to the lack of modes II with frequencies between the Väissälä frequency and the acoustic cut-off frequency. When the altitude increases, the frequency of the acoustic maximum is
shifted towards the high frequencies, while that of the gravity maximum is shifted towards the low frequencies. The ratio of the amplitude of the acoustic peak to the gravity one increases with the altitude.

The temporal power spectrum of the horizontal velocity field (Figure 5') is a decreasing function of the frequency with a small bump in the tail. The interpretation of
Fig. 5. Temporal power spectrum of the horizontal velocity field at various levels $z$. The notations are the same as in Figure 5.

We have plotted the normalized function

$$Y = \frac{(Y_{\text{max}}(z) - Y_{\min}(z))}{(Y_{\text{max}}(0) - Y_{\text{min}}(0))}$$

with $Y(\omega, z) = \hat{g}(\omega, z)$, $\Phi_{\omega}(\omega, z)$, $\hat{g}(0, z)$, $\Phi_{\omega}(0, z)$ at $z = L/2$; $z = 0$; $z = L$. 

© Kluwer Academic Publishers • Provided by the NASA Astrophysics Data System
Fig. 6. Spatial power spectrum of the vertical velocity field at various levels $z$. We have plotted the normalized function

$$Y = \frac{y(k, z) - y_{\text{min}}(z)}{y_{\text{max}}(z) - y_{\text{min}}(z)}$$

with

$$y(k, z) = q(z) \Phi_{v_z}(k, z).$$

(--- $z = 0$; ----- $z = L/2$; --- $z = L$).

Fig. 6'. Spatial power spectrum of the horizontal velocity field at various levels $z$. The notations are the same as in Figure 6.
this curve in relation to the distribution of the modes I in the $k-\omega$ diagram is not straightforward.

The spatial power spectrum of the vertical velocity field (Figure 6) is a monotonic decreasing function of the horizontal wave number. When the altitude increases, the importance of the large wave number increases; this means that the horizontal characteristic length of the oscillating elements decreases contrarily to the result obtained in the case of a semi-infinite isothermal atmosphere for isothermal perturbations (Souffrin, 1966).

The spatial power spectrum of the horizontal velocity field (Figure 6') has a maximum which is shifted towards the large $k$ for increasing altitudes; this leads us to the same conclusion as before, i.e. a decrease of the horizontal characteristic length. The fact that the small $k$ do not contribute to the spatial power spectrum of the horizontal velocity field is a consequence of the horizontal momentum conservation equation.

These results are obtained for an isothermal slab. It is likely that for an atmosphere with a more realistic distribution of temperature, the shape of the spatial and temporal power spectra at each altitude and their evolution with the altitude will depend on the variation of the temperature through the atmosphere. Thus the conclusion relatively to the evolution of the horizontal characteristic length of the oscillating motions may be modified.

4. Conclusion

From the preceding study of the response of an atmosphere, bounded above by a free surface upper boundary condition, excitated at its basis by a non-resonant way, the following conclusions emerge:

The shape of the spatio-temporal spectra of the velocity field at the level $z$, namely the location in the $k-\omega$ plane of the zeros and of the relative maxima (modes I and II) merely depend on the structure of the atmosphere between the level $z$ and the top of the atmosphere. The same remark is valid for the evolution with the altitude. The fact that this result is independent of the properties of the source spectrum justifies this kind of approach for the solar case where the form of the excitation remains unknown.

The shape of the excitation influences the amplitude of the relative maxima of the velocity power spectra and eventually determines an absolute maximum for some frequency. An essential characteristic of this non-resonant excitation is that it provides a velocity field with opposite behaviour for horizontal or vertical motions, i.e. a relative maximum of the spatio-temporal power spectrum of the vertical velocity corresponds to a zero of the horizontal one and conversely. The temporal power spectrum of the vertical velocity field shows two peaks which positions in $\omega$ vary with the altitude and the spatial power spectra of the velocity field give a decrease of the horizontal characteristic length. But these results obtained in the isothermal case will be modified when the temperature varies with the altitude.

What can be the relevance of the present discussion to the solar case? The complete calculation of the influence of the deep layers on the region where the oscillation is
observed is not yet made: as long as the discussion is limited to the models approximating the excitation by an 'oscillating lower boundary', it is important to test if the observed solar motions are compatible with an excitation resonant or not. On the basis of the present study, we see that the location of the maxima of the velocity in the $k$--$\omega$ plane varies with the altitude for a non-resonant excitation, while it is constant for a resonant one. These maxima coincide for the vertical and horizontal velocity in the resonant case while a maximum of vertical velocity corresponds to a zero of horizontal velocity and inversely in the non-resonant case. These two properties can provide a test to select one of the two kind of excitation. This implies that the observations relative to different altitudes have a spatio-temporal resolution good enough to obtain the spatio-temporal power spectra of the velocity field.

Most of the observations are made at the center of the solar disk, hence the derived information is relative to the vertical velocity. Actually observations on large solar surfaces are performed (Fossat and Ricort, 1973) but as the observed profiles are due to the radial component of the velocity field and moreover integrated on large solar surfaces, no information about the horizontal velocity can be derived.

The spatio-temporal power spectrum of the vertical velocity field has been derived by Mein (1966), Frazier (1968), Tanenbaum et al. (1969), Edmonds and Webb (1972a,b) from the observation of several spectral lines at the center of the solar disk. But the obtained informations on the variation with the altitude are not sufficiently precise to test if the observed solar motions are compatible with a non-resonant excitation.

The observations of Deubner (1972), Stix and Wöhl (1974) provide the spatio-temporal spectrum of the velocity field for a spectral line as a function of the position on the disk: from the center to the limb, a decrease of the five-minute oscillatory motions occurs and a low frequency component appears, which increases when the contribution of the horizontal velocity to the radial one increases. These results are compatible with a non-resonant excitation as can be seen in Figures 1 and 2: Figure 1 shows that the contribution of the low frequencies to the vertical motion is very small, while it is important for the horizontal motion (Figure 2).

The observations generally give a temporal power spectrum showing a peak about 300 s, shifted towards the high frequencies when the altitude increases because the low frequencies are damped (Noyes and Leighton, 1963). We note that the observed temporal power spectrum differs from the one defined in (9) by integration on all spatial frequencies, because the observation across a circular surface constitutes a low-pass spatial filtering, for which the transmission coefficient is an Airy function. The spatial power spectrum has not yet been derived with good precision: Fossat and Ricort (1973) intended to make future solar observations using optical spatial filters perfected by Fossat and Martin (1972), which will provide a better spatial resolution.

The preceding results obtained for an isothermal atmosphere must be considered as preliminary. They may be modified by the dependence of the temperature on the altitude. Calculations with a realistic solar model are in progress.
Acknowledgements

The author thanks Dr P. Souffrin for suggesting and discussing this problem, and Dr G. Berthomieu for her constant help and encouragement throughout this work.

References