The problem of diagnosing flare particle acceleration mechanisms from hard X-ray bursts is discussed, and it is argued that the electron trap model of bursts is more amenable to observational investigation at present than models of thick-target type. It is then shown that data for the large X-ray burst of 1972 August 4 are consistent with the source electrons being trapped in a very large vibrating coronal magnetic bottle. Furthermore, the observations show that the burst time profile is not dominated by collisional losses.

It is proposed instead that the entire profile is essentially determined by betatron action of the varying trap field on the electrons. This betatron model is then analyzed in detail and shown to predict very well the observed correlation of electron flux and spectral index in this event when it is supposed that the electrons are initially produced by runaway in a direct electric field. Comparison of the model with observations permits inference of the approximate form of magnetic field evolution in the trap. Finally the physics behind this field evolution is briefly considered.

Subject headings: flares, solar — X-rays, solar

I. INTRODUCTION

Solar hard X-ray bursts are widely recognized as a potentially valuable key to understanding particle acceleration processes in flares and as being particularly important since the emitting electrons (E = 20–200 keV) comprise the bulk of the energy in flare particles and sometimes in the entire flare (see, e.g., reviews by Kane 1974; Lin 1974; Brown 1975). Nevertheless, hard X-ray burst studies have in the main not led as yet to any major steps forward in quantitative understanding of electron acceleration, for several reasons. First, the greater majority of bursts studied are small and of short duration so that the count statistics, and the instrumental time resolution used, limit interpretation mostly to establishing general trends in the spectra and intensities of bursts (e.g., Kane 1974; Datlowe, Elean, and Hudson 1974). Second, when large events are observed with good count statistics and high time resolution, it is necessary to use a rapid reduction technique in order to obtain the dynamic X-ray spectra, with full instrumental time resolution, which are necessary for detailed quantitative inference of electron acceleration parameters (Brown 1971). This procedure has not generally been followed, instead, it has been replaced by considerations of only general burst trends over sample time intervals (e.g., Frost 1969; van Beek, de Feiter, and de Jager 1973). And, above all, on the theoretical front there is a lack of quantitative models predicting the intensity and spectral evolution of bursts, to which observations can be compared.

In this paper we develop such a model of electron acceleration to describe the main phase of a very large burst observed by the hard X-ray spectrometer aboard the ESRO TD-1A satellite (van Beek 1973; van Beek et al. 1973). The event in question was sufficiently large and prolonged to provide observations with excellent statistics, permitting accurate spectrum determination with the full instrumental time resolution of 1.2 s. Actual reduction of the data to over $10^3$ instantaneous spectra was made feasible by utilising the rapid pulse-height inversion technique established by Hoyng and Stevens (1974).

Two distinct types of model of electron acceleration in X-ray bursts prevail in the literature. First, there are those involving continuous injection of electrons into a target from an unspecified “black box” acceleration region. In these the burst time profile is determined purely by the modulations of the black-box electron source since the X-ray emission lifetime after injection into the target is very short, either because of energy loss in a dense plasma (thick target—e.g., Brown 1971) or because of escape from the emitting region (thin target—e.g., Datlowe and Lin 1973). Thus though the thick-target model, for example, is of interest in connection with optical and ultraviolet flare heating and in providing a useful reference standard (cf. § II), it does not lead to a model for the electron acceleration because this process is decoupled from the X-ray source. (This is equally true for a thin target.)

The second class of burst model is that in which energetic electrons are initially produced in a rather brief acceleration phase (Takakura and Kai 1966), again unspecified, and subsequently emit X-rays while trapped in a coronal magnetic bottle. According to this electron-trap model, the burst time profile is governed, after initial
Fig. 1.—Observed count rates (channels 1, 3, 5, and 6) for the large event of 1972 August 4. Time resolution is 1.2 s for channels 1–4 and 4.8 s for higher channels.
accleration, by the development of the fast electron flux and spectrum within the X-ray source itself, and so provides a direct diagnostic of conditions in the source. In particular, the simple impulsive (single spike) time profiles of some small events (Kane 1974) may be attributed to the collisional decay of the fast electrons in a static trap, the decay time yielding the ambient plasma density. Second, the softening of some such events in their decay phase (Kane and Anderson 1970) may be attributed to nonuniformity of the ambient plasma density in combination with an energy-dependent pitch-angle distribution of the injected electrons (Brown 1972). Larger and more complex events, however, cannot be interpreted in this way since their time profiles display multiple peaks, contrary to the monotonic decay characteristic of collisions. Brown (1973) has pointed out that, in these cases, the X-ray burst profile may be determined by oscillations of the trapping field which modulate the density of the trapped plasma and, more importantly, drive the trapped fast electrons. That is, the trap model must in fact invoke continuous acceleration of electrons to explain the profiles of large bursts but, distinct from continuous injection situations, the acceleration occurs as part of the behavior of the X-ray source itself and not as a decoupled mechanism.

In the present paper we elaborate the electron trap model further. First we show that the general features of the large 1972 August 4 event are compatible with a trapped electron X-ray source modulated by Alfvén oscillations, subsequent to a short initial acceleration. Then we derive quantitatively the effect of field changes on the trapped electron flux and spectrum in the trap model and show that these predictions are in good agreement with the observed interrelationship of the instantaneous flux and spectrum in the August 4 event (Hoyng et al. 1975) during the trapping phase. (The mechanism of acceleration in the initial phase is left unspecified.) This enables us to estimate the magnetic field and density evolution in the trap directly from the burst observations. Finally we briefly discuss possible origins of these field variations and their relationship to other observations.

II. THE 1972 AUGUST 4 EVENT

At 06:20 UT on 1972 August 4 one of the largest events of the present solar cycle occurred. The entire event was observed by the hard X-ray spectrometer, aboard TD-1A, which has 12 channels, with approximately logarithmically spaced boundaries—viz., 29, 41, 53, 71, etc., keV—this burst being detectable up to channel 8 (i.e., about 350 keV). Time profiles for channels 1, 3, 5, 6 are shown in Figure 1.

Hoyng et al. (1975) have carried out a detailed temporal analysis of this and other events using the full 152 time resolution of the instrument. First, using the technique established by Hoyng and Stevens (1974), the counts were converted to photon spectra for each instrumental integration period. Next, though slight deviations from a single power-law were present in these spectra (cf. van Beek et al. 1974; see also § IV), the instantaneous best-fit power laws

\[ I(\epsilon) = a(t) \epsilon^{-\gamma(t)} \]

were determined. As part of the routine data reduction, the thick-target electron flux was then derived according to (Brown 1971)

\[ \mathcal{F}(t) = 4.1 \times 10^{25} \epsilon^{-\gamma} \left( 25^{-\gamma} B(\gamma - 1) \right)^{-1} \epsilon^{-1}, \]

with \( a(t) \), \( \gamma(t) \) obtained from equation (1). (Equation [2] and subsequent equations correct a numerical error in Brown [1971] but omit the factor due to bremsstrahlung on ions other than protons.) \( \mathcal{F} \) is the instantaneous rate of injection of electrons with energy exceeding 25 keV, into a thick target, which would be needed to produce the burst. (The figure of 25 keV is adopted merely as a convenient reference energy not involving significant spectral extrapolation beyond the directly observed energy range.) Use of the thick-target parameter \( \mathcal{F} \) in the initial reduction was motivated by two factors. First, since the electrons injected into a thick target lose all their energy collisionally, the bremsstrahlung efficiency is a maximum and \( \mathcal{F} \) sets lower limits to the total number and energy of electrons needed for burst production (Brown 1975; Hoyng et al. 1975). Second, and more important here, the thick-target flux \( \mathcal{F}(t) \) together with \( \gamma(t) \) suffices to define the time evolution of any burst—i.e., the thick-target model is a two-parameter description. (This is because, by definition, the thick-target density is high enough for total collisional braking of electrons, resulting in a bremsstrahlung flux independent of the actual high density [Brown 1971].) Any other model requires a third parameter—the ambient density \( n \) for an electron trap and the target column thickness \( \Delta N \) for a thin target (Brown 1971, 1975). Now we saw in § I that the electron trap model can only explain a complex time profile if in oscillation, implying \( n = n(t) \). Therefore, since the X-ray flux from \( n \) trapped electrons depends on \( n(t) N(t) \), presentation of burst data in terms of variation of energetic electron numbers would require, in a trap model, specification, in advance, of a particular form for \( n(t) \). This masks the time evolution of \( N(t) \) behind a function \( n(t) \) of time which \( a \) priori may be arbitrary. We have therefore followed the procedure of reduction to the two parameters \( \mathcal{F}(t) \), \( \gamma(t) \) being the parameters of an equivalent thick target. This allows easy intercomparison of different bursts and also ready deduction of the parameters for any specific models in a particular burst, using the model interrelationships derived by Brown (1971, 1975)—e.g., of \( N(t) \), \( n(t) \) for a trap—see § III. That is, we use the thick-target flux as a reference and as a computational device in data reduction, but without implying that the thick target is the correct source model.

Figure 2 shows the time development of \( \mathcal{F} \) and of \( \gamma \) through the August 4 event, together with the count rate in channel 1. Note that, for clarity, only every fifth data point (i.e., 6 s spacing) has been included in these profiles.
Fig. 2.—Time development of the X-ray spectral index $\gamma$ and thick target electron flux, $\mathcal{F}$ computed from the data in Fig. 1 using linear interpolation to fill small data gaps. In the $\mathcal{F}$ profile, the integrals show respectively the total number and energy of electrons injected ($E_i > 25$ keV) on a thick-target interpretation.

Channel 1 data are again shown at the top of the figure for comparison purposes, and letters A–E refer to corresponding points in Fig. 3. The bottom profile refers to ambient field and density variations inferred from the betatron model (see §§ II and IV).
Also shown in Figure 2 are the total number and energy of electrons above 25 keV which have to be injected into a thick target to produce the entire event.

The relatively minor influence of statistical fluctuations in the observations is evident from the smoothness of variation of the fitted spectral index $\gamma$. In particular, this permits comparison of instantaneous values of the electron flux $\Phi$ and the X-ray spectral index $\gamma$ with the result shown in Figure 3 as a plot of $\Phi(t)$ against $\gamma(t)$ (after Hoyng et al. 1975). What is most striking in this figure is its division into the “dogleg” portion $AB$, swept out by the event during the brief initial rising phase $AB$ in Figure 2, and the portion $CE$ indicating strong correlation of $\Phi$ and $\gamma$ throughout the remainder of the event. The time sequence through this latter (main) portion, is also quite distinctive. After the initial rise $AB$, the event follows a series of counterclockwise loops through Figure 3, centered about the general line of correlation, each loop corresponding to one of the large oscillations visible in the original time profile (Fig. 2). As the event profile decays (subsequent to about 06h32m) and the oscillations die out, the loops in Figure 3 transform into a monotonic path down the curve $DE$.

Physical interpretation of the part $CE$ of Figure 3 in terms of an electron trap is the principal aim of the present paper. As already indicated in § I, interpretation in terms of a thick target will require theoretical treatment of, for example, stochastic acceleration in a plasma turbulent region of field reconnection. No solution of this problem seems imminent. In an electron trap, on the other hand, the electron acceleration is governed only by time variations.

Fig. 3.—Correlation diagram obtained by eliminating time between $\Phi(t)$ and $\gamma(t)$ in Fig. 2, again showing only every fifth data point for clarity. The solid line (based on all data points) indicates the smoothed path of the event development, discussed fully in the text. Note that point $C$ is passed several times and so does not refer to a definite time in Fig. 2 but to an extremum flux.

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in the trapping field intensity, and this is amenable to a much simpler description. Before proceeding to obtain this description in § III, we present some general evidence that an electron trap may indeed have been involved in the August 4 event, and derive some constraints on the parameters involved.

First, the sharp division in form between the main event CE in Figure 3 (lasting about 15 minutes) and the rapid initial rise AB (lasting about 1 minute) is indicative of distinct physical processes governing the two phases and, in particular, is consistent with the trap-model postulate of an impulsive acceleration phase followed by prolonged burst development under quite different conditions.

Second, Hoyng et al. (1975) have found real periodicities in the time profile of this event with periods up to about 120 s, the period apparently increasing with time. (Periodicities have previously been reported in large burst [Frost 1969; Parks and Winckler 1969]). According to our trap interpretation this 120-s period should correspond in order of magnitude to the travel time of a magnetoacoustic wave across the typical dimension L (cm) of the trap. We take this for the present to have the Alfvén speed (cf. § V) which, if the field is $H$ (gauss) and the plasma density $n$ (cm$^{-3}$), implies the relation

$$\left(\frac{L}{H}\right)n^{1/2} \approx 2.6 \times 10^{13}$$

(3)

General oscillatory behavior is in fact observable in the original time profiles (Fig. 1) and is seen to persist, though with decreasing amplitude, right to the end of the event (note especially the 53–76 keV channel). As discussed in § I, such behavior is incompatible with a collisionally governed time profile. Therefore, we have the further condition that the mean density $n$ should be sufficiently small not to totally obliterate the burst oscillations observed right down to 30 keV. This requires that the energy loss time $t_L$ of a 30-keV electron should not be less than 1000 s so that, using the formula for $t_L$ given by Brown (1972),

$$n \leq 2 \times 10^7 \text{ cm}^{-3}.$$  

(4)

We will see later that the model requires that $n$ should decrease substantially toward the end of the event. We estimate that the initial density $n_0$ of the plasma in which the electrons are initially accelerated (phase $AB$) could be as high as

$$n_0 \approx 4 \times 10^7 \text{ cm}^{-3}$$

(5)

without violating the secondary role to be played by collisions throughout the event.

This figure determines the total number $N$ of electrons above 25 keV which must be present in the trap to produce the observed burst intensity. The nonthermal emission measure $nN$ deduced for this event (Hoyng et al. 1974) was about $5 \times 10^{38}$ so that

$$N \approx 10^{38},$$

(6)

which corresponds to a total energy of about $5 \times 10^{31}$ ergs of electrons. If the magnetic field is to be capable of containing these particles, we obtain a final condition that its energy density $H^2/8\pi$ should be least comparable to theirs, viz.,

$$HL^{3/2} \geq 3 \times 10^{16}.$$  

(7)

The conditions (3)–(7) imply $L \geq 1.1 \times 10^{10}$. However, if we take $L \approx 10^{10}$, then the number density of fast electrons $n_L = N/L^3$ is larger than the ambient plasma density $n$, which seems implausible. We are therefore driven to adopt a large value for $L$, giving, as a typical set of parameters,

$$L \approx 5 \times 10^{10} \text{ cm}, \quad H \approx 12 \text{ gauss}, \quad n_0 = 4 \times 10^7 \text{ cm}^{-3}, \quad n_L = 8 \times 10^9 \text{ cm}^{-3}.$$  

(8)

These parameters, though involving a strikingly large volume, are quite compatible with direct observations of coronal active features obtained by various means—cf. Newkirk (1967); Vaiana et al. (1973); McQueen et al. (1974). These observations are also consistent with our assumption, in formula (7), that the trap volume is $L^3$, i.e., that the trap has a thickness comparable to its length. Furthermore, it must be reiterated that this flare was one of the largest on record. Interpretation of certain large events in terms of coronal traps has already been suggested by Hudson (1973) on the grounds of the visibility of X-rays even when the associated chromospheric flare is far behind the limb, a prime example being the event of 1969 March 30 (Frost and Dennis 1971) which shows similarities to the event we are studying here. Both the limb occultation and data on the closely related radio phenomena (Hudson 1973) indicate an X-ray source extending very high in the corona. We show further in the Appendix that for the above parameters, the model predicts the GHz radio observations quite well.

More definite figures for the trap parameters will, however, have to be based on a proper assessment of the motion of waves along the trap. In particular, the energy density $n_L\dot{E}$ of fast electrons is seen from equations (8) to be at least comparable, and probably in excess of, the thermal energy density $n_LkT$ of the ambient plasma. Parker (1965) has shown (in connection with galactic cosmic rays) that, under these conditions, wave modes exist propagating at near the sound velocity in the suprathermal gas. With the parameters given in (8) and with $\dot{E} \approx 20$
keV, this sound velocity exceeds the Alfvén speed by an order of magnitude. To obtain the same oscillation time (cf. eq. [7]), the trap would then also have to be that much larger. However, this would imply $n_1 \tilde{E} \ll n_0 k T$ and a situation again controlled by Alfvénic motions, so that the true dimensions of the trap must be somewhere between these limits.

It should be emphasized, of course, that neither the above considerations nor the following trap analysis and its agreement with observations precludes a thick target interpretation of events. The observational status of both models has been reviewed by Kane (1974) and Brown (1975) among others, with the conclusion that they may both be reconciled qualitatively with the data. Indeed, the inconclusive nature of this qualitative approach seems to demand the development of quantitative analyses of models and comparison with good data (in particular the acceleration characteristics). Here we hope to initiate this approach by treatment of the comparatively simple case of a vibrating trap but with the intention that future analyses will render the thick target amenable to the same quantitative comparison with observations.

### III. THE BETATRON MODEL

In describing the effect of the changing magnetic field on the trapped plasma and energetic electrons we take account only of changes in the longitudinal field—i.e., we suppose the electrons to be confined in a cylindrical region whose principal time variation is a changing cross section [$H = H(t)$] only $H \approx H(S)$ where $S$ is the coordinate along the tube]. Since the tube is of finite length, there must in fact be a transverse field component to trap the electrons and, since the tube ends are tied at the photosphere, this transverse field must vary with time in the trapping region, as shown schematically in Figure 4. These moving transverse field components will give rise to some Fermi acceleration of the trapped electrons, but we show shortly that this may be negligible compared with acceleration by the changing longitudinal component and so concentrate on the latter in the present paper (§IVb).

Since we are neglecting collisional energy losses and since the electron Larmor radii are very small compared with the scale distance of field variation, the electron energy changes due to longitudinal field variations are given by the adiabatic invariance of their magnetic moments (e.g., Spitzer 1962)—just as in a betatron accelerator. That is,

$$E_{1} / E_{1o} = H / H_{o},$$

where $E_1$, $E_{1o}$ are the energies in transverse motion ($=p_{r}^{2}/2m$) when the field has values $H$, $H_{o}$, respectively. Thus betatron acceleration occurs on an $e$-folding time scale $\tau_B$ equal to that for magnetic field changes in the trap, that is,

$$\tau_B \ll L / v_A,$$
where \( v_A \) is the Alfvén speed. Fermi acceleration by reflection between traveling transverse field changes, on the other hand, will occur on an e-folding time scale \( \tau_F \) which can be as long as

\[
\tau_F \approx \frac{v}{4v_A} \frac{L}{v_A} \approx \frac{v}{4v_A} \tau_B
\]

—e.g., by modification of Sweet’s (1969) expression to nonrelativistic energies—depending on the geometry of the magnetic mirror motions. It follows that \( \tau_F \gg \tau_B \) and Fermi acceleration may be neglected (cf. § IVb).

To relate equation (9) to the changes produced in the flux and spectrum of all the fast electrons, we require to know first the flux and spectrum at one instant, and second the relationship between the transverse energy \( E_L \) and total energy \( E \) of the electrons (i.e., their pitch angle distribution) at some instant. We take this reference instant to be the time at which the initial phase of acceleration of electrons in the trap ceased—i.e., the end of phase \( AB \) in Figure 3, namely, at 06h22m54s. Quantities pertaining to this instant are given the subscript zero, i.e., the field is \( H_0 \), the plasma density \( n_0 \), and so forth. Directly from the observations we infer \( \gamma_0 = 3.4 \), \( S_0 = 4.6 \times 10^{36} \text{ s}^{-1} \), while the distribution of energetic electrons (electrons per unit total energy \( E_0 \)) is

\[
N_0(E_0) = (\delta_0 - 1) \frac{\mathcal{N}_0}{E_1} \left( \frac{E_0}{E_1} \right)^{-\delta_0},
\]

where \( \mathcal{N}_0 \) is the total initial number of electrons present with energies greater than the reference energy \( E_1 \) (which we later set equal to 25 keV to correspond to \( \mathcal{S} \)) while the electron spectral index \( \delta_0 \) is related to the X-ray spectral index \( \gamma_0 \) by (Brown 1971)

\[
\delta_0 = \gamma_0 - \frac{1}{3}.
\]

The effect of the field changes on the total energy \( E \) is given by

\[
E = E_\perp + E_\parallel = (H/H_0)E_{t0} + E_{t0} = E_0(1 + hE_{t0}/E_0),
\]

where

\[
h = H/H_0 - 1
\]

and \( E_{t0}/E_0 = \sin^2 \alpha_0 \) where \( \alpha_0 \) is the initial electron pitch angle.

As regards the initial distribution of pitch angles, we cannot predict this in detail without a quantitative model of phase \( AB \) of the burst. Here we suppose the initial acceleration to have occurred by the action of a large-scale electric field. Under these conditions the acceleration occurs by increase of the energy \( E_\parallel \) along the magnetic field only, \( E_\perp \) being unchanged. It does not seem unreasonable, therefore, to treat \( E_{t0} \) as a constant for all fast electrons, to be interpreted as the mean transverse energy of the electrons that run away under the action of the electric field on the plasma. Since it is outside the scope of the present paper to treat this process quantitatively, we henceforth treat \( E_{t0} \) as a free parameter and write

\[
E^* = E_{t0}.
\]

Combining equations (12), (14), and (16), we thus obtain, by continuity, the spectrum \( N(E, h) \) of trapped electrons at any time subsequent to the injection phase (\( h \) parametrizing time)

\[
N(E, h) = N_0(E_0) \frac{dE_0}{dE} = (\delta_0 - 1) \frac{\mathcal{N}_0}{E_1} \left( \frac{E - hE^*}{E_1} \right)^{-\delta_0}.
\]

From equation (17) we now obtain the total number of electrons and their effective spectral index, at any time.

The total number of electrons above energy \( E_t \) now becomes

\[
\mathcal{N}(h) = \int_{E_t}^{\infty} N(E, h) dE = \mathcal{N}_0(1 - hE^*/E_1)^{-\delta_0 + 1}.
\]

Since the spectrum (17) is not exactly a power law, we must define a power law index \( \delta \) to be associated with it. For this we take the logarithmic point slope at an energy \( E_t \) near to the lowest channel boundary of the X-ray spectrometer used to obtain the data—this corresponds as closely as possible to the technique actually used in fitting the X-ray data to a power law. That is, we take (using eq. [17])

\[
\delta(h) = -\left( \frac{\partial \log N}{\partial \log E} \right)_{E=E_t} = \delta_0(1 - hE^*/E_1),
\]

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which is related to the X-ray index $\gamma$ at the same instant by (cf. eq. [13])

$$\delta = \gamma - \frac{1}{2}. \quad (20)$$

For purposes of comparison with Figure 3, we first need to relate $N$ to the thick-target flux parameter $F$ used in reducing the observations. Using the equations of Brown (1971), we have

$$F = \frac{\gamma - 3/2}{\gamma - 3/2} \frac{n}{n_0} N. \quad (21)$$

As far as the ambient density $n$ is concerned, with our assumption of one-dimensional trap geometry, this is simply given by

$$n/n_0 = H/H_0 = 1 + h. \quad (22)$$

Now $N/N_0$ is given by equation (18) as a function of $h$; substitution of this and equation (22) in (21) would give $F/F_0$ as a function of $h$. However, we actually require to have $F/F_0$ as a function of $\gamma$. We can express $h$ in

![Graph showing theoretical $(F, \gamma)$ correlation lines based on the betatron model for three values of the parameter $E*$, superposed on the observed data. According to the basic model, the point $(\gamma, F)$ is constrained to move on one of these lines, the actual motion depending on $h(t)$.](image)

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BETATRON ACCELERATION IN A SOLAR X-RAY BURST

Fig. 6.—Electron spectra (eq. [17]) due to betatron action on the trapped electrons for initial and extreme values of the field ratio.

\[ h = \frac{\gamma - \gamma_0 E_i}{\gamma - \frac{1}{2} E^*} \]  

(23)

and consequently we obtain

\[ \frac{F}{F_0} = \frac{\gamma - 3/2}{\gamma_0 - 3/2} \left[ 1 + \frac{\gamma - \gamma_0 E_i}{\gamma - \frac{1}{2} E^*} \right] \left[ 1 - \frac{\gamma - \gamma_0 E_i}{\gamma - \frac{1}{2} E^*} \right]^{\gamma_0 + 3/2}. \]  

(24)

Using the initial values of \( \gamma_0, F_0 \) (at 06h22m54s) already quoted and setting \( E_i = 25 \text{ keV} \) (which defined \( F \) in Fig. 3), \( E_i = 30 \text{ keV} \) (to correspond to the lowest channel boundary), we show in Figure 5 the results of prediction (24), as compared with the data, for values of the parameter \( E^* = 7.5, 15, \) and \( 20 \text{ keV} \). Evidently the model predicts the observed \( (F, \gamma) \) correlation well and is not highly sensitive to the choice of \( E^* \). The best agreement with observations is obtained for \( E^* \approx 15 \text{ keV} \) which, interpreted as a runaway energy, implies an initial plasma with a thermal energy \( kT \) a few times smaller than \( E^* \), i.e., \( T \approx 3-5 \times 10^7 \text{ K} \), a result in very satisfactory agreement with soft X-ray measurements of the high-temperature plasma of large flares (Neupert 1969).

In addition, the range of \( h \) covered by the correlation in Figure 5 permits us to calculate the extremes of deviation of the electron spectrum (17) from a pure power law. In Figure 6 we show that the electron spectrum attained at the extreme peaks of the burst (\( h \approx 0.2 \)) is very slightly concave upward while near the end of burst decay (\( h = -0.8 \)), and to a lesser extent at burst minima, the electron spectrum shows an increase in \( \gamma \) of about 0.7 in the index over the energy range 25–150 keV. Bearing in mind that these have to be integrated over the bremsstrahlung cross section to obtain X-ray spectra and that we have neglected relativistic effects (among others, cf. § IV), we conclude that the predicted spectrum does not differ significantly from observations.

IV. COLLISIONAL AND OTHER CORRECTIONS

We conclude from Figure 5 that betatron acceleration of electrons injected into a coronal trap by a direct electric field could be a realistic model of the 1972 August 4 event. The basic reason for the qualitative agreement of the
model with the observed \((\mathcal{F}, \gamma)\) correlation is that, since electric field acceleration results in smaller pitch angles for higher-energy electrons, these are least affected by betatron action on the transverse energy. Thus as the magnetic field increases, lower-energy electrons are accelerated most and the spectrum softens \((\gamma\) increases) while the flux increases—and conversely for decreasing field—just as observed. The fact that the degree of energy/pitch-angle dependence, needed to fit the observations quantitatively, leads to a realistic value for the runaway energy \(E^*\) lends support to the model.

However, the agreement with our prediction of \(\mathcal{F}(\gamma)\) (eq. [24]) can be seen to be imperfect. In particular, at higher values of the flux \(\mathcal{F}\) (i.e., of the field \(H\)), the data points mostly fall at smaller \(\gamma\) than our predictions—i.e., the burst does not simply sweep back and forth along the predicted correlation line. Though we do not propose to pursue refinements and variations of the model in detail here, we may check whether the relaxation of various idealizations we have made could explain the form and magnitude of the deviations from the model.

\(\text{a) Collisions}\)

A complete treatment of the effect of collisions would require solution of the full continuity equation for the energetic electrons, allowing for collisional energy loss and scattering and for the time dependence of the ambient density. Since, however, collisional effects are, by hypothesis, small, we can obtain an estimate of their effect from a perturbational correction to equation (14). This perturbation solution yields

\[
E \approx E_0 + hE^* - K\frac{E_0 - E^*}{E_0^{3/2}} \tilde{m}(t),
\]

where \(\tilde{m}\) is the mean plasma density through the event and \(m(t)\) is a factor of order unity, depending on the detailed form of \(h(t)\), \(t\) being the time after initial injection and \(K \approx 2\pi e^4 \Lambda(2m_e)^{1/2}\) is the constant in the Coulomb energy-loss cross section, with \(e, m_e\) the electron charge and mass, respectively, and \(\Lambda\) the Coulomb logarithm. If \(m\) is approximated to unity, inclusion of the collision term in equation (17) leads to the results

\[
\mathcal{F}' = 1 - \frac{(\gamma_0 - 3/2)}{(1 - hE^*/E_1)^{3/2}} \tau
\]

and

\[
\gamma' - \gamma = -\frac{3}{2} \frac{E_0^{1/2}}{E_1^{1/2}} \frac{\gamma_0}{(1 - hE^*/E_1)^{5/2}} \tau,
\]

with

\[
\tau = K\tilde{m}/E_1^{3/2}.
\]

Here, \(\mathcal{F}', \gamma'\) are the results including collisions, \(\mathcal{F}, \gamma\) the results without collisions, and \(\tau\) is a dimensionless time. Elimination of \(\tau\) between equations (26) and (27) shows the effect of collisions on Figure 5 to be displacement of each point on the correlation curve (24) along a curve (in fact, a straight line in a [\log \mathcal{F}, \gamma]-diagram) to smaller \(\mathcal{F}\) and \(\gamma\)—i.e., to the left and downward, the displacement increasing with time. Taking \(\tilde{m} \approx 2 \times 10^7\) cm\(^{-3}\) and \(t = 200\) s in the above equations yields \(\gamma' - \gamma \approx -0.5\) and \(\mathcal{F}'/\mathcal{F} \approx 0.7\) as the order of the displacement due to collisions during the three highest peaks in the burst (Fig. 2). Thus collisions move the upper part of the predicted correlation curve (Fig. 5) in the right sense and by the right order of magnitude to explain the spread of data points to the left.

\(\text{b) Nonuniform Field}\)

In § III we took the trap field to be purely longitudinal. The curvature actually present in the trap field will modify some of our equations based on one-dimensional geometry (e.g., both [9] and [22]). We do not attempt to evaluate this effect here but merely note that spatial variation of our equations through the X-ray source will add to the spread in Figure 5. As noted in § III, however, the existence of a transverse component in the oscillating field also results in some Fermi acceleration of the trapped electrons. Though this may be slow (eq. [11]) compared with the betatron process, the actual rate depends on the proportions of “head-on” and “overtaking” collisions of electrons with the moving transverse field components and so on the wave forms on the trap. We postpone details of this Fermi process for separate analysis, but note here that it increases the longitudinal component of electron energy. Its effect will therefore be to move points to the left in the upper part of Figure 5.

The actual path taken by the burst through the correlation diagram (Fig. 3) is not explained by these general considerations of effects (a) and (b). This we leave for future consideration, but note that the incorporation of these refinements in the model appears capable of explaining the small discrepancy of the results of § III from the observations.
V. THE MAGNETIC FIELD CHANGES

The actual behavior of $h$, $\mathcal{F}$, and $\gamma$ as functions of time has so far not been brought into our physical description of the betatron model, time entering the equations only via $h$. [In order to derive $h(t)$ theoretically, we would have to solve a very complicated initial-value MHD problem.] If the actual burst data followed the predicted correlation line in Figure 5 exactly, however, we could derive completely the evolution of $h(t) = H(t)/H(0)$ by evaluating $h$ from equation (23) and the observed $\gamma(t)$. In practice this procedure will lead to some spurious element in the line in Figure 5 exactly, however, we could derive completely the evolution of $h(t) = H(t)/H(0)$ by evaluating $h$ by the ATM coronograph (McQueen et al. 1974). Field oscillations have, on the other hand, been invoked in compared with the expansion of coronal magnetic bubbles on a larger time and distance scale observed directly to solve a very complicated initial-value MHD problem. If the actual burst data followed the predicted correlation indeed comprise an oscillatory component (due to wave reflection at the feet of the tube) superposed on an overall problem would seem to need investigation most urgently. Specifically, though the response may be expected to trapping flux tube to the injection of a large amount of energy in the form of fast electrons. This aspect of the

Field oscillation (due to the fast particle pressure), it is unclear why the expansion time scale should be long compared to the oscillation time (Fig. 2), since naively those might both be expected to be $\sim \frac{1}{\tau_{\text{coll}}}$.

Proper analysis of this problem, will, however, have to take account first of the interaction of the flux tube and the surrounding active region atmosphere and second of the presence of the suprathermal wave modes (Parker 1965) in the trap.

Finally it is of interest that, in the model we have proposed, the energy initially released in the form of energetic electrons, presumably by dissipation of the active-region field, is ultimately returned to the field (minus a small collisional loss) when it expands under the pressure of the trapped particles. This state of affairs reduces the amount of energy which has to be dissipated ultimately from the active region field as a whole in order to produce a hard X-ray burst. The problem of the huge number of electrons involved in the burst remains, however (cf. Brown 1975; Hoyng et al. 1975), and suggests that the electrons may be supplied from the low dense regions of the arch in the initial phase (cf. Fig. 4).

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APPENDIX

EVALUATION OF EXPECTED GHz RADIO FLUX

Takakura and Scalise (1970) have computed the volume emissivity of cyclotron radiation of an isotropic electron distribution in a homogeneous field. Since these computations were performed for a spectrum $N(E) \sim E^{-3}$, we may readily adapt the results to estimate the expected flux density from our electron trap model. Taking an aspect
angle of $60^\circ$ and neglecting reabsorption and plasma effects (which limits the application to high frequencies), one finds

$$F_v \sim 2 \times 10^{-39} \mathcal{N} v_H (v_H / v)^2 - 5$$

(29)

where $F_v$ is in solar flux units ($10^{-22}$ W m$^{-2}$ Hz$^{-1}$) and $v_H$, the cyclotron frequency, in MHz; from (6) and (8): $\mathcal{N} \approx 10^{10}$ and $v_H = 34$ MHz. In (29) an extreme relativistic approximation is made, limiting its validity to $v/v_H \lesssim 10^2$ or $v \lesssim 4$ GHz. Comparing the observations (Croom and Harris 1973) with the predictions, one gets

$$F_{19\text{GHz}} \approx 1.2 \times 10^5 \quad \text{(observed: } \sim 2 \times 10^5)$$

$$F_{37\text{GHz}} \approx 6 \times 10^4 \quad \text{(observed: } \sim 1 \times 10^4)$$

The agreement is within an order of magnitude, which is very good considering the number of approximations involved.

In this frequency range the similarity in the observed hard X-ray and GHz time profiles themselves is also very good. The agreement is less good between both the time profiles and the fluxes at both higher and lower frequencies (the latter partially attributable to absorption effects). By and large, however, it can be said that the GHz observations are consistent with the electron trap model. The radiation at these high frequencies originates from harmonic number $\sim 10^3$ and is emitted exclusively by the (extreme) relativistic electrons, $E > 500$ keV.

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