A POSSIBLE WIDTH-LUMINOSITY CORRELATION OF THE Ca II K1 AND Mg II k1 FEATURES

T. R. Ayres, J. L. Linsky, and R. A. Shine
Joint Institute for Laboratory Astrophysics, University of Colorado and National Bureau of Standards, Boulder, Colorado

Received 1974 November 7

ABSTRACT

Existing high resolution stellar profiles of the Ca II and Mg II resonance lines suggest a possible width-luminosity correlation of the K minimum features. We show that such a correlation can be simply understood if the continuum optical depth of the stellar temperature minimum, τc*, is relatively independent of surface gravity as suggested by three stars studied in detail.

Subject headings: atmospheres, stellar — Ca II emission — chromospheres, stellar — line profiles

I. INTRODUCTION

Several strong resonance and subordinate lines in the spectra of late-type stars show width-luminosity correlations. The most notable of these correlations are the Wilson-Bappu effect obeyed by the Ca II 3934 Å K-line emission core (Wilson and Bappu 1957); an analogous effect shown by the ultraviolet Mg II 2796 Å k-line (Kondo et al. 1972; Moos et al. 1974); and a weaker correlation seen in the Hα absorption core (Kraft, Preston, and Wolff 1964; LoPresto 1971; Fosbury 1973). In addition, Lutz, Furenlid, and Lutz (1973) have shown that the extensive damping wings of the Ca II H and K lines broaden with increasing luminosity in much the same way as the line cores.

In this letter we present evidence for a width-luminosity relation obeyed by the K1 and k1 minimum features (see, e.g., fig. 1a) in the inner wings of the Ca II and Mg II resonance lines. The K1 width, ΔλK1, is physically significant because it is a rough measure of the location of the stellar temperature minimum in line optical depth units. We suggest that the observed correlation of ΔλK1 with luminosity can be attributed to the same mechanism that broadens the line wings—the well-known variation of photospheric mass column density (g cm⁻²) with gravity (Aller 1963; Avrett 1972; Peytremann 1972; Lutz et al. 1973)—if the continuum optical depth of the stellar temperature minimum, τc*, is roughly independent of gravity.

II. OBSERVATIONAL CORRELATIONS

The observations discussed here include all the high-resolution data presently available. Although these observations were made with different instruments and techniques, some photometric and some photographic, the inhomogeneity of the data sources is not a serious drawback because the K1 and k1 widths depend only on the relative intensities in each profile and on the instrumental functions. Hopefully we have minimized the latter influence by limiting our sample to high-resolution data. Furthermore, we consider only spectral types F–K and luminosity classes III–V in order to avoid possible misinterpretation of distorted profiles, such as those produced by mass loss and bulk motions in the extended envelopes of late-type supergiants (Liller 1968).

The K1 widths for seven stars and the analogous k1 widths for six stars are given in table 1, and are plotted against absolute visual magnitude in figure 1b. The K1 widths for β Gem and α Tau were measured at lower dispersion than the other five stars, but the α Tau emission width W0, measured from the same profile, is in good agreement with the value given by Fosbury (1973), so instrumental corrections for α Tau and β Gem are probably unnecessary.

We find that the data in figure 1b can be represented by a relationship of the form

\[ M_V \approx (-12 \pm 2) \log \Delta \lambda_{K1} + \text{const}, \]

which is consistent with the six k1 widths for

\[ \log (\Delta \lambda_{k1}/\Delta \lambda_{K1}) \approx 0.40-0.45. \]

We can convert this empirical dependence of ΔλK1 on M_V to a more useful form using the method of Reimers (1973), which applies a bolometric correction to M_V to obtain ΔλK1 as a function of \( M_{bol} \), and then inverts \( L \approx 10^{-0.4(M_{bol}+5)} \propto \rho R^2 T_{eff}^4 \) via a mass-luminosity relation to get

\[ \Delta \lambda_{K1} = \Delta \lambda_{K1}(\rho, T_{eff}), \]
TABLE 1
Stellar Parameters, $K_i$ and $k_i$ Widths

<table>
<thead>
<tr>
<th>Star</th>
<th>Spectral Type</th>
<th>$K_V$*</th>
<th>log $K_i$(km s$^{-1}$)</th>
<th>Approximate Spectral Resolution (Å)</th>
<th>Approximate Spectral Resolution (Å)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sun</td>
<td>G2 V</td>
<td>+4.83</td>
<td>$1.37 \pm 0.02^{(1)}$</td>
<td>$\leq 0.04$</td>
<td>$1.89 \pm 0.05^{(1)}$</td>
</tr>
<tr>
<td>α Boo</td>
<td>K2 II</td>
<td>-0.20</td>
<td>$1.82 \pm 0.03^{(2)}$</td>
<td>$\leq 0.04$</td>
<td>$1.93^{(3)}$</td>
</tr>
<tr>
<td>α Cen A</td>
<td>G2 V</td>
<td>+4.35</td>
<td>$1.47 \pm 0.05^{(7)}$</td>
<td>$\sim 0.06$</td>
<td>$1.96^{(4)}$</td>
</tr>
<tr>
<td>α Cen B</td>
<td>K5 VI</td>
<td>+5.69</td>
<td>$1.50 \pm 0.02^{(2)}$</td>
<td>$\sim 0.06$</td>
<td>$2.29^{(5)}$</td>
</tr>
<tr>
<td>α CMi</td>
<td>F5 V</td>
<td>+2.65</td>
<td>$1.60 \pm 0.02^{(3)}$</td>
<td>$\sim 0.04$</td>
<td>$2.29^{(6)}$</td>
</tr>
<tr>
<td>α Tau</td>
<td>K5 III</td>
<td>-0.70</td>
<td>$1.92^{(11)}$</td>
<td>$\sim 0.10$</td>
<td>$2.27^{(12)}$</td>
</tr>
<tr>
<td>β Cas</td>
<td>F2 IV</td>
<td>+1.5</td>
<td>...</td>
<td>...</td>
<td>$2.15 \pm 0.01^{(10)}$</td>
</tr>
<tr>
<td>β Gem</td>
<td>K0 III</td>
<td>+0.95</td>
<td>$1.71 \pm 0.02^{(12)}$</td>
<td>$\sim 0.10-0.20$</td>
<td>$2.12^{(13)}$</td>
</tr>
</tbody>
</table>

* Allen 1973, except as noted.

Notes.—(1) Linsky and Avrett 1970; flux profile.
(3) Brinkmann, Green, and Barth 1966.

where $g \propto M/R^2$ is the stellar surface gravity and $T_{eff}$ is the effective temperature. We find that our data are consistent with

$$
\Delta \lambda_{K_i} \propto g^{-0.27 \pm 0.04} T_{eff}^{1.42 \pm 0.2}.
$$

III. ANALYSIS

For simplicity we ignore the dependence of $\Delta \lambda_{K_i}$ on $T_{eff}$, because the variation of $T_{eff}$ (about 40 percent) over spectral types F–K is small compared with the factor of about 5 variation of $g^{-0.27}$ between main-sequence stars and giants. We now consider the consequences of our premise that the mean continuum optical depth of the stellar temperature minimum is relatively independent of surface gravity.

a) The Continuum Optical Depth of $T_{min}$

The dominant source of photospheric continuous opacity at the wavelengths of maximum emergent flux for normal late-type stars is the $H^-$ ion. The frequency-integrated $H^-$ $b$-$f$ opacity is proportional to the electron pressure (Gingerich 1964); i.e.,

$$
\kappa^{H^-}(cm^2 g^{-1}) \propto P_e.
$$

Over the temperature range we are considering, $3000^\circ K \leq T_{min} \leq 5000^\circ K$, hydrogen is predominantly neutral and $P_e$ is dominated by metal ionization. Hence (e.g., Mihalas 1970),

$$
P_e = \text{const } P_A A_{\text{met}} = \text{const } g m A_{\text{met}},
$$

where $P_e$ is the gas pressure, $A_{\text{met}}$ is the abundance of easily ionized metals relative to hydrogen, $g$ is the surface gravity, and $m$ is the mass column density (increasing inward). The second equality above is ob-
tained from hydrostatic equilibrium. Integrating equation (4) over \( m \), we find that the continuum optical depth of the stellar temperature minimum \( \tau_{\text{c}} \) scales as

\[
\tau_{\text{c}}^* = A_{\text{met}} m^* g^\frac{1}{2},
\]

where \( m^* \) is the column density at \( T_{\text{min}} \).

If \( \tau_{\text{c}}^* \) is roughly independent of gravity, we have

\[
m^*(g \text{ cm}^{-2}) \approx \text{const } A_{\text{met}}^{-1/2} g^{-1/2}.
\]

In physical terms, low gravity stars require more mass above their photospheres than high gravity stars for similar continuum optical depths, because of the approximately quadratic pressure dependence of \( H^\circ \) formation.

\( \text{b) The Formation of the } K_1 \text{ Feature} \)

Detailed studies of the Ca II line formation problem for a variety of model atmospheres (Ayres, Linsky, and Shine 1974; Shine and Linsky 1974) usually show that the \( K_1 \) minimum feature is formed in the radiation damping dominated portion of the absorption profile, and that the location of \( K_1 \) in \( \Delta \lambda \) is often a good indicator of the location of the temperature minimum in physical depth. In fact, the \( K_1 \) minimum appears roughly in that portion of the inner wings where monochromatic optical depth \( \tau_{\Delta \lambda} \approx 2/3 \) occurs at the temperature minimum (e.g., an Eddington-Barbier relation for the flux).

The numerical models on which the above notions are based used the heretofore almost universal assumption of complete redistribution (CRD) in the line transfer calculation. Some recent work using the more physical “partial redistribution” approximation (PRD) has shown significant departures from CRD, especially at the low densities characteristic of stellar temperature minima (Milkey and Mihalas 1973, 1974; Milkey, Ayres, and Shine 1974). However, our experience with a two-level representation of Ca II and the Milkey, Ayres, and Shine results for Mg II suggest that even within the framework of PRD, \( K_1 \) retains its sensitivity to the location of \( T_{\text{min}} \). Furthermore, the scaling with \( m^* \) is essentially unchanged from the CRD result derived below. Hence, for clarity, we assume CRD in what follows, recognizing that the resulting scaling laws for \( \Delta \lambda_{K_1} \) also should be applicable in the PRD approximation.

In order to exploit the Eddington-Barbier relation above, we must calculate the monochromatic optical depth at \( T_{\text{min}} \). Ignoring van der Waals broadening compared with radiative damping we find:

\[
\tau_{\Delta \lambda}^* = \text{const } N_{\text{Ca}^+}/\Delta \lambda^2
\]

Here \( N_{\text{Ca}^+} \) is the column density of \( \text{Ca}^+ \) ions at \( T_{\text{min}} \), \( A_{\text{Ca}} \) is the calcium abundance relative to hydrogen, and \( \Delta \lambda \) is the wavelength (Å) from line center. The proportionality constant contains various atomic parameters such as the oscillator strength and radiative damping width. In the second line of equation (8) we have assumed that all of the calcium atoms are in the ground state of \( \text{Ca}^+ \).

Taking \( \tau_{\Delta \lambda}^* \approx 2/3 \), we obtain

\[
\Delta \lambda_{K_1} = \text{const } A_{\text{met}} m^*/\Delta \lambda^2,
\]

where we have assumed that the abundance of the other easily ionized metals scales as \( A_{\text{met}} \).

Finally, substituting equation (7) for \( m^* \) above, we obtain

\[
\Delta \lambda_{K_1} \approx \text{const } A_{\text{met}}^{-1/4} g^{-1/4},
\]

which shows essentially the same dependence on gravity as the empirical result of equation (3) in § II, as well as a weak dependence on metal abundance.

We note that a completely analogous argument applies to the \( k_1 \) features of Mg II, except that the \( k_1 \) width in (km s\(^{-1}\)) will be larger than the \( K_1 \) width by roughly the factor

\[
\left( \frac{\Delta \lambda_{k_1}}{\Delta \lambda_{K_1}} \right) \approx 3.5,
\]

in reasonably good agreement with the empirical value of 2.5–2.8 derived in § II and the solar value of 3.2 ± 0.5.

We suspect that our apparent overestimate of \( \Delta \lambda_{K_1}/\Delta \lambda_{k_1} \) is caused by the density dependence of the relative departure of Mg II and Ca II from CRD. This dependence arises from the effect of the Ca II metastable states which tend to mix a fixed fraction of incoherence (i.e., CRD) into an essentially pure coherent scattering problem. Since the incoherence fraction, \( \lambda_s \), for Mg II has no such lower bound, and since the departure from CRD is a sensitive function of \( \lambda_s \), the \( \Delta \lambda_{K_1}/\Delta \lambda_{k_1} \) ratio is likely to show an observable dependence on gravity.

\( \text{IV. DISCUSSION} \)

We have demonstrated that the simple assumption that \( \tau^* \approx \text{constant independent of gravity} \) can reproduce the observed increase of \( \Delta \lambda_{K_1} \) (and hence also \( \Delta \lambda_{k_1} \)) with decreasing \( M_\odot \). If our assumption is not correct and we say \( \tau^* \approx g^\alpha \), then the effect on equation (10) is to introduce a multiplicative factor of \( g^\alpha \). However, our choice of \( \tau^* \approx \text{constant} \) is not completely ad hoc, but is motivated by semi-empirical upper photosphere–low chromosphere models for the Sun (Linsky and Avrett 1970; Vernazza, Avrett, and Loeser 1973), Procyon (Ayres et al. 1974), and Arcturus (Ayres and Linsky 1975). These models show a slow increase in \( \tau^* \) with decreasing \( g \), amounting to possibly a factor

© American Astronomical Society • Provided by the NASA Astrophysics Data System
of 6 as the gravity decreases by a factor of 600. Thus $n$ might be $\approx -0.28$ and the power of $g$ in equation (10) $-0.32$ instead of $-0.25$, but our results are not substantially altered.

Finally, since the empirical $\Delta \lambda K - M_g$ correlation is quantitatively similar to the width-$M_g$ relation obeyed by the K-line emission half-width, $W_0$ (e.g. Reimers 1973), we can speculate that either:
1) $W_0$ measures a point in the K-line profile that may be controlled by radiation damping rather than Doppler broadening (e.g. Chipman 1972); or
2) $W_0$ is broadened by the Doppler effect, and the variation of chromospheric turbulence with gravity is coincidentally the same as the "mass" effect which broadens $K_1$ and the entire damping wings of $K$.

Although we must admit the possibility of the latter situation, it would be a remarkable coincidence indeed. If the former situation is true, then the many attempts to link the Wilson-Bappu effect to an increase in chromospheric turbulence with increasing luminosity (e.g., Hoyle and Wilson 1958; Kraft 1959; Reimers 1973) may be based on an oversimplified physical picture of the formation of the K-line emission core and inner wings.

This work was sponsored in part by the National Aeronautics and Space Administration under grants NAS5-23274 and NGR 06-003-057 to the University of Colorado.

REFERENCES
