A FACULAR MODEL BASED ON THE WINGS OF THE Ca II LINES

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Abstract. We develop a relatively simple procedure for deriving models of upper photospheric regions based on the damping wings of the Ca II resonance and infrared triplet lines. The procedure is used to derive a facular model but can also be applied to late-type stars. We compare our model to that of Chapman.

1. Introduction

Previous studies of photospheric faculae fall into one of two categories. The first method is to use observations of the center-to-limb contrast at one or several points in the visible continuum to obtain the temperature versus continuum optical depth. The models proposed by Reichel (1953), Kozhevnikov and Kuz’minykh (1964), Chapman (1970), and Wilson (1971) are examples of this approach. The other method is to use observations of photospheric lines. Curve of growth analyses have been applied by Mitropol’skaya (1954) and Voikhanskaya (1966) who find excitation temperatures higher in faculae than in the average Sun. Chapman and Sheeley (1968) studying equivalent width differences and Stellmacher and Wiehr (1971) studying the profiles of five magnetically insensitive lines both concluded that faculae are hotter than the normal photosphere.

Both of these approaches have disadvantages as methods for determining the vertical structure of faculae. Reliable continuum measurements are difficult to obtain beyond \( \mu = 0.3 \) and are also susceptible to geometric effects (e.g., shadowing) near the limb. In addition one is forced to analyze faculae statistically instead of individually since a given active region may change significantly during the time it passes from disk center to the limb. Although an analysis based on profiles of weak Fraunhofer lines does not suffer from these difficulties, such lines depend strongly on non-thermal velocity fields as well as the temperature structure. Also, if the lines are not formed completely in LTE, they may have a complicated dependence on the electron density and other radiation fields as well.

A better approach may be to use the extensive damping wings of the Ca II K and \( \lambda 8542 \) lines as an atmospheric probe. We show how the Ca II line wings can be used to directly analyze photospheric temperature structures and apply this method to the Ca II profiles for a facula given by Shine and Linsky (1972).

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2. Formation of the Ca II Wings

The opacity in the wings of the K line is, to a good approximation, given by:

$$\kappa_{\Delta \lambda} = 1.17 \times 10^{-26} N_I \left[ \Gamma_A + \Gamma_{vW} \left( \frac{T}{5000} \right)^{0.3} N_H \right] / \Delta \lambda^2,$$

where \(\kappa_{\Delta \lambda}\) is the opacity at a wavelength \(\Delta \lambda\) Å from line center, \(N_I\) is the number density of the ground state of Ca II, \(\Gamma_A\) is the radiative damping width (\(\approx 1.5 \times 10^8\) for both K and \(\lambda 8542\)), \(\Gamma_{vW}\) is the coefficient for collisional broadening from neutral hydrogen, and \(N_H\) is the neutral hydrogen density. Since non-LTE calculations made with the assumption of complete redistribution (e.g., Linsky and Avrett, 1970) show LTE to be a good approximation for the Ca II source functions until slightly below the temperature minimum, we will assume LTE throughout. The possible importance of departures from complete redistribution will be considered elsewhere.

We use Lambert and Warner's (1968) Ca abundance of \(2.14 \times 10^{-6}\) relative to hydrogen. For the Van der Waals collisional broadening coefficients the theoretical values are evidently too low (de Jager and Neven, 1970; Holweger, 1972) and therefore we have empirically estimated values from the observed average K and \(\lambda 8542\) profiles given in Shine and Linsky (1972) and the Harvard-Smithsonian Reference Atmosphere (HSRA) (Gingerich et al., 1971). We find that a value of \(\Gamma_{vW} = 1.6 \times 10^{-8} \pm 10\%\) gives the best reproduction of both K and \(\lambda 8542\). This is about twice the theoretical value obtained using Unsöld's (1955) formula and is therefore in approximate agreement with Holweger's (1972) empirical enhancement factor of 1.7 for a number of Ca II lines. It is about 10\% lower than de Jager and Neven's (1970) empirical values for the IR lines if we substitute our values of the Ca II abundance and \(f\) values into their derivation.

If we now assume: (a) \(I_\lambda = S_\lambda\) at \(\tau_\lambda = 1\) (an Eddington-Barbier relation), (b) \(N_{\text{He}} = 0.10 N_H\), and that in the region of formation: (c) the gas pressure equals the total pressure, (d) \(N_e \ll N_H\), and (e) \(N_I = N_{\text{Ca II}} = N_{\text{Ca}} = 2.14 \times 10^{-6} N_H\), and (f) ignore the variation of temperature in the equation of hydrostatic equilibrium for each \(\Delta \lambda\) (i.e., assuming a constant scale height), we obtain for the K line the following relationship among \(\Delta \lambda\), radiation temperature \(T\) at \(\Delta \lambda\), and \(N_H\) in the region of its formation:

$$N_H = \left( \frac{5000}{T} \right)^{0.3} \left\{ 8.672 \times 10^{31} + 2.11 \times 10^{36} \Delta \lambda^2 \left( \frac{T}{5000} \right)^{0.3} \left( \frac{g}{g_\odot} \right) \right\} \times \left( 1 - \frac{\tau_c}{T} \right)^{1/2} - 9.313 \times 10^{15},$$

where \(g\) is the gravitational acceleration and \(g_\odot\) is the solar value. The \((1 - \tau_c)\) term is a correction for continuous opacity. It can only be used if the continuous optical depth \(\tau_c\) is known as a function of \(T\). If \(N_H\) is less than \(10^{15}\) cm\(^{-3}\), it is better to use:

$$N_H = 1.2 \times 10^{20} \frac{\Delta \lambda^2}{T} \frac{g}{g_\odot}$$
which allows only for radiative damping. The column mass density \( m \) will be
\[
m = 1.1 \, N_H \, kT/g
\] (4)
and we can therefore obtain \( T \) as a function of \( m \). The formula for \( \lambda 8542 \) corresponding to Equation (2) is:
\[
N_H = \left( \frac{5000}{T} \right)^{0.3} \left[ \left\{ 8.672 \times 10^{31} + 1.18 \times 10^{36} \Delta \lambda^2 \left( \frac{T}{5000} \right)^{0.3} \left( \frac{g}{g_\odot} \right) \times \right. \right. \\
\left. \left. \times \left( 1 - \tau_e \right) \theta(T)/T \right\}^{1/2} - 9.313 \times 10^{15} \right], \quad (5)
\]
where
\[
\theta(T) = \frac{3}{5} (1 + 0.2 \exp[19730/T]). \quad (6)
\]
Here we have replaced assumption (e) above with an assumption that the population of the lower level \( (N_3) \) of the \( \lambda 8542 \) transition is
\[
N_3 = N_{Ca}/\theta(T) = 2.14 \times 10^{-6} \, N_H/\theta(T). \quad (7)
\]
This is very nearly the LTE value.

As an indication of the usefulness and accuracy of this approach, we find that using the observed \( K \) and \( \lambda 8542 \) wings in the formulas above will reproduce the middle and upper photosphere of the HSRA to better than 50K.

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**Fig. 1.** The dashed lines are theoretical contours of constant \( m \) for the \( K \) line labeled by the value of \( \log m \). The solid lines are the radiation temperatures in the wing of the \( K \) line for active and quiet regions as labeled.
Fig. 2. The dashed lines are theoretical contours of constant $m$ for $\lambda8542$ labeled by the value of $\log m$. The solid lines are the radiation temperatures in the wing of $\lambda8542$ for active and quiet regions.

Fig. 3. Temperature structures of the HSRA, our facular model, and that of Chapman (1970). The first two have nearly equal values of $\tau_{5000}$ for a given $m$ which are given by the lower $\tau_{5000}$ scale. The upper $\tau_{5000}$ scale is for the Chapman model.
Our approach can be applied to the interpretation of stellar flux data by replacing the \((1 - \tau_c)\) terms in Equations (2) and (5) with the term \((2/3 - \tau_c)\), which follows from using the Eddington-Barbier flux relation instead of the intensity relation.

3. A Facular Model Based on the Ca II Wings

We can now use the low angular resolution photoelectric observations in Shine and Linsky (1972) to derive the spatially averaged differences between faculae and the quiet Sun. These were obtained with a slit covering an area of 14000 by 350 km on the Sun. The radiation temperatures obtained are indicated in Figures 1 and 2 for the K and \(\lambda 8542\) lines. The dashed lines are curves of constant \(m\) computed from the formulas above. The continuum optical depth term was used for \(\lambda 8542\) but is not important for K in the range shown in the figure. Using these temperature differences we obtain the facular model shown in Figure 3 together with the HSRA. A complete photospheric model can then be computed using the same methods employed in the HSRA. Such a model is given in Table I. It reproduces the observed radiation temperature differences within 10%. The quantity \(Z\) is the depth in km \((Z=0\) at \(\tau_{5000}=1)\), \(P\) is the pressure, and \(\Delta P\) is the pressure minus the pressure at the same depth in the HSRA.

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Also plotted in Figure 3 is Chapman’s (1970) facular model. It is intended to represent the bright elements in an inhomogeneous model which partly accounts for the higher temperature. The temperature rise at \( m \approx 2 \) is nevertheless incompatible with high resolution Ca II K observations. It predicts, for example, that the brighter elements of a facula would have K1 minima at about 4 Å from line center [using Equations (2), and (4) which are valid in this part of Chapman’s model]. But none of the examples of fine structure shown in Liu and Smith’s (1972) photographic study show K1 beyond about 0.5 Å. The problem probably lies in Chapman’s assumption that the small bright features near the limb measured by Rogerson (1961) represented isolated bright points. It is more likely that they were only the more prominent edge of a larger cluster. Hence the ‘shadowing’ assumed by Chapman would be less severe and lower temperatures would suffice to explain their contrast.

On the other hand, our model based on the Ca II wings and a plane-parallel approximation will not reproduce Chapman’s continuum contrast curve, predicting contrasts too small by almost a factor of 3 for \( \mu \leq 0.6 \). This could be due to our averaging over a 20° long slit but the differences may be too large for this to be the complete explanation. It could instead be a breakdown of some of our assumptions. Stellmacher and Wiehr’s (1971) model also suggests the same kind of inconsistency between the contrasts in the Doppler cores of weaker lines at disk center and the center-to-limb continuum contrasts. Simultaneous observations in the optical continuum and the Ca II wings could answer this question.

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References