COMMENTS ON ‘SOLAR POLAR SPINDOWN’,
BY KENNETH SCHATTEN
(Research Note)

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Schatten (1973) purports to show that it is possible for the observed surface differential rotation of the Sun to be explained as being due to differential solar wind torques. I wish to argue in this note that the conclusions Schatten draws are untenable. Schatten ignores a number of important observations, ignores much of the published literature concerning theoretical fluid dynamics applied to the solar convection and differential rotation problem, and makes some highly unreasonable assumptions concerning the nature of convective turbulence. The result is a gross underestimate of the strength of turbulent mixing in the convection zone. My points are as follows:

1. Solar wind torques can compete with turbulent stresses in the solar convection zone only if (as Schatten tries to argue) the effective viscosity is many orders of magnitude smaller than that suggested by mixing length theory. Theorists familiar with the mixing length concepts will readily concede their limitations, but it is most unlikely the theory would overestimate the turbulent viscosity by $10^6$, or so, which is what Schatten requires. Schatten tries to argue that the dynamic viscosity $\eta$ should be roughly constant throughout the convection zone, near its surface value of about $10^5$ gm cm$^{-1}$ s$^{-1}$. By definition the dynamic viscosity $\eta = \varrho \nu$, in which $\nu$ is the kinematic viscosity, $\varrho$ is the fluid density. Since $\varrho$ increases by $10^6$ or so to the bottom of the convection zone, Schatten requires the kinematic viscosity to fall off by the same factor. In mixing length theory, it is usually assumed that $\nu$ can be related to properties of the motion by $\nu = CVL$ in which $V$ is a typical velocity of the flow, $L$ the scale of convection, or mixing length. $C$ is a dimensionless factor usually assumed to be between 0.1 and 1 (Schatten chose 0.1). In order to decrease $\nu$ strongly enough with depth, Schatten supposes both the convective velocity and the mixing length decrease with depth (each would have to decrease by a factor of a thousand at the bottom of the convection zone). While mixing length theory for convection does suggest the velocity may fall off somewhat with increasing density (perhaps by a factor of 10), virtually all experience with convection indicates the mixing length or convection scale should strongly increase with depth. In laboratory convection experiments, convection cells fill the depth of the container, and the scale is defined by the total depth – the larger

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the depth, the larger the convection cells are. Such large deep cells dominate because they are able to release the most potential energy to drive their motion. With compressibility as in the Sun, it is thought the mixing length and vertical scale may be bounded by the local scale height (in pressure or density), but certainly is not much less than the scale height. Results of Baker (see Kraft, 1969, Figure 26) indicate that if the mixing length were much less than the scale height, stellar models would predict no convection zone at all for the Sun and the separation of F stars into those with convection zones and those without, which is well known, would not be predicted.

Since the scale height is larger by at least a factor of 100 at the bottom of the convection zone than at the top, the kinematic viscosity may actually be larger there than at the surface, but is certainly not much smaller. This means the dynamic viscosity would be at least $10^6$ times larger at the bottom than Schatten can allow. For an illustrative table of eddy viscosity values based on mixing length, as well as convenient discussion in the context of differential rotation, the reader (and Schatten) are referred to Cocke (1967), which is based on earlier work by Baker and Temesvary (1966) and Böhm-Vitense (1958). Köhler (1970) gives a simple quantitative argument showing how, with reasonable eddy viscosity values, solar wind torques can produce no more than about $10^{-5}$ of the observed differential rotation.

Schatten argues (at the bottom of page 324) that the convective velocity may decrease with depth as fast as $q^{-1}$. This is based on the assumption that the temperature fluctuations associated with the convection vary little with depth. In a compressible fluid, these fluctuations represent small differences between the actual temperature and the adiabatic value (actually their gradients) which differences may in fact change radically with depth. Mixing length models such as that used by Baker and Temesvary (1966) give $V$ decreasing much more slowly with depth than $q^{-1}$.

2. Schatten ignores the large amplitude fluctuations in the differential rotation seen in observations of Howard and Harvey (Howard and Harvey 1970; Howard 1971) as well as the evidence the report of organized global scale motions seen in the residuals left after the daily best fit differential rotation has been subtracted out. These fluctuations and residual motions are of roughly the same amplitude as the shears in the mean differential rotation, are of global scale, and have a very broad spectrum in time, from days to years. Some of these fluctuations may not represent real solar motions, but most probably do. Because they are comparable in amplitude and scale to the mean differential rotation, they must interact with it in strongly nonlinear ways, determining its amplitude and shape. The accelerations associated with these motions are far larger than any solar wind torque could produce. It is probable the motions extend deep into the convection zone (at least the more slowly changing ones) and so contribute everywhere to strong mixing of momentum, at a rate many orders of magnitude faster than can be effected by differential solar wind torques. In crude models, their effects could be included in estimates of eddy viscosity, but it is preferable to calculate their effects explicitly since they may actually build up differential rotation (turbulent mixing, if it
has a degree of organization, or a degree of two dimensionality, need not lead to solid rotation). These effects have been studied in simplified ways in the now rather sizable theoretical literature on giant cell convection on the Sun and their implications for differential rotation (see Gilman 1972, 1974, for references), which Schatten has also ignored. The motions modelled in these works may be the fluctuations seen in Howard's observations. The Reynolds stresses generated in these models, with motions of similar amplitude to those in Howard's observations, are orders of magnitude larger than solar wind torques.

3. If Schatten were correct that solar wind torques and large scale magnetic stresses in the convection zone have a strong influence on the differential rotation, large changes in differential rotation should occur with the evolution of the solar cycle, since the subsurface and surface distribution of magnetic fields obviously changes substantially with the cycle. However, neither sunspot statistics nor Doppler measurements show evidence of this. Howard (1973) sees only a small correlation of rotation with the presence of active regions, and Ward (private communication) was not able to find any systematic solar cycle effect in sunspot rotations. If the magnetic stresses are substantially weaker than other stresses, then even if they fluctuate strongly, they will only cause slight modulations of the differential rotation. Parker (1971) argues that the solar wind torques are several orders of magnitude less even than the magnetic stresses in the convection zone, whose influence is very modest at best, compared to eddy viscous stresses.

4. Schatten's notions concerning the solar dynamo, fast rotating core, and magnetic fields threading into the interior are at least inconsistent. He claims that if the Sun is not pulsating, but has a fast rotating interior (with period of, say, between 0.5 and 5 days), magnetic fields must thread into the interior and extract angular momentum from the core. The magnetic field threading in this way requires, he says, a substantially modified solar dynamo.

The trouble with these arguments is that, if any significant field (even much less than 1 G) does thread into the interior, all regions it reaches will rather quickly come into corotation with the convection zone (in part the convection zone will spin up) because large azimuthal electromagnetic body forces will be built up due to stretching of the field lines by the large differential rotation. Large differences would be smoothed out in a period very short compared to the lifetime of the Sun. Since we know the rotation rate at the surface, this means if fields do thread into the interior, the interior must also be rotating at about the surface rate. Some idea of this hydromagnetic spin down can be obtained from Benton and Clark (1974) and earlier references cited therein.

In addition, Schatten's dynamo argument is circular, with the system largely feeding on itself for energy, and therefore quite implausible. In particular, the energy used in producing toroidal field lines from poloidal fields, by the stretching due to differential rotation (the 'Babcock process') must, in the first instance, come from the differential rotation. But in Schatten's picture, these torques arise almost entirely from the pres-
ence of the magnetic field, since without the field, solar wind angular momentum loss can slow down the remaining solar mass only by viscous torques at the surface. It seems far more likely the differential rotation has an energy source which is not of magnetic origin, namely, the turbulent convection, influenced by the mean rotation. Parker (1971) arguing that the solar wind torques, even with a magnetic field present, are too weak to do anything to the surface differential rotation was also led to the conclusion that differential rotation is of nonmagnetic origin.

5. To sum up, the convection zone (and, because of strong vertical coupling by waves and magnetic fields, the photosphere above) should appear essentially as a solid shell for solar wind torques, because of the high mixing within. Consequently, solar wind torques can influence only the mean rotation rate of the entire shell, not its differential rotation, as Schatten concludes. This mean rotation presumably does decrease slowly with time due to solar wind torques, acting in near balance with weak stresses from the solar interior, but Schatten adds little that is new to this subject.

References