ON THE DETERMINATION OF NOISE IN PHOTOGRAPHIC MEASUREMENTS OF SOLAR VELOCITIES AND MAGNETIC FIELDS

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(Received in final form 7 February, 1973)

Abstract. The noise in photographic measurements of solar velocities and magnetic fields is assumed to be essentially determined by the granularity of the film, its gamma, the scanning spot size and the parameters of the specific spectral line. A formula is derived which serves for a quantitative estimate of the rms velocity and magnetic field noise when evaluating spectrograms and spectroheliograms. Four typical examples are treated and show that the estimate is correct within 20%.

1. Introduction

One of the fundamental limitations in the photographic recording of solar spectra and spectroheliograms is given by the granularity of the film. With this assumption we shall derive a formula that allows a quantitative estimate of the lower limit to which Doppler velocities and magnetic fields can be measured with present photographic means. It seems essential that the noise be calculated before rather than determined experimentally after the exposure and scanning process, so that the experimental parameters can be optimized and other sources of noise be ruled out. The formula is illustrated using experimental data quoted by Mattig et al. (1969), Sheeley and Bhatnagar (1971a, b), Beckers and Schröter (1968), and Leighton (1959). The effect of the film's modulation transfer function and granularity on the spatial resolution had been discussed earlier by Brandt (1967).

2. Granularity of the Film

First, we show how film granularity as quoted by Kodak (1967) can be interpreted in terms of an intensity fluctuation $\Delta I/I$ from the knowledge of the film characteristic curve. The granularity of a specific film usually is defined as 1000 times the rms density variation $\sigma_1$ (dimensionless) for an area $F_1$ of the scanning spot and a density $D_1$. Using the relation derived by Siedentopf (1937)

$$\sigma \sim \sqrt{\frac{D}{F}}$$

one can easily transform the granularity $\sigma_1$ which Kodak (1967) gives for a scanning spot of 48 $\mu$ diam ($F_1$) and density $D_1=1.0$ to a value $\sigma_2$ for a different area $F_2$ (in

* Mitteilungen aus dem Fraunhofer Institut Nr. 118.

Solar Physics 31 (1973) 75–80. All Rights Reserved
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and density $D_2$:

$$\sigma_2 = \frac{24\sigma_1 \sqrt{\pi D_2}}{\sqrt{F_2}}. \quad (2)$$

A density variation $\sigma_2$, however, via the slope of the characteristic curve $\gamma$ is converted to an rms intensity variation $\Delta I/I$ in the following way:

$$\sigma_2 \times 10^{-3} = \Delta D = \gamma \Delta \log_{10} I = \gamma (\log_{10} e) \ln \left(1 + \frac{\Delta I}{I}\right) \approx \gamma (\log_{10} e) \frac{\Delta I}{I}.$$

Therefore,

$$\frac{\Delta I}{I} \approx 2.30 \frac{\sigma_2 \times 10^{-3}}{\gamma}. \quad (3)$$

3. Shift of the Spectral Line

From the Doppler relation a calibration factor $C_\nu$ can be derived which denotes the velocity (in m s$^{-1}$) causing a lineshift of 1 mÅ. Similarly the formula for the longitudinal Zeeman effect leads to the definition of a calibration factor $C_H$ indicating the magnetic field strength necessary for a lineshift of 1 mÅ.

From the known gradient in the wing of a spectral line ($((1/I) (dI/d\lambda))$, with $I=$ intensity in the wing of the line) a measured ‘true’ intensity variation $\Delta I/I$ can be interpreted as a certain lineshift $\Delta \lambda$ and this in turn as a given Doppler velocity or magnetic field. However, in exactly the same way (cf. Figure 1) the spurious rms intensity variation caused by the film’s granularity (and calculated from (3)) must be inter-

![Fig. 1. The effect of noise (introduced by film granularity) on the determination of the lineshift.](image)
interpreted in terms of a Doppler noise \( \Delta V_{\text{rms}} \) and magnetic field noise \( \Delta H_{\text{rms}} \), resp. Allowing for the statistical behaviour of the granularity phenomenon it can be shown easily that for \( n \) independent measuring points

\[
\Delta V_{\text{rms}} = 2.30 \frac{\sigma_2 \times 10^{-3}}{\gamma} C_V \left( \frac{1}{I \ dl} \right)^{-1} \frac{1}{\sqrt{n}}.
\]

\( \Delta H_{\text{rms}} \) follows from the same equation if \( C_H \) is inserted at the place of \( C_V \).

If the whole line profile is used for the determination of \( V \) or \( H \) average values of \( \sigma_2, D_2 \) and the gradient must be adopted.

4. Examples

In the following, four types of photographic velocity and magnetic field measurements shall be treated – one in somewhat more detail, the others only very briefly. Where experimental film data (summarized in Table I) by the authors were missing, data by Kodak (1967) were adopted; unfortunately, the gradients of the line profiles could only be estimated from the Utrecht Atlas and thus seem to represent the weakest point in the overall accuracy of the estimate.

Considering e.g. the analysis of ‘line wiggle’ spectrograms carried out by Mattig et al. (1969) and inserting the appropriate film data and scanning area into Equations (2) and (3) yields \( \Delta I/I = 1.34 \times 10^{-2} \). With this value and the parameters of the specific line one obtains from (4) for 12 independent measuring points per line profile

\[
\Delta V_{\text{rms}} = 36 \text{ m s}^{-1}.
\]

However, Mattig et al. (1969) in a part of their analysis (method(b)) measured velocities with reference to an adjacent terrestrial line, which means that the errors of both measurements must be added statistically. Calculating the noise for the terrestrial line \((1/I) (dI/d\lambda) = 6.5 \times 10^{-3} \text{ (mA)}^{-1} ; D_2 = 1.2 \) leads to 53 m s\(^{-1} \) – a somewhat higher value, mainly because only 4 points per profile can be used. This results in an overall rms noise of approx. 64 m s\(^{-1} \) which is well within the range quoted by the authors.

It is interesting to note that \( \Delta V_{\text{rms}} \) depends on the gradient of the specific line – in contradiction to the statement by Berniere et al. (1962) that it is constant for a given sort of film. If one plots these authors’ values of \( \Delta V_{\text{rms}} \) as a function of the Rowland intensity, a clear decrease of the noise figure with increasing Rowland intensity shows up.

The second example are the Doppler spectroheliograms shown by Sheeley and Bhatnagar (1971a, b). Using their experimental data and assuming a scanning area of 0.5° × 0.5° in the final spectroheliogram one obtains

\[
\Delta V_{\text{rms}} = 65 \text{ m s}^{-1}
\]

when two spectroheliograms in opposite line wings are subtracted \((n=2)\). The usual trade of spatial resolution for photometric accuracy can be achieved by choosing a
### TABLE I

Experimental data and noise figures for various types of photographic velocity and magnetic field measurements

<table>
<thead>
<tr>
<th>Author</th>
<th>Mattig et al. (1969)</th>
<th>Sheeley and Bhatnagar (1971a, b)</th>
<th>Beckers and Schröter (1968)</th>
<th>Leighton (1959)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of measurement: velocity ( V ), magn. field ( H )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Technique</td>
<td>spectrogram</td>
<td>spectroheliogram</td>
<td>spectrogram</td>
<td>spectrohel.</td>
</tr>
<tr>
<td>Image scale (arcs mm(^{-1}))</td>
<td>5.9</td>
<td>5.0</td>
<td>7.6</td>
<td>11.3</td>
</tr>
<tr>
<td>Dispersion (mm/Å)</td>
<td>7</td>
<td>1.92</td>
<td>7</td>
<td>0.7</td>
</tr>
<tr>
<td>Scanning area ( (F_2) ) (( \mu^2 ))</td>
<td>50 ( \times ) 50</td>
<td>100 ( \times ) 100(^a)</td>
<td>130 ( \times ) 490(^a)</td>
<td>180 ( \times ) 180(^a)</td>
</tr>
<tr>
<td>Scanning area ( (\text{private communication}) )</td>
<td>0.3 ( \times ) 10 ( \text{arcs} ( \times ) mÅ)</td>
<td>0.5 ( \times ) 0.5(^a) (\text{arcs}(^3))</td>
<td>1 ( \times ) 70(^a) (\text{arcs} ( \times ) mÅ)</td>
<td>2 ( \times ) 2(^a) (\text{arcs}(^3))</td>
</tr>
<tr>
<td>Wavelength (Å)</td>
<td>6302.5</td>
<td>4924</td>
<td>6173</td>
<td>6102.8</td>
</tr>
<tr>
<td>( C_V ) (m/s mÅ) (ms(^{-1}) mÅ(^{-1}))</td>
<td>47</td>
<td>59</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>( C_H ) (G mÅ) (G mÅ(^{-1}))</td>
<td>–</td>
<td>–</td>
<td>22.5</td>
<td>31</td>
</tr>
<tr>
<td>Gradient of line profile ( (1/\lambda) ) (d/( d\lambda )) (mÅ(^{-1}))</td>
<td>5 ( \times ) 10(^{-3})(^a)</td>
<td>7.5 ( \times ) 10(^{-3})(^a)</td>
<td>4.8 ( \times ) 10(^{-3})(^a)</td>
<td>4.5 ( \times ) 10(^{-3})(^a)</td>
</tr>
<tr>
<td>Film</td>
<td>Kodak IV-E</td>
<td>Kodak 2485</td>
<td>Kodak IV-E</td>
<td>Kodak II-D, II-F</td>
</tr>
<tr>
<td>Gamma (( \gamma )) (Kodak data)</td>
<td>3.5</td>
<td>3.5</td>
<td>3.5</td>
<td>3.2</td>
</tr>
<tr>
<td>Granularity (( \sigma_1 )) (Kodak data)</td>
<td>24</td>
<td>50</td>
<td>24</td>
<td>36</td>
</tr>
<tr>
<td>Average density ( (D_0) )</td>
<td>1.0(^a)</td>
<td>0.7(^a)</td>
<td>0.7(^a)</td>
<td>0.7(^a)</td>
</tr>
<tr>
<td>Number of independent measuring points ( (n) ) (private commun.)</td>
<td>12</td>
<td>2</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>rms noise quoted by author</td>
<td>60–75 (m s(^{-1}))</td>
<td>–</td>
<td>5 (G)</td>
<td>20 (G)</td>
</tr>
<tr>
<td>rms noise calculated</td>
<td>64 (m s(^{-1}))</td>
<td>65 (m s(^{-1}))</td>
<td>5.2 (G)</td>
<td>17.6 (/G)</td>
</tr>
<tr>
<td>Remarks</td>
<td>Terrestrial line as reference!</td>
<td>For 1( \times ) 1( \times ) 1( \times ) scanning area noise ( \approx ) 33 (m s(^{-1}))</td>
<td>Noise only twice the value of comparable photoelectric method</td>
<td>Large scale noise is greater</td>
</tr>
</tbody>
</table>

\(^a\) Where no exact data were given by the authors reasonable assumptions were made (cf. text).

scanning area of 1\( ''\) \( \times \) 1\( ''\) which yields \( AV_{\text{rms}} \approx 33 \text{ m s}^{-1} \) – sufficient for the velocity range of granulation and oscillation (0.15 to 0.9 km s\(^{-1}\)).

The effect of reducing the noise due to film grain by increasing \( n \) is nicely demonstrated in the ‘averaged spectroheliograms’ shown by Sheeley (1971) which additionally have the advantage that shortlived features are suppressed.

A third example deals with the investigation of magnetic knots by Beckers and Schröter (1968) who claim to have obtained an rms noise of 5 G for the longitudinal magnetic field component. Inserting their experimental data and assuming a scanning
area of the order of 1" by 70 mÅ (corresponding to 130 × 490 μ²) finally leads to
\[ \Delta H_{\text{rms}} = 5.2 \, G, \]
where \( n = 4 \), since the results were derived from measurements made in opposite line wings in two spectra. This is only about 5 percent more than the value which these authors determined experimentally from a defocussed spectrum.

Finally Leighton's (1959) method of mapping solar magnetic fields photographically should be discussed briefly. Taking the photographic data given by this author combined with film specifications of Kodak (1967) and assuming a scanning area of 2" × 2" on the final magnetogram (cf. Table I) one obtains \( \Delta I/I = 5.11 \times 10^{-3} \). If through the cancelling process no additional noise is introduced – a questionable assumption – and if by the use of altogether \( n = 4 \) independent exposures (see p. 369 in Leighton's paper) the effect of granularity is reduced due to its statistical nature – a probable assumption – then the rms magnetic field noise calculated from (4):
\[ \Delta H_{\text{rms}} = 17.6 \, G. \]

The discrepancy to the value determined by Leighton (20 G) may easily be accounted for by a slightly altered scanning area and in any case is well within the accuracy and assumptions involved in this method. This noise figure, however, only applies to small scale features. As Leighton himself points out, the large scale noise due to plate inhomogeneities and third order aberrations of the spectrograph is higher.

5. Conclusion

With the method suggested here the knowledge of three film parameters (mean density, rms granularity, gamma) and one line parameter (intensity gradient in the wing) allows the calculation of the rms noise in photographic velocity and magnetic field measurements with a known scanning spot size. The method seems to be useful for three purposes in different stages of an investigation:
- planning of the exposure by choosing the right kind of image scale, spectral line, film material, development, etc.,
- evaluation of the exposed film by selecting an appropriate scanning spot size, step width, number of data points, etc.;
- discussion of the significance of results and pinpointing other possible sources of noise (like noise or mechanical jitter in a scanner).

As the examples show a prediction of the noise figure should be reliable within 20% – if all parameters are well known.

Acknowledgements

The problem was brought to our attention by Dr J. M. Beckers, when discussing his experimental results. Furthermore we gratefully acknowledge some helpful discussions with Drs F.-L. Deubner, U. Grossmann-Doerth and W. Mattig.
References