THE MOON AS A PROPOSED RADIOMETRIC STANDARD FOR MICROWAVE AND INFRARED OBSERVATIONS OF EXTENDED SOURCES

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ABSTRACT

Measured values of the average midnight and morning terminator infrared brightness temperatures of the central portion of the lunar disk can quite accurately determine the mean surface temperature despite likely horizontal and vertical inhomogeneities of the thermal properties of the lunar soil. These data together with laboratory measurements on lunar soil and in situ temperature measurements in and on the lunar surface lead to a mean surface temperature of 220.5° K ± 2.5 percent and a mean temperature 35° K hotter at a depth of 1 m. The monthly average brightness temperature is then estimated as a function of wavelength between 10 μm and 100 cm taking into account likely temperature dependencies of the thermal conductivity and loss tangent, and the variation of emissivity with wavelength. The accuracy with which the Moon can be used as an absolute radiometric standard for extended sources is estimated based on the likely range of lunar thermal and electromagnetic properties.

Subject headings: infrared — Moon — radio radiation — radio radiation, planetary

1. INTRODUCTION

Observations in the infrared, millimeter, and microwave portions of the spectrum have been important in the study of a wide range of astronomical objects. For some problems only relative data are necessary, but for others absolute fluxes or equivalent brightness temperatures are essential. In particular, observations of the solar chromosphere and the background radiation field generally require absolute flux data.

A common procedure for absolute radiometric calibration involves a determination of atmospheric transmission (especially at wavelengths less than several millimeters), antenna gain and spill, transmission losses to the receiver, and calibration of the receiver using a well-matched load at known temperature. Each of these steps introduces a significant and often unknown amount of error. As an alternative to this procedure, one can observe a known signal either from a transmitter in the far zone of the antenna or from an extraterrestrial source. An example of the first alternative is the work of Krotikov, Porfirev, and Troitsky (1961). This particular approach requires corrections for terrestrial radiation diffracted by the observed disk (Tseytlin 1963, 1964). The present paper pursues the second alternative as first suggested by Low and Davidson (1965) by considering the accuracy with which the Moon may be considered a known source.

The Moon exhibits an inhomogeneous relief as well as diurnal and spatial variations in its infrared and microwave thermal flux. In this respect, it might appear to be a poor object for consideration as a celestial radiometric standard. However, if we confine our attention to the center of the lunar disk, and consider only the mean brightness temperature \( \langle T_b(\lambda, t) \rangle \) averaged over a lunar synodic period, the Moon has three very favorable characteristics: (1) the physical principles determining the mean...
temperature are well known; (2) the mean temperature does not vary appreciably with time; and (3) the mean temperature, as we will show, should not vary appreciably with wavelength from the onset of the thermal regime at, say, 5 µm to at least 10 cm. We can think of no other bright astronomical object with these properties.

To evaluate \( \langle T_B(\lambda, t) \rangle \), we consider in turn the thermal and electromagnetic properties of the lunar surface. We shall show that the use of the mean brightness temperature for calibration considerably reduces the effect of uncertainties in the electromagnetic properties since the mean physical temperature \( \langle T(x, t) \rangle \) is unlikely to vary greatly with depth below the surface. The main uncertainty in \( \langle T_B(\lambda, t) \rangle \) results from assumptions concerning the inhomogeneity of thermal properties across the lunar surface and the temperature dependences of the thermal conductivity and microwave opacity. In this paper we do not attempt to minimize these uncertainties but, rather, show that their effects on the deduced values of \( \langle T_B(\lambda, t) \rangle \) are small.

We emphasize that we are proposing the concept of using the central portion of the lunar disk as a radiometric standard for extended sources. In time, better values for lunar thermal and electromagnetic properties will lead to revisions in the mean brightness temperatures we now propose. As such revisions are made, previous observations based on the Moon as a radiometric standard may be easily recalibrated. Also, this paper may be useful in observations of point sources by providing a system calibration except for antenna gain.

II. MEAN SURFACE TEMPERATURE \( \langle T(0, t) \rangle \)

a) Outer Limits

For completeness, we derive the possible outer limits on \( \langle T(0, t) \rangle \). For this purpose we can assume the lunar surface to be plane-parallel and homogeneous with no internal sources of heat. Under these conditions, the surface temperature \( T(0, t) \) is controlled by the balance of insolation and thermal reradiation with the general result that

\[
\langle T^4(0, t) \rangle = \frac{f}{4\pi\sigma} E_B/E_{IR}.
\]

Here \( f \) is the solar constant at 1 a.u. and \( \sigma \) the Stephan-Boltzmann constant. The lunar bolometric emissivity \( E_B \) is defined as a spectral average weighted by the insolation

\[
E_B = \left[ \int_0^\infty E_v f(v)dv \right]/\left[ \int_0^\infty f(v)dv \right],
\]

and the lunar thermal emissivity \( E_{IR} \) is defined as a spectral and time average weighted by the Planck function at the surface temperature

\[
E_{IR} = \left\{ \frac{1}{P} \int_0^P \int_0^\infty E_v B_v[T(0, t)]dvdt \right\}/[\sigma \langle T^4(0, t) \rangle].
\]

Both emissivities are averages over all angles of incidence (if applicable) and of emission.

We may deduce an upper limit to \( \langle T(0, t) \rangle \) by assuming the surface material to be a perfect thermal conductor, in which case the surface temperature is constant with phase and \( \langle T(0, t) \rangle \) equals \( \langle T^4(0, t) \rangle^{1/4} \). If we assume values of 1.95 cal cm\(^{-2}\) min\(^{-1}\) for the solar constant, 0.88 for \( E_B \), and 0.89 for \( E_{IR} \), the upper limit on \( \langle T(0, t) \rangle \) is 296° K.
To deduce a lower limit on the mean surface temperature we assume instead the surface material to be a perfect insulator, in which case \( T(0, t) \) is zero during the night and

\[
T(0, t) = \left[ \frac{E_B f}{E_{1R} \sigma} \cos \left( \frac{2\pi t}{P} \right) \right]^{1/8}
\]
during the lunar day. Here we set \( t = 0 \) at noon and \( P \) is the lunar synodic period. The lower limit on \( \langle T(0, t) \rangle \) is 164° K.

\( b) \) The Homogeneous Approximation

Clearly, no meaningful estimate of \( \langle T(0, t) \rangle \) can be made without recourse to a detailed description of thermal conduction in the lunar surface. Simply stated, the problem is to solve the equations of thermal conduction and, if necessary, radiative transfer in the lunar surface subject to the constraints of geometry, a radiating surface boundary condition, and time-independent temperature deep in the surface. The simplest model for the lunar surface, used by Wesselink (1948), assumes a plane-parallel surface geometry and homogeneous, temperature-independent thermal properties. We refer to this as the classic model of the lunar surface.

For all reasonable thermal properties and surface geometries, the surface temperature \( T(0, t) \) follows the insolation closely during the day and then cools during the night at a rate determined by the thermal conductivity \( \kappa \), density \( \rho \), and specific heat \( c \) of the material. For specified values of the emissivities, the solar constant, and selenocentric latitude, and when the thermal properties are constant, the surface temperature is only a function of \( \gamma = (\kappa \rho c)^{-1/2} \). This thermal parameter has typical values of \( \geq 1000 \) for fine powders in a vacuum, 200 for sand, 100 for porous rock, and 20 for most solid rocks. Computation of \( T(0, t) \) as a function of \( \gamma \) can be found in Jaeger (1953) and also in figure 1. Parameters and surface temperatures for these and other models are given in table 1. Since the daytime temperature is nearly independent of \( \gamma \), determinations of lunar thermal properties are best made by measurement of the midnight (solar apex) temperature \( T(0, 180°) \) or of the minimum temperature \( T(0, 270°) \) reached at the morning terminator just before sunrise. Likewise, the most crucial and difficult measurements needed for \( \langle T(0, t) \rangle \) are these nighttime temperatures.

For specified values of the emissivities and the solar constant and assuming the classic model described above, \( T(0, 180°) \), \( T(0, 270°) \), and \( \langle T(0, t) \rangle \) are unique functions of \( \gamma \) at the center of the disk. We may, therefore, think of \( \langle T(0, t) \rangle \) as a function of \( T(0, 180°) \) or \( T(0, 270°) \) with \( \gamma \) as an independent parameter. These relations, which we call calibration curves, are given in figure 2 for the above assumed values of \( E_{1R} \), \( E_R \), and \( f \). These computations are for the center of the disk and are similar to those by Linsky (1966; hereafter referred to as Paper I). Within the range \( 300 < \gamma < 2000 \), these calibration curves are linear with slopes almost precisely 0.50. These properties follow directly from the relative independence of daytime surface temperatures to the thermal properties of the medium and the similar shape of nighttime cooling curves (see fig. 1) which differ only by a parallel displacement to cooler temperatures as the medium becomes more insulating (larger \( \gamma \)). Thus, no matter how complex our representation of the lunar surface and its thermal properties might be, if the surface temperatures are similar to those of the classic model during the lunar day and parallel to one such model during the night, then the calibration curves are the same as those in figure 2. We may therefore be able to accurately determine \( \langle T(0, t) \rangle \) from

\[ \text{We use time and phase angle measured relative to local noon interchangeably.} \]
T(0, 180°) or T(0, 270°) even if all of the assumptions of the classic model are incorrect. We will make use of this argument often.

The only measurement of the minimum temperature T(0, 270°) to date is Low's (1965) infrared observations in the 17.5–22-μm window taken with an 18 arc sec beam-width. He finds a mean value for the minimum brightness temperature of 90° K, which implies for the assumed value of the emissivity T(0, 270°) = 91.5° K and from figure 2 ⟨T(0, t)⟩ = 220° K. More recently, Mendell and Low (1970) have obtained lunar nighttime temperatures using a new instrument with a 22-μm effective wavelength and a 2.4 arc min beam. With a beam of this size, no reliable measurement of T(0, 270°) can be made, but Mendell (1971) cites a value of 100 − 101 ± 3° K as the midnight brightness temperature on one of their scans along the lunar equator. We take T(0, 180°) = 102 ± 3° K, which according to figure 2 implies that ⟨T(0, t)⟩ = 221 ± 1.5° K. As these two independent results are in agreement, we take 220.5° K as the present best estimate for ⟨T(0, t)⟩ at the center of the lunar disk. We assign a formal error of ± 1.5° K to the mean temperature and consider the effects of errors in the assumed emissivity below.

We consider seven potential sources of error in this experimental-theoretical value of the mean surface temperature. We first investigate five which appear in the homo-
| Model  | Depth [cm] | \( \rho \) [g cm\(^{-3}\)] | \( \kappa_c \) [cal cm\(^{-1}\) (deg-K\(^{-1}\)] | \( R_{350} \) | \( \gamma_{o50} \) | \( \gamma \) | \( \langle T(0, t) \rangle \) [\( ^* K \)] | \( T(0, 180^\circ) \) [\( ^* K \)] | \( T(0, 270^\circ) \) [\( ^* K \)] | \( \langle \Delta T \rangle \) [\( ^* K \)] | \( \Delta T_r \) (1 m) [\( ^* K \]) |
|--------|-----------|------------------------|------------------------|---------|---------|---------|----------------|----------------|----------------|----------------|----------------|----------------|
| 300    | All       | ...                    | ...                    | ...     | 300     | ...     | 235.3         | 132.0           | 121.1           | 0.0             | 1.42            |                 |
| 600    | All       | ...                    | ...                    | ...     | 600     | ...     | 225.6         | 111.9           | 102.3           | 0.0             | 5.68            |                 |
| 1000   | All       | ...                    | ...                    | ...     | 1000    | ...     | 219.8         | 100.0           | 91.2            | 0.0             | 15.8            |                 |
| 1400   | All       | ...                    | ...                    | ...     | 1400    | ...     | 216.1         | 92.2            | 83.9            | 0.0             | 31.0            |                 |
| 2000   | All       | ...                    | ...                    | ...     | 2000    | ...     | 212.5         | 85.2            | 77.5            | 0.0             | 63.2            |                 |
| 1000/250 | 0-5      | ...                    | ...                    | ...     | 1000    | ...     | 220.1         | 100.5           | 96.7            | 0.0             | 1.2             |                 |
| 1000/250 | > 5       | ...                    | ...                    | ...     | 250     | ...     | ...           | ...             | ...             | ...             | ...             | ...             |
| 1000-1 | All       | ...                    | ...                    | ...     | 1.00    | 710     | 1000          | 221.6           | 103.8           | 94.5            | 25.1            | 12.3            |
| 1000-2 | All       | ...                    | ...                    | ...     | 2.00    | 580     | 1000          | 223.2           | 107.5           | 98.2            | 36.0            | 19.6            |
| 11(1)  | All       | 1.30                   | 3.40 \times 10^{-8}    | 0.52    | 850     | ...     | 217.9         | 95.7            | 86.6            | 14.0            | 20.3            |                 |
| 11(2)  | All       | 1.64                   | 4.46 \times 10^{-9}    | 0.52    | 660     | ...     | 220.9         | 101.8           | 92.2            | 13.3            | 15.5            |                 |
| 12     | All       | 1.64                   | 2.20 \times 10^{-9}    | 1.48    | 830     | ...     | 217.1         | 94.1            | 85.5            | 31.6            | 24.3            |                 |
| 15(1HR) | 0-5       | 1.64                   | 2.00 \times 10^{-9}    | 0.16    | 370     | ...     | ...           | ...             | ...             | ...             | ...             | ...             |
| 15(1HR) | > 8       | 1.64                   | 3.30 \times 10^{-9}    | 0.10    | 290     | ...     | 211.9         | 92.1            | 87.6            | 29.0            | 3.5             |                 |
| 15(2HR) | 0-5       | 1.30                   | 4.46 \times 10^{-9}    | 1.48    | 580     | ...     | ...           | ...             | ...             | ...             | ...             | ...             |
| 15(2HR) | > 8       | 1.64                   | 3.30 \times 10^{-9}    | 0.10    | 290     | ...     | 216.7         | 102.5           | 97.8            | 26.4            | 2.4             |                 |
| 15(3HR) | > 8       | 1.30                   | 4.46 \times 10^{-9}    | 1.48    | 580     | ...     | ...           | ...             | ...             | ...             | ...             | ...             |
| 15(4HR) | > 8       | 1.64                   | 3.30 \times 10^{-9}    | 0.16    | 370     | ...     | 217.8         | 105.3           | 100.1           | 24.5            | 2.2             |                 |
| 15(4HR) | > 8       | 1.64                   | 3.30 \times 10^{-9}    | 0.16    | 370     | ...     | 217.3         | 104.0           | 99.0            | 30.5            | 1.8             |                 |
| 15(4)  | 0-4       | 1.64                   | 3.30 \times 10^{-9}    | 0.16    | 370     | ...     | 222.5         | 105.5           | 100.3           | 33.5            | 1.8             |                 |
| 15(4)  | > 8       | 1.64                   | 3.30 \times 10^{-9}    | 0.10    | 290     | ...     | ...           | ...             | ...             | ...             | ...             | ...             |
Fig. 2.—Calibration curves relating midnight surface temperatures $T(0, 180^\circ)$ and morning terminator surface temperatures $T(0, 270^\circ)$ to the lunation mean surface temperature $\langle T(0, t) \rangle$. These calibration curves are for classical models, i.e., models with homogeneous, temperature-independent thermal properties, and are characterized by a thermal parameter $\gamma = (\kappa pc)^{-1/2}$ with values cited. Models designated by points A–F are described in text.

The difficulty of infrared absolute radiometry of cold sources is mitigated in part by the relative insensitivity of brightness temperature measurements to errors in the measured flux. Also, where their data overlap, Mendell and Low's (1970) cooling curves agree well with those of Murray and Wildey (1964) and Wildey, Murray, and Westphal (1967) but disagree with those of Shorthill and Saari (1965). Without going into a detailed error analysis of the infrared observations, which is complex and best done by the original observers, we summarize the reasons why we consider the data of sufficient quality to permit their use in determining $\langle T(0, t) \rangle$. (a) Large flux errors at 20 $\mu$m translate into small brightness temperature errors for cold objects. (b) The
slope of the calibration curves in figure 2 are such that the resultant error in $\langle T(0, t) \rangle$ is half that of $T(0, 180^\circ)$ or $T(0, 270^\circ)$. (c) The mean temperature deduced from the minimum temperature agrees with that deduced from the midnight temperature within its stated error brackets.

ii) Emissivities and the Solar Constant

In computing the calibration curves in figure 2, we have assumed $E_B = 0.88$, $E_{IB} = 0.89$, and $f = 1.95 \text{ cal cm}^{-2} \text{ min}^{-1} (1.36 \times 10^8 \text{ Wm}^{-2})$. We estimate the uncertainty in the value of $\langle T(0, t) \rangle$ deduced from nighttime measurements by analyzing the consequences of assuming reasonable outer limits on $E_B$, $E_{IB}$, and $f$. We assumed $f = 1.95 \text{ cal cm}^{-2} \text{ min}^{-1}$ based on Labs and Neckel (1971), who have recently reviewed the literature. They estimate an accuracy of $\pm 1$ percent in this value due primarily to systematic errors. However, the effective solar constant at the lunar surface varies by $\pm 3.37$ percent due primarily to the ellipticity of the Earth's orbit around the Sun (cf. Tyler 1961). Models with $f = 2.016$ (point B) and $f = 1.884$ (point C) show that this effect, which is much more important than the error in the solar constant itself, can introduce a variation of only $\pm 1.5^\circ \text{K}$ in the value of $\langle T(0, t) \rangle$ in the course of a year as deduced from the mean temperature corresponding to $T(0, 180^\circ)$ or $T(0, 270^\circ)$ on the calibration curves. We treat this as a random error, although it formally is not, because its effect on radio emission is complex and will not be analyzed later.

The average fraction of this flux absorbed by the Moon near the center of the disk $E_B$ is not precisely known. It can be estimated by convolving the intermediate bandpass albedo measurements of the whole disk (Harris 1961) with the solar energy distribution. This results in $E_B = 0.89$. Comparison of visual brightness with 8–13 $\mu$m lunar emission (Lucas et al. 1969) yields $E_B$ typically 0.93 for Surveyor landing sites in mare regions and 0.83 for the Surveyor 7 site in the rock environs of Tycho. The value of $E_B$ for Sinus Midii (Surveyor 6) is 0.916. Hemispherical reflectance spectra of Apollo 11 returned samples from Mare Tranquilitatus by Birkebak, Cremers, and Dawson (1970) and Conel and Nash (1970) result in values of $E_B$ of 0.89 for rock, 0.84 for breccia, and 0.93 for soil samples. For Apollo 11 (Mare Tranquilitatus) soil sample 10084,68 Cremers, Birkebak, and White (1972) find $E_B = 0.90 \pm 0.01$, while for Apollo 12 soil Birkebak, Cremers, and Dawson (1971) find $E_B \approx 0.89$. It thus appears as though $E_B$ is somewhat smaller in rocky and mountainous regions than in mare regions. For the central part of the lunar disk, which is part mare and part mountainous, Saari and Shorthill (1972) find a mean value of $E_B = 0.88$ based on simultaneous visual and thermal measurements. We adopt this value as representative and estimate the range of mean values for the center of the lunar disk as 0.92–0.84. In figure 2 are plotted the results of models with $E_B = 0.92$ (point A) and $E_B = 0.84$ (point D). The resultant error in the deduced value of $\langle T(0, t) \rangle$ is $\pm 2.0^\circ \text{K}$ as determined by the value of $\langle T(0, t) \rangle$ on the calibration curves corresponding to $T(0, 180^\circ)$ and $T(0, 270^\circ)$ for points A and D.

To our knowledge the only laboratory measurement of the infrared $E_{IB}$ of lunar soil is that of Cremers et al. (1972). They find $E_{IB} = 0.90 \pm 0.02$ for Apollo 11 mare soil in the temperature range 270–370 K. Laboratory measurements of simulated lunar soils (e.g., Van Tassel and Simon 1964) suggest $E_{IB} = 0.88$ and that the emissivity is nearly constant in the wavelength range 7–14 $\mu$m. From observations of a 3 x 3 arc min area at the center of the lunar disk, Murray, Murray, and Williams (1970) find $E \approx 0.89$ and nearly constant in the spectral range 10.5–13.5 $\mu$m assuming unit lunar emissivity near 8 $\mu$m. From the Murray et al. (1970) data, it seems reasonable

2 Very recently Birkebak and Abdulkadir (1972) have found for Apollo 12 and 14 soil samples $E_{IR} \approx 0.93$ at $400^\circ$ K and 0.975 at $100^\circ$ K.
to assume $E_{IR} \approx 0.89$, as Conel (1969) finds that the silicate rock powders of micron size exhibit Christiansen frequencies near $8 \mu m$, at which the material is black, and broad plateaus of nearly constant emissivity at larger wavelengths.

The above laboratory and observational data refer to the normal emissivity and not, strictly speaking, to $E_{IR}$, which is a global average quantity. For the present, however, we assume the infrared emissivity to be isotropic and discuss the effect of departures from isotropy due to small-scale roughness below. We take as the most likely value for $E_{IR} 0.89$ and for reasonable outer limits 0.93 and 0.85. In figure 2 are plotted the surface temperatures of models with $E_{IR} = 0.93$ (point F) and $E_{IR} = 0.85$ (point E). The primary effect of increasing $E_{IR}$ is to lower daytime temperatures and thus decrease $\langle T(0, t) \rangle$ much more than decreasing nighttime temperatures. The resultant uncertainty in $\langle T(0, t) \rangle$ is $\pm 2.0^\circ K$. A countervailing effect is that an increase in $E_{IR}$ increases the nighttime surface temperature deduced from the observed brightness temperatures at the same time as the surface temperature itself is lower. The effect is to reduce the resultant uncertainty in $\langle T(0, t) \rangle$ to $\pm 1.7^\circ K$.

iii) A Partly Transparent Surface

Ulrichs and Campbell (1969) have called attention to the possibility that the lunar surface material may be somewhat transparent to infrared radiation with the result that the observed thermal emission would be produced in a finite layer rather than at the surface only. The effect is to decrease daytime emission and enhance nighttime emission, especially when there are steep thermal gradients just below the surface. The critical question is whether this effect produces calibration curves relating apparent nighttime surface temperatures to the mean surface temperature different from those of figure 2. Since Ulrichs and Campbell (1969) compute no lunation cooling curves, we attempt to answer this question by referring to their eclipse cooling curve. Here the effect of considering volume rather than surface emission is to raise the umbral emission. But the computed umbral cooling curve is parallel to the computed curve assuming only surface emission and apparently can be reproduced by assuming surface emission only and smaller $\gamma$. We conclude, therefore, that our previously deduced value for $\langle T(0, t) \rangle$ is unaffected by the existence of a partly transparent surface.

iv) Radiative Conductivity

For material as thermally insulating as that at the lunar surface, Wesselink (1948) suggested that radiation should play a role in transporting heat through the spaces between particles. This idea has been studied in more detail by Wechsler and Simon (1966), Wechsler, Glaser, and Fountain (1971), Linsky (Paper I), Halajian and Reichman (1969), and Winter and Saari (1969), among others. We have computed two models similar to the classical model with $\gamma = 1000$ except that a term proportional to $T^3$ is added to the conductivity such that the ratio of radiative to contact conductivity is 1.0 and 2.0, respectively, at 350° K. The resultant cooling curves in figure 1 are parallel to the nonradiative ones, and these models therefore lie on the calibration curves (cf. fig. 3). Since Halajian and Reichman (1969) and Winter and Saari (1969) also find no difference in shape between cooling curves with and without radiative conductivity, we conclude that the existence of radiative conductivity introduces no errors into our deduced value for $\langle T(0, t) \rangle$.

v) Two-Layer Models

A two-layer model of the lunar surface, consisting of a shallow insulating layer over more conducting material, was first suggested by Piddington and Minnet (1949). To date, no measurements of the change in thermal properties with depth are available.
Fig. 3.—Inhomogeneous lunar surface models compared with the calibration curves of the classical homogeneous models of fig. 2. Dots, two-layer models consisting of an upper layer with $\gamma = 1000$ and a lower layer with $\gamma = 250$. The upper layer thicknesses are (from top to bottom) 0.5, 1, 2, 3, and 5 cm, respectively. Circles, composite models for a lunar surface consisting of part $\gamma = 1000$ and part $\gamma = 300$ material. From top to bottom, the fraction of $\gamma = 300$ material is 75, 50, 30, 20, 10, and 3 percent, respectively. Triangles, same except mare insulating component is $\gamma = 2000$ material. Crosses, radiative models with $\gamma = 1000$ and $R_{330} = 1.0$ (lower cross) and $R_{330} = 2.0$ (upper cross). Squares, from top to bottom, Models 15(4), 11(2), 11(1), and 12.

other than the conclusion that the lunar soil at the Apollo 11 landing site extends to a depth of 4.1 m (Shoemaker et al. 1970). If this depth is representative for the Moon, then the infrared thermal emission is unaffected by the properties of the base material. We compute a series of models in which a layer of $\gamma = 1000$ material of various thicknesses overlaps $\gamma = 250$ material, since such models are known (Jaeger 1953) to exhibit very flat nighttime cooling curves. One such model is given in figure 1. As expected, these models lie off the calibration curves of figure 3, much more so for the minimum temperature than the midnight temperature. We conclude that the midnight temperature is a more accurate means of determining $\langle T(0, t) \rangle$ and that an increase in the conductivity of lunar material with depth could introduce an uncertainty of only 1° K in the deduced value of $\langle T(0, t) \rangle$. 
c) The Effects of Inhomogeneity at the Lunar Surface

The major difficulties in estimating the mean surface temperature arise from the heterogeneous nature of the lunar surface, including variations in thermal properties across the surface and roughness. The cause of the problem is that such inhomogeneities imply temperature differences across the surface, especially during the night. Since the effective infrared brightness temperature is an average in which the hotter temperature regions are very strongly weighted by the Planck function, small portions of the surface which are relatively hot during the night can significantly raise the apparent nighttime infrared temperatures (Hopfield 1967) and at the same time not materially affect the average lunation temperature. Thus, the calibration curves used could be significantly in error. As before, we are concerned with an average region on the disk near the sub-Earth point and not the anomalous regions which appear hot during the night and during eclipses and which are often associated with young craters and certain maria.

i) A Smooth Surface with Heterogeneous Thermal Properties

We consider first only the effects of thermal heterogeneity by assuming that the lunar thermal emission consists of radiation from distinct regions where $\gamma = 1000$ and 300, respectively. In this idealized problem, the effective wavelength for observing is assumed to be that used by Mendell and Low (1970), namely 22 $\mu$m, and the fraction of the surface covered by $\gamma = 300$ material is varied between 3 and 75 percent. Each of these models lies within $1^\circ$ K of the calibration curves of figure 3.

Low (1965) observed spots on the lunar disk as cold or colder than 70$^\circ$ K. A more realistic test of the errors induced by small-scale heterogeneity is to assume that the surface consists of regions where $\gamma = 2000$ and 300, respectively, corresponding to minimum temperatures of 77$^\circ$ and 121$^\circ$ K. The resultant errors in $\langle T(0, t) \rangle$, as shown in figure 3, are 1.3$^\circ$ K from minimum temperature determinations and 0.5$^\circ$ K from midnight temperatures. This calculation has a more general application than may first appear, because the effect of ordered and disordered roughness, as described below, is often to produce several regions which nearly behave as plane-parallel surfaces with different values of $\gamma$.

ii) Exposed Bare Rocks

Fremlin (1959) and Hopfield (1967) have noted that the existence of only a relatively small number of rocks on the lunar surface can radically affect the nighttime emission, as $\gamma = 20$ rocks would have a midnight surface temperature of 220$^\circ$ K and emit 70,000 times as much flux at 12 $\mu$m as $\gamma = 1000$ soil at 100$^\circ$ K. This is true only for a plane-parallel surface consisting of rock, but not necessarily true (Roelof 1968) for exposed individual rocks. Roelof finds that since the $e$-folding distance of thermal wave propagation into rock during a lunation is 60 cm, submeter rocks have nearly isothermal interiors during most of a lunation and are thus near equilibrium with radiation from the lunar surface and the Sun. In short, only rocks greater than 1 m in size are 100$^\circ$ K hotter than their surroundings at night, and the temperature enhancement of smaller rocks is much less. Roelof concludes that neither large nor small rocks on the Moon are sufficiently abundant to affect terrestrial-based infrared observations at night. Winter (1970a) comes to the same conclusion after a more detailed calculation using the rock-size distributions observed by Surveyor spacecraft. He reaches the opposite conclusion for the Surveyor 7 landing site near Tycho, which is considerably more rocky, especially for large-size rocks, than the other sites. This region of the Moon is known to be anomalously hot during the night, with a color temperature and effective emissivity (Allen and Ney 1969) explicable by such a rock-size distribution.
iii) Small-Scale Craters

Buhl and co-workers (Buhl 1967; Buhl, Welch, and Rea 1968) have reported computations of the thermal behavior of craters during a lunation and an eclipse. They find that craters deeper than the $e$-folding distance of thermal wave propagation during a lunation, roughly 16 cm for $\gamma = 1000$ material, can remain as much as $15^\circ$--$20^\circ$ K hotter than their flat surroundings during the night. Hemispherical craters during a lunation simulate flat material of $\gamma/2$, and relatively deeper craters simulate flat material of even smaller $\gamma$. From their results we find that the maximum overestimate of $\langle T(0, t) \rangle$ obtained by assuming the observed $90^\circ$ K morning terminator brightness temperature as produced by a flat rather than a partially cratered surface is about $4^\circ$ K. This error itself is probably much too large, as Winter (1970) suggests, because Buhl and co-workers did not properly allow for subsurface heat exchange below the craters. Winter finds that during the night the craters are warm but their environs are anomalously cool. Thus, a cratered surface cools more nearly like a flat surface than previously suggested. Also, most craters are less steep than hemispherical and thus behave even more like a flat surface. These arguments lead us to conclude that cratering on or greater than a decimeter scale is not a significant source of error in the deduced value of $\langle T(0, t) \rangle$, except possibly in very young regions on the disk such as Tycho Crater and its environs.

iv) Centimeter Scale Roughness

A wide variety of phenomena, including photometric phase functions, polarization, radar reflectivity, and such infrared anomalies as limb darkening of the subsolar point, imply that the lunar surface is not smooth. As before, roughness on the centimeter scale is especially effective in altering the thermal emission properties of the surface, because on this scale lateral conduction does not completely obliterate small-scale temperature differences. Bastin and Gough (1969) have solved the thermal conduction problem in a highly idealized two-dimensional structure consisting of a parallel array of grooves or troughs. During the lunar night, the ridges cool more rapidly and the troughs less rapidly than a plane-parallel surface with the same thermal properties. The nighttime thermal emission is very forward-directed, with the result that near the center of the disk apparent nighttime radiation temperatures are enhanced while the monthly average radiation is unaffected or decreased. For the particular example of a surface completely covered with grooves 4 cm deep, 4 cm wide, and 8 cm apart, the midnight radiating temperature is enhanced at 20 $\mu$m by about $7^\circ$--$8^\circ$ K while $\langle T(0, t) \rangle$ is decreased about $3^\circ$ K. The value of $\langle T(0, t) \rangle$ inferred from $T(0, 180^\circ)$ and the calibration curves of figure 2 assuming plane-parallel geometry is $7^\circ$ K too high. If the grooves are less deep or more widely spaced, the effect is decreased.

We suspect that the above calculation overstates the error in the deduced value of $\langle T(0, t) \rangle$ due to roughness, for two reasons. (a) The lunar surface roughness is certainly disordered rather than ordered. The regularity of the pattern used by Bastin and Gough (1969) tends to enhance the directionality of nighttime emission and thus the enhancement of $T(0, 180^\circ)$. (b) In Surveyor 1–6 and Apollo 11 pictures, most of the centimeter scale relief appears to be due to shallow craters rather than to rocks lying above the surface. If so, then the mean slope at this scale is much less than the $45^\circ$ which characterizes the Bastin and Gough model. From Ranger VII photographs of a mare region, Shoemaker (1966) finds a mean slope of $5^\circ$ at a scale of 1 m. This is certainly a lower limit on the mean slope at a scale of a few centimeters. A definitive test would be a determination of the mean slope and especially the slope frequency distribution on scales of 1–10 cm from Surveyor 6 photographs of the Sinus Medii area at the center of the disk. It seems likely from Bastin and Gough's (1969) computations that if the grooves were 12 cm apart instead of 8 cm, corresponding to a
decrease in mean slope from 45° to 36°, then the error in the deduced value of \( \langle T(0, t) \rangle \) would be more than halved.

d) Direct Measurements and Models for the Apollo Sites

At the Hadley Rille site of Apollo 15, thermocouples were attached to heat flow probes inside and above the lunar surface. Langseth et al. (1972) have obtained a mean surface temperature at the site of 217° ± 3° K from theoretical daytime surface temperatures and from nighttime surface temperatures derived from the temperatures of those thermocouples above the surface. Since the site is at approximately 26° N latitude, the corresponding temperature for the equator must be somewhat higher. We estimate \( \langle T(0, t) \rangle \) at the equator by assuming a 10 percent decrease in \( f \) to account for the change in latitude. This corresponds to a 5° K change in \( \langle T(0, t) \rangle \) and thus an equatorial mean temperature \( \langle T(0, t) \rangle = 222° ± 3° K \).

Sufficient thermal measurements of lunar soil returned from the Apollo 11 landing site (Mare Tranquililitus) and the Apollo 12 site (Oceanus Procellarum) are now available to allow the construction of tentative models. Robie, Hemingway, and Wilson (1970) have measured the specific heat of a rock and of a soil sample from the Apollo 11 area in the temperature range 90°-350° K. The specific heats of the two samples are nearly identical, and a one-term power law fit to their data on the soil sample gives

\[
C(T) = 0.209[T/350]^{0.9} \text{ cal g}^{-1} \text{ (deg-K)}^{-1}. \tag{5}
\]

This simple expression should be adequate for our purposes, as a temperature dependence to the specific heat cannot contribute to an increase in \( \langle T(x, t) \rangle \) with depth (cf. Paper I).

Cremers (1971) and Cremers et al. (1972) have measured the thermal conductivity of Apollo 11 soil sample number 10084,68,2 between 160° and 440° K. They found that an effective thermal conductivity of the functional form

\[
\kappa(x, T) = \kappa_e(x) + \kappa_R(x)T^3 \tag{6}
\]

quite adequately fit their vacuum data at all densities as it does most rock powders (Watson 1964; Wechsler et al. 1971). In particular, Cremers (1971) finds for a density of 1.30 g cm\(^{-3}\)

\[
\kappa(T) = 3.40 \times 10^{-6} + 4.12 \times 10^{-14}T^3 \text{ cal cm}^{-1} \text{ (deg-K)}^{-1} \tag{7}
\]

and for a density of 1.64 g cm\(^{-3}\)

\[
\kappa(T) = 4.46 \times 10^{-6} + 5.33 \times 10^{-14}T^3 \text{ cal cm}^{-1} \text{ (deg-K)}^{-1}. \tag{8}
\]

The lower density was that measured when the material was poured lightly under terrestrial gravity, while the latter was the average density of two core samples. Most likely, the lower density more closely represents conditions in the upper 10 cm of the lunar surface, as the taking of a core sample must cause some settling of the loose material. Wechsler et al. (1971) note that the lower lunar gravity should not appreciably change the thermal conductivity.

Of great interest is the relative importance of radiation to conduction as characterized by \( R_{350} \), the ratio of the second term in equation (6) at 350° K to the first term. This quantity is 0.52 at 1.30 g cm\(^{-3}\) and 1.64 g cm\(^{-3}\), but decreases to 0.35 for Cremers's (1971) data at 1.95 g cm\(^{-3}\). Intuitively one would expect \( R_{350} \) to decrease with increasing density, as the contact area for conduction must increase and the void spaces where only radiation can transport energy must decrease. Since this change in
$R_{350}$ does not occur between 1.30 and 1.64 g cm$^{-3}$, some other quantity such as mean grain size must be important.

We have computed lunar models taking the specific heat from equation (5) and the thermal conductivity from equations (7) and (8). We find that for Model 11(1) [$\rho = 1.30$, $\kappa(T)$ given by eq. (7)] $\langle T(0, t) \rangle$ is 217.9° K, whereas for Model 11(2) [$\rho = 1.64$, $\kappa(T)$ given by eq. (8)] $\langle T(0, t) \rangle$ is 220.9° K. Surface temperatures for these models are given in table 1. In this table the quantity $\gamma_{350}$ is the thermal parameter $(\kappa \rho c)^{-1/2}$ evaluated at 350° K.

The Apollo 12 site, like the Apollo 11 site, is covered with a thick layer of fines. Cremers and Birkebak (1971) measure for loosely poured fines (density 1.30 g cm$^{-3}$)

$$\kappa(T) = 2.20 \times 10^{-6} + 7.62 \times 10^{-14}T^3 \text{cal cm}^{-1} \text{(deg-K)}^{-1}. \quad (9)$$

For this sample, radiation plays a larger role in heat transfer with $R_{350}$ equal to 1.48. Cremers and Birkebak (1971) feel that the increased role of radiation in this sample as opposed to the Apollo 11 sample is due to larger particles present in the Apollo 12 material, as Watson (1964) has found $R_{350}$ to increase with particle size. We have also computed a model based on equations (5) and (9) and assuming a density of 1.30 g cm$^{-3}$. For this Model 12, $\langle T(0, t) \rangle$ is 217.0.

At the Hadley Rille site of Apollo 15, Langseth et al. (1972) found the mean temperature at a depth of 1 m to be 35° K hotter than the mean surface temperature. As discussed in § III, this is due to the effect of a steady-state heat flow from the interior and a temperature dependence of the thermal conductivity. Langseth (1971) estimates that 3°-5° K of the 35° K is due to the steady-state heat flow and the rest, say 31° K, to temperature-dependent conductivity. This temperature increase places an important constraint on the thermal conductivity of the first few centimeters of the surface material. Another constraint is provided by their measured surface brightness cooling curve during a lunar night (see fig. 4), in which they find very rapid cooling for the first 80 hours and then much slower cooling with $T_a(0, 180°) \approx 103° K$ and $T_a(0, 270°) \approx 98° K$. They interpret the slow cooling past 80 hours as due to an increase in $\rho$ and $\kappa$ below 8 cm. Their direct measurements between 50 and 138 cm show that the thermal conductivity increases with depth from $3.3 \times 10^{-5}$ to $6.0 \times 10^{-5}$ cal cm$^{-1}$ (deg-K)$^{-1}$. These values are much larger than the above-mentioned measurements on surface fines.

We have constructed several models for the Hadley Rille site aiming to satisfy the constraints of a 31° K increase with depth due to temperature-dependent conductivity and the measured nighttime surface brightness temperatures. For comparison with the cooling curve in figure 4, we assume $E_{IR} = 0.89$. In general, we find that if the thermal conductivity does not increase until a depth of 8 cm, then no appreciable flattening of the cooling curve occurs at 80 hours. A depth of 4 or 5 cm is more reasonable. For all our models, we take $\kappa_e = 3.3 \times 10^{-5}$ cal cm$^{-1}$ (deg-K)$^{-1}$ below 8 cm and an intermediate layer of $\kappa_e = 2.0 \times 10^{-5}$ cal cm$^{-1}$ (deg-K)$^{-1}$. Both layers are assumed to have a density of 1.64 g cm$^{-3}$, and the specific heat is taken to be that in equation (5). In the lower and intermediate layers, we take $\kappa_R$ to be that found by Cremers and Birkebak (1972) for Apollo 11 soil on the assumption that $\kappa_R$ is not appreciably density dependent.

In Model 15 (1HR) we take for the upper 5-cm layer the density and thermal conductivity of Apollo 12 soil. This nearly produces the required increase in mean temperature with depth $\Delta T = 29.0° K$ but at the expense of very low surface temperature $\langle T(0, t) \rangle = 211.9° K$ and very low nighttime brightness temperatures (fig. 4). The surface layer chosen was thus much too insulating. In Model 15 (2HR) the effect of increasing $\kappa_e$ in the upper layer to $4.46 \times 10^{-6}$, the value of $\kappa_e$ that Cremers (1971) found for Apollo 11 soil at 1.64 g cm$^{-3}$, is to increase $\langle T(0, t) \rangle$ to a reasonable value.
Fig. 4.—Surface brightness temperatures at the Hadley Rille site of Apollo 15 measured by Langseth et al. (1972) compared with the prediction of several models. Parameters for these models are given in table 1.

and the nighttime brightness temperatures almost to their measured values. However, $\Delta T$ is decreased to 26.4° K. Most likely this is due to two effects: (a) the increase in $\gamma_{350}$ in the upper layer decreases the amplitude of diurnal temperature variations in the layer and thus the ability of $\kappa(T)$ to increase $\langle T(x, t) \rangle$ (see § IIId of this paper and fig. 2 of Paper I), and (b) the increase in $\gamma_{350}$ in the upper layer means that the lower layers with larger thermal conductivities are more closely coupled to the upper layer and thus also decrease diurnal temperature variations in the upper layer. The effect of making the surface layer thinner from 5 to 4 cm is shown by Model 15 (3HR). Now the agreement with $\langle T(0, t) \rangle$ and nighttime brightness temperatures is excellent, but $\Delta T$ is 6.5° K too low. Clearly, the solution is to decrease $\kappa_c$ and raise $R_{350}$ in the upper layer. An acceptable solution is $\kappa_c = 3.76 \times 10^{-6}$ cal cm$^{-1}$ (deg-K)$^{-1}$ and $R_{350} = 2.07$ in the upper layer. For this Model 15 (4HR) $\langle T(0, t) \rangle = 217.5°$ K and $\Delta T = 31.0°$ K. Finally, we obtain Model 15(4) by placing the previous model at the lunar equator instead of Hadley Rille. For this final model $\langle T(0, t) \rangle$ is 222.5 and $\Delta T$ is 33.2° K. Note that Model 15(4) is essentially on the calibration curve for $T(0, 180°)$ versus $\langle T(0, t) \rangle$ in figure 3 but departs greatly from the calibration curve.
MOON AS PROPOSED RADIOMETRIC STANDARD

TABLE 2
SURFACE TEMPERATURE ERRORS DUE TO UNCERTAINTIES IN PARAMETERS

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>ASSUMED VALUE [cal cm$^{-2}$ min$^{-1}$]</th>
<th>RANGE</th>
<th>ERROR IN $\langle T(0, t) \rangle$ from:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f$</td>
<td>1.95</td>
<td>1.913-2.047</td>
<td>Measured $T(0, 270^\circ)$ Measured $T(0, 180^\circ)$</td>
</tr>
<tr>
<td>$E_B$</td>
<td>0.88</td>
<td>0.84-0.92</td>
<td>$\pm 1.5$</td>
</tr>
<tr>
<td>$E_{IR}$</td>
<td>0.89</td>
<td>0.85-0.93</td>
<td>$\pm 2.0$</td>
</tr>
<tr>
<td></td>
<td>Sum if errors additive</td>
<td>...</td>
<td>$\pm 5.2$</td>
</tr>
<tr>
<td></td>
<td>Sum if errors random</td>
<td>...</td>
<td>$\pm 3.0$</td>
</tr>
</tbody>
</table>

for $T(0, 270^\circ)$ versus $\langle T(0, t) \rangle$. This is due to the change in slope of the cooling curve at 80 hours into the night and suggests that the midnight rather than morning terminator temperatures can more accurately determine $\langle T(0, t) \rangle$.

e) A Realistic Estimate of $\langle T(0, t) \rangle$

At the present time we cannot give a definitive value for $\langle T(0, t) \rangle$ characteristic of the central portion of the lunar disk. We can, however, estimate its value and place realistic errors on this estimate. Assuming the classic model and our values of the radiation parameters $E_B$, $E_{IR}$, and $f$, nighttime observations yield a mean surface temperature of $220.5^\circ \pm 1.5^\circ$ K. We summarize in tables 2 and 3 the previous discussion on errors in the radiation parameters and those due to departures from the classical model assumptions. Errors in the radiation parameters can probably be expected to add together at random and thus introduce an error of $\pm 3.0^\circ$ K. Errors due to departures from the classical assumptions tend to be systematically negative, that is, the mean temperature would be less than that deduced by the classic model. The primary source of error here is that due to centimeter scale roughness. We think that a reasonable estimate of the error that may be associated with it is about $-3^\circ$ K. The sum of these errors if additive is $4.6^\circ$ K if $T(0, 180^\circ)$ is used to estimate $\langle T(0, t) \rangle$.

At this point the choice of a best estimate of $\langle T(0, t) \rangle$ and the error in its value is a matter of taste. We could choose a value of $218^\circ \pm 4.5^\circ$ K by assuming that half of the error associated with departures from the classical assumptions are systematic and half are random. If we assume that all the errors in tables 2 and 3 add in a random

<table>
<thead>
<tr>
<th>DEPARTURE FROM CLASSICAL ASSUMPTIONS</th>
<th>ERROR IN $\langle T(0, t) \rangle$ from:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transparency at thermal wavelengths</td>
<td>~0</td>
</tr>
<tr>
<td>Radiative conductivity</td>
<td>$\pm 0.3$</td>
</tr>
<tr>
<td>Two-layer models</td>
<td>~3</td>
</tr>
<tr>
<td>Heterogeneity on surface</td>
<td>~1.3</td>
</tr>
<tr>
<td>Small-scale craters</td>
<td>~0</td>
</tr>
<tr>
<td>Exposed bare rocks</td>
<td>~0</td>
</tr>
<tr>
<td>Centimeter scale roughness</td>
<td>~7.6</td>
</tr>
<tr>
<td>Sum if errors additive</td>
<td>~0</td>
</tr>
</tbody>
</table>

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fashion, then the resultant \( \langle T(0, t) \rangle \) is 220.5° ± 5.5° K. In either case, Models 1000, 11(1), 11(2), 12, and 15(4) are consistent with this mean surface temperature. At a depth of 1 m the Langseth et al. (1972) data suggest that the mean temperature should be 37° K hotter.

III. DEPTH DEPENDENCE OF THE MEAN TEMPERATURE \( \langle T(x, t) \rangle \)

Since a dielectric medium like the lunar surface material is not opaque to microwave radiation, any variation in the mean thermal radiation with wavelength must be due either to a variation in the mean temperature with depth or to some variation of the opacity with depth or temperature. We consider in this section the conditions under which the mean temperature can vary with depth. We assume that the lunar medium is either homogeneous or quasi-homogeneous as defined by Winter and Saari (1969). This means that differential volume elements contain a representative sample of solid material and void such that these volume elements have a unique kinetic temperature.

\[ a) \text{ The Classical Model} \]

As described above, the classical model assumes a plane-parallel surface geometry and homogeneous, temperature-independent thermal properties. We also assume for the moment no internal sources of heat. Although this case has been solved in Paper I, we summarize the derivation in order to generalize later. At any given time \( t \) and depth \( x \) (increasing inward), the thermal flux \( F(x, t) \) (defined to be positive if directed outward)

\[ F(x, t) = \kappa(x) \frac{\partial T(x, t)}{\partial x} \]  \hspace{1cm} (10)

and heat content per unit volume of the material \( Q(x, t) \) are related by the heat conduction equation

\[ \frac{\partial Q(x, t)}{\partial t} = \frac{\partial F(x, t)}{\partial x}. \]  \hspace{1cm} (11)

We take the time average (over a synodic period) of both sides of this equation, symbolized by the operator \( \langle \cdots \rangle \). This operator commutes with \( \partial/\partial t \) and \( \partial/\partial x \). In the absence of secular variation in the insolation or internal heat sources, \( \langle Q(x, t) \rangle \) must be constant and thus

\[ \langle F(x, t) \rangle = \langle \kappa(x) \partial T(x, t)/\partial x \rangle = \text{constant}. \]  \hspace{1cm} (12)

The further assumption of homogeneity implies that the conductivity is constant so that the mean temperature is independent of depth

\[ \langle T(x, t) \rangle = \langle T(0, t) \rangle. \]  \hspace{1cm} (13)

Thus, variations in the density and specific heat with depth or temperature are inconsequential.

\[ b) \text{ Inhomogeneity} \]

We consider now whether any form of inhomogeneity can change the simple result that \( \langle T(x, t) \rangle \) is depth independent. If we allow a general variation of conductivity
with depth but assume a plane-parallel geometry, equation (12) is still satisfied and the constant remains zero. Thus,

$$\kappa(x)\left(\frac{\partial}{\partial x}\langle T(x, t) \rangle \right) = 0 ,$$

(14)

and $\langle T(x, t) \rangle$ remains independent of depth.

Horizontal inhomogeneities such as variations in thermal properties, roughness, cratering, and boulders imply horizontal variations in the mean surface temperature. At depths larger than the scale of these inhomogeneities, horizontal variations in $\langle T(x, t) \rangle$ will be damped out by lateral conduction and $\langle T(x, t) \rangle$ will be the geometrically weighted average of the mean surface temperatures. We expect that at shallower depths the weighted mean of $\langle T(x, t) \rangle$ will be the same as that below. Since thermal microwave emission is proportional to temperature, we therefore expect it to reflect the geometrically weighted average of $\langle T(0, t) \rangle$ no matter at what depth such radiation originates.

c) Steady-State Heat Flow

Superimposed on the transient diurnal heat flow is a steady flow due to the thermal gradient between the surface and a hot interior. At the Hadley Rille site, Langseth et al. (1972) measure an equilibrium flux $\langle F(x, t) \rangle$ of $7.9 \times 10^{-7}$ cal cm$^{-2}$ sec$^{-1}$. This flux is one-half the terrestrial value and much larger than the $2.3 \times 10^{-7}$ cal cm$^{-2}$ sec$^{-1}$ estimated by MacDonald (1963) for a Moon consisting of purely condritic material. The effect of this steady-state heat flux is to produce a thermal gradient proportional to the flux and inversely proportional to the thermal conductivity. For a thermal conductivity of $4.1 \times 10^{-3}$ cal cm$^{-1}$ (deg-K)$^{-1}$ measured at a depth of 91 cm by Langseth et al. (1972), the gradient is 0.019° K cm$^{-1}$, whereas for a mean thermal conductivity near the surface of $3.0 \times 10^{-6}$ cal cm$^{-1}$ (deg-K)$^{-1}$ (the data of Cremers and Birkebak at 220° K), the gradient becomes 0.26° K cm$^{-1}$. In computing $T(x, t)$ in our models, we include a time-independent steady-state contribution due to the mean flux $\Delta T_F(x)$ given by

$$\Delta T_F(x) = \langle F(x, t) \rangle \int_0^x \frac{dx'}{\kappa(x, \langle T(x', t) \rangle)} .$$

(15)

d) Temperature-dependent Thermal Properties

As mentioned above, we have reason to believe that radiation may carry a significant portion of the flux in the lunar surface material. If so, an equivalent conductivity (eq. [6]) can be defined which includes a temperature-dependent term due to the fourth-power temperature dependence of the Stephan-Boltzmann law. For a more detailed discussion of radiative conductivity, see Clegg, Bastin, and Gear (1966), Troitsky, Burov, and Alyoshiva (1968), and Troitsky (1969). The significance of a temperature-dependent conductivity, as initially pointed out by Muncey (1958), is that the mean of a function of the temperature rather than the temperature itself is invariant with depth. Thus, the mean temperature will in general be depth dependent even if there is no net conductive flux.

In Paper I we derived analytical expressions for the change in mean temperature with depth. Here we extend the method to include conductivity of the form of equation (6). Substituting equation (6) for $\kappa(x, T)$ in equation (12), we find

$$\frac{\partial}{\partial x} \left[ \frac{4\kappa_c(x)}{\kappa_E(x)} \langle T(x, t) \rangle + \langle T^4(x, t) \rangle \right] = \frac{4\langle F(x, t) \rangle}{\kappa_E(x)} ,$$

(16)
which integrates to
\[
\frac{4\kappa_c(x)}{\kappa_R(x)} \langle T(x, t) \rangle + \langle T^4(x, t) \rangle = 4 \langle F(x, t) \rangle \int_0^\infty \frac{dx'}{\kappa_R(x')} + \frac{4\kappa_c(0)}{\kappa_R(0)} \langle T(0, t) \rangle + \langle T^4(0, t) \rangle. \tag{17}
\]

Since the insolation is periodic, we expand the temperature in a Fourier series,
\[
T(x, t) = T_0(x) + \sum_{n=1}^\infty T_n(x) \cos [n \omega t - \phi_n(x)], \tag{18}
\]
where \( T_0(x) = \langle T(x, t) \rangle \), \( \phi_n(x) \) is the phase lag of the \( n \)th harmonic, \( \omega = 2\pi/P \), and \( T_n(x) \) is the amplitude of the \( n \)th harmonic. These amplitudes, which are all positive in practice, have the property of decreasing rapidly with depth and \( n \). Substituting equation (18) into equation (17) leads to the expression
\[
\frac{4\kappa_c(x)}{\kappa_R(x)} T_0(x) + T_0^4(x) = \frac{4\kappa_c(x)}{\kappa_R(x)} \langle T(0, t) \rangle + \langle T^4(0, t) \rangle + 4 \langle F(x, t) \rangle \int_0^\infty \frac{dx'}{\kappa_R(x')} \]
\[
- \{4T_0^2(x)[T_1^2(x) + T_2^2(x)] + \frac{5}{3}T_0(x)T_1^2(x)T_3(x) + \frac{4}{3}[T_1^4(x) + T_2^2(x)] + T_1^2(x)T_2^2(x) + \cdots \}, \tag{19}
\]

where we have dropped higher-order terms. Equation (19) states that the mean temperature increases monotonically with depth due to a net flux \( \langle F(x, t) \rangle \) and asymptotically to a limit as the term in brackets approaches zero.

The magnitude of the increase in \( \langle T(x, t) \rangle \) between its surface and its asymptotic value when there is no net conductive flux depends primarily on the relative importance of radiative to contact conduction (see fig. 2 of Paper I). In Paper I we found that the model which yielded the best fit to infrared and microwave observations was one in

![Fig. 5.—Increase with depth of the lunation mean temperature \( \langle T(x, t) \rangle \) for several models. Parameters for these models are given in table 1.](image-url)

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which \( \gamma_{350} = 885 \) and \( R_{350} = 1.0 \). For this model the increase in \( \langle T(x, t) \rangle \) between its surface and its asymptotic value is 24° K.

The scale over which this increase in mean temperature occurs is approximately half that of the e-folding distance of thermal wave attenuation (Troitsky et al. 1968). In the above model this scale is about 2.5 cm. As examples of the increase in mean temperature one might expect in lunar soil, \( \langle T(x, t) \rangle \) is given in figure 5 as a function of depth for Models 1000, 1000–1, 1000–2, 11(2), 12, and 15(4). The increases in \( \langle T(x, t) \rangle \) due to the radiative conductivity alone are 0°, 24.0°, 36.0°, 13.4°, 32.0°, and 33.2° K, respectively. A significant aspect of radiative conductivity is that the effect on \( \langle T(x, t) \rangle \) is only in the first few centimeters beneath the surface.

When radiation plays any role at all in heat transport, the functional form is usually that of equation (6). This is true even for a rough surface model consisting of deep holes (Bastin and Gough 1969). If the pore sizes in the material are very much less than the mean free path of thermal photons, the radiative term in equation (6) will be proportional to \( T^2 \) rather than to \( T^3 \) (Troitsky et al. 1968). This will reduce the magnitude but not affect the scale of the increase in mean temperature with depth.

IV. PHYSICAL PARAMETERS AND PROCESSES DETERMINING THE MEAN BRIGHTNESS TEMPERATURE \( \langle T_b(\lambda) \rangle \)

In discussions of the microwave thermal emission from the Moon (cf. Krotikov and Troitsky 1964; Hagfors 1970) it is customary to express the emergent flux (or brightness temperature) of a ray leaving the surface at an angle \( \phi'(=\cos^{-1} \mu') \) from the normal in the form

\[
T_b(\lambda, t, \mu') = [1 - R(\mu_i)] \int_0^\infty T(x, t)k(\lambda)\exp\left\{ -\frac{k(\lambda)\mu}{\mu_i} dx\right\}. \tag{20}
\]

In this expression \( k(\lambda) \) is the absorption coefficient, \( \phi'(=\cos^{-1} \mu') \) the angle of the internal ray with respect to the inwardly directed normal given by Snell's law, and \( R(\mu_i) \) is the internal reflectivity for this ray. The term \( [1 - R(\mu_i)] \) represents the microwave emissivity of lunar material. Since the above equation has been assumed in all analyses of lunar microwave emission, we list the assumptions upon which it is based:

1. The surface geometry is plane-parallel. 2. The medium has homogeneous properties so that rays traverse straight lines. 3. The attenuation coefficient is purely absorptive with no scattering component. 4. The medium behaves as a solid dielectric. 5. The Rayleigh-Jeans approximation is valid. The latter approximation, although valid at wavelengths greater than about 1 mm, is a poor one in the infrared. We avoid this approximation by replacing \( T(x, t) \) and \( T_b(\lambda, t, \mu') \) with the corresponding Planck functions at these temperatures and then determining \( \langle T_b(\lambda) \rangle \) from the corresponding mean value of the Planck function on the left-hand side of equation (20).

In the following two sections, we estimate the range of values of the emissivity and absorption coefficient likely for lunar surface material. We also touch on the validity of certain of the above assumptions and the qualitative effect of departures from these assumptions on \( \langle T_b(\lambda) \rangle \).

a) Microwave Emissivity

i) The Relationship of Emissivity to Reflectivity

We seek a physically plausible description of the emissivity for lunar material at microwave and far-infrared wavelengths. The problem is a complex one involving considerations of the lunar soil dielectric constant \( \epsilon \), density, roughness of the surface, and the proper interpretation of thermal polarization and radar data. A most important question is how to relate the emissivity...
to a measurable quantity, the external reflectivity. It is this question to which we now address ourselves.

In the limit of wavelengths much larger than the scale of roughness and particle sizes, the lunar surface may be considered locally a plane-parallel dielectric medium with essentially unit permeability and zero conductivity. In this approximation the internal reflectivity $R(\mu_i)$ is the sum of the Fresnel coefficients for polarized radiation with the electric vectors parallel and perpendicular to the plane of incidence. It is then a function only of $\varepsilon$ and $\mu_i$. In particular, for vertically directed internal rays

$$R(\mu_i = 1) = \left(\frac{\sqrt{\varepsilon} - 1}{\sqrt{\varepsilon} + 1}\right)^2.$$ \hspace{1cm} (21)

Moreover, external rays incident on the surface are specularly reflected with the external reflectivity $\rho(\mu')$ equal to the internal reflectivity $R(\mu_i)$ by the reversibility principle of optics (see, e.g., Rossi 1957 for a proof). We thus find that the emissivity $E(\mu')$ and external reflectivity are related by

$$E(\mu') \equiv 1 - R(\mu_i) \hspace{1cm} (22a)$$

$$= 1 - \rho(\mu'). \hspace{1cm} (22b)$$

Difficulties arise when the lunar surface reflects diffusely rather than specularly as occurs at millimeter, and most likely at shorter, wavelengths (see below). Now the relation of emissivity to external reflectivity is no longer straightforward. Conel (1969) has suggested a “cloudy atmosphere” model in which the small particles of the surface material are considered as independent scatterers suspended in a nondispersive medium. He derives the reflectivity for such an atmosphere in the Eddington two-stream approximation and assumes, but does not derive, a relationship of emissivity to reflectivity the same as the first part of equation (19).

Siegel and Howell (1968) have shown that for an opaque surface, equation (22b) is a natural consequence of Kirchhoff’s law. When scattering is not specular, the quantity $\rho(\mu')$ is the directional hemispherical reflectivity, defined in analogy with the specular case, as the ratio of energy reflected in all directions to the energy incident from a direction $\mu'$. Elsewhere (Linsky 1972) we have shown that this generalized Kirchhoff’s law is valid for a plane-parallel isothermal atmosphere in which the isotropic scattering is coherent or there is complete redistribution within a line. Linsky and Mount (1972) considered whether equation (22b) is valid in relating emissivity and reflectivity for the lunar surface. They find that the isotropic coherent scattering assumption is consistent with the millimeter radar data but that diurnal thermal gradients in the lunar surface are sufficiently large to produce some departures from this relation at centimeter and long millimeter wavelengths. These departures are the result of scattering and low opacity.

Aronson et al. (1969) have partially confirmed this generalized Kirchhoff’s law for powders of various sizes in the range 5–30 μm at wavelengths of 7–20 μm. The experimental conditions are such that the ratio of wavelength to particle size ranges from $\frac{1}{2}$ to 4. Their experiment is not a proof of this generalized Kirchhoff’s law in that the emissivity at 45° ($\mu' = 0.7$) is compared with the external reflectivity measured at 20° ($\mu' = 0.94$) from hemispherical illumination. Herein we assume equation (22b) to be valid.

ii) Measurements of the Dielectric Constant

The work of Campbell and Ulrichs (1969) is the most comprehensive recent set of dielectric constant measurements of terrestrial rock powders at 67 cm and 0.86 cm. They find that for solid rocks, $\varepsilon$ is essentially the same at both frequencies and independent of temperature. Since the density variation of $\varepsilon$ for the rock powders apparently
moons as proposed radiometric standard

obeys the classical Rayleigh mixing formula, any temperature and density depend-

encies of ε for the powders are also unimportant. At a density of 1.0 g cm−3, ε for these
powders clusters in the range 1.9–2.1. Using similar techniques, Gold, Campbell, and
O'Leary (1970) measured values of ε on Apollo II sample powders at 67 cm. They
found ε = 1.8 and 2.5 at densities of 1.0 and 1.6 g cm−3, respectively, and that the
dielectric constant of lunar soil also obeys the Rayleigh mixing formula. Cremers et al.
(1971) find for lunar soil a density of 1.265 g cm−3 when a soil sample is deposited
under terrestrial gravity conditions. Since the true soil density near the surface is
likely to be less than this value, we assume densities in the range 1.0–1.3 g cm−3.

From the Gold et al. (1970) measurements, we estimate ε = 1.97 ± 0.17, from which
it follows by equation (21) that R(1) = 0.028 (±0.004, −0.007).

iii) Analysis of the Microwave Thermal Emission

For a number of years typical cited values for the lunar surface dielectric constant
have been 1.5 in the Russian literature and 2.7 in the Western literature. Since the
resultant normal internal reflectivities R(1) are 0.01 and 0.06, respectively, for a smooth
surface, a resolution of this discrepancy is imperative. The value of 1.5 cited in
Krotikov and Troitsky's (1964) review paper is based mainly on two methods utilizing
the lunar thermal emission. The first ascribes the difference between the mean bright-
ness temperature <Tb(λ)> and the mean surface temperature <T(0, t)> to the surface
loss R(1). At 3.2 cm, the data cited yield ε ≈ 1.5. Aside from the inherent inaccuracy
of deriving a small quantity from the difference between two large quantities, this
method is useless because the mean temperature observed at 3.2 cm is not <T(0, t))
but, rather, the mean temperature many centimeters below the surface, which can
differ considerably from <T(0, t)> as we have seen.

The second approach cited by Krotikov and Troitsky (1964) is the analysis of linear
polarization in the thermal emission near the limb, which arises from an enhancement
of the Fresnel internal reflectivity for the electric vector in the plane of incidence
relative to that perpendicular to the plane. The resultant linear polarization at each
position on the disk depends only on ε if the surface layer is homogeneous and plane
parallel. In practice, the polarization increases rapidly as θout approaches 90° near the
lunar limb. Thus, the true polarization must be deconvoluted out of the apparent
polarization seen by a broad antenna pattern. Also, gentle slopes with scales large
compared with a wavelength and depolarizing scattering due to roughness on the
scale of a wavelength add further uncertainty to the analysis. In the wavelength
range 0.86–3.7 cm, polarization analyses cited by Hagfors (1970) and Krotikov and
Troitsky (1964) yield ε in the range 1.5–1.8 corresponding to R(1) ~ 0.016. At longer
wavelengths 6–21 cm, where the deconvolution problem is worse, such analyses yield
larger values of ε 1.92–2.3 and R(1) ~ 0.032. We see no way of realistically estimating
the errors in such results other than by comparison with independent techniques.

iv) Analysis of the Radar Data

To date, the electrical properties of the lunar surface have been probed by radar
from the Earth in the wavelength range 0.86–2000 cm, by Surveyor spacecraft at
2.3 cm, and by the Explorer 35 lunar orbiter at 220 cm. Except for the latter set of
bistatic reflection measurements, there is no possibility of sampling the lunar external
reflectivity completely, as the angles of incidence and reflection must be equal. We
consider first the radar observations at the largest wavelengths, where the interpretation
is simplest.

Tyler's (1968) analysis of the Explorer 35 radar reflection experiment off the lunar
surface provides some unambiguous data on the dielectric constant as the cross-
section for specular scattering is to first order unaffected by roughness and there is no
detected diffuse component. Consequently, the null reflected signal at 60°.0 ± 1° is a
true measure of the Brewster angle, with the result that $\epsilon = 3.0 \pm 0.2$ for the average Moon, 2.8 for highlands, and 3.2 for maria. As the attenuation length for photons is typically $10\lambda$ (Gold et al. 1970), there is probably no marked change in $\epsilon$ in the depth range 1–40 m. Tyler (1968) argues that the existence of a deep Brewster null and the lack of a decrease in terrestrial-based radar reflectivity at shorter wavelengths implies no decrease in $\epsilon$ except in the upper few tens of centimeters of the surface.

Terrestrial-based radar exhibits two significant features: (1) essentially constant cross-section of $\sim 7.5$ percent of the disk area independent of wavelength (Hagfors 1970) and (2) a monotonic increase in the diffusely reflected component seen in time delay measurements from about 20 percent at 68 cm to 85 percent at 0.86 cm (Evans and Hagfors 1964). The power returned from a short pulse typically contains an initial “quasi-specular” peak due to specular reflection from a statistical distribution of plane facets oriented with respect to the mean lunar surface and, at all time delays, a diffuse component due to scattering by roughness or inhomogeneities on the scale of a wavelength. The crux of the difficulty in determining $\rho$ lies in inferring from the data how much the roughness enhances the returned signal over what it would be if the Moon were a smooth sphere of similar dielectric properties.

The basic quantity of interest is the radar cross-section $\sigma_{\varphi, \varphi'}(\phi, \phi', \theta)$ defined such that $\sigma_{\varphi, \varphi'}(\phi, \phi', \theta)/4\pi$ is the power reflected into 1 sterad solid angle per square meter of target surface for an incident flux of 1 watt m$^{-2}$ (cf. Evans and Hagfors 1968). Here $\phi, \phi'$, and $\theta$ are the angles of incidence, reflection, and the azimuth angle between them. The cross-section is usually assumed azimuth-angle-independent, and for terrestrial-based radar $\phi$ and $\phi'$ are always equal. The total measured cross-section, the intercepting area of a perfectly conducting sphere which would reflect back an equal amount of energy to the antenna as the actual target. The measured total cross-section and the normal reflectivity are related by

$$\sigma = 2\pi R_M^2 \int_0^{\pi/2} \sigma_{\varphi, \varphi'}(\phi, \phi' = \phi) \sin \phi d\phi,$$

is the intercepting area of a perfectly conducting sphere which would reflect back an equal amount of energy to the antenna as the actual target. The measured total cross-section and the normal reflectivity are related by

$$\sigma = g\rho(1)\pi R_M^2,$$

where the directivity factor $g$ represents the enhancement of backscattering relative to an isotropically scattering sphere. Thus, $g = 1.0$ for a smooth dielectric sphere. When this sphere is replaced by one with a statistical distribution of plane facets producing the “quasi-specular” return, $g = (1 + n\alpha^2)$, where $n$ is of order unity and $\alpha$ is the rms surface slope (cf. Evans and Hagfors 1964). The term $\alpha$ decreases with wavelength so that at $1\,\text{m}, \alpha \approx 5^\circ$ and $g \approx 1$. Since the diffuse component is small for $\lambda > 10\,\text{cm}$ (so that $g \approx 1$) and the observed cross-section is very uncertain beyond 3 m, we first deduce the normal reflectivity in this wavelength range on the assumption of a smooth spherical Moon. From the data cited by Hagfors (1970), the cross-section and thus $\rho(1)$ have values of $0.075 \pm 0.013$ (probable error), and consequently $\epsilon = 3.10 (+0.21, -0.31)$. This value for $\rho(1)$ is probably valid in the limit of long wavelengths.

In the limit of pure diffuse radar reflection there is another straightforward means of deriving $\rho(1)$ and thus the normal emissivity. Rea, Hetherington, and Mifflin (1964) and Evans and Hagfors (1968) show that

$$\sigma = G_M\bar{\rho}\pi R_M^2,$$

where $\bar{\rho}$ is the spherical albedo or reflectivity averaged over a sphere

$$\bar{\rho} = \int_0^{\pi/2} d\phi \int_0^{\pi/2} d\phi' \sigma_{\varphi, \varphi'}(\phi, \phi') \sin \phi \sin \phi'.$$
and the directivity $G_M$ is given by

$$G_M = 2 \left[ \int_0^{\pi/2} \sigma_0(\phi, \phi') \sin \phi \, d\phi \right] \left[ \int_0^{\pi/2} \sin \phi \, d\phi \int_0^{\pi/2} d\phi' \sigma_0(\phi, \phi') \sin \phi \sin \phi' \right]. \quad (27)$$

The normal reflectivity expressed in these terms is

$$\rho(1) = 2\pi \int_0^{\pi/2} \frac{\sigma_0(0, \phi')}{4\pi} \sin \phi' \, d\phi'. \quad (28)$$

We assume that $\sigma / \pi R_M^2 = 0.075 \pm 0.013$ in the millimeter and submillimeter portions of the spectrum as at longer wavelengths. The only remaining datum needed is a valid functional form for $\sigma_0(\phi, \phi')$. Diffuse scattering often follows a Lambert phase function $\sigma_0(\phi, \phi') \sim \cos \phi \cos \phi'$ or $\sigma_0(\phi, \phi) \sim \cos^2 \phi$ for the backscattered signal, but this particular form of $\sigma_0(\phi, \phi)$ has never been observed in lunar time delay echoes. Instead, the diffuse component appears to be of the form $\sigma_0(\phi, \phi) \sim \cos^{3/2} \phi$ at wavelengths of 23 cm or longer and $\sigma_0(\phi, \phi) \sim \cos \phi$ at 0.86 and 3.6 cm. According to Evans and Hagfors (1968), the latter set of data is uncertain; thus, whether the angular dependence of the backscattered signal is $\cos \phi$ or $\cos^{3/2} \phi$ at short wavelengths is in doubt. Also, whether these forms are valid at wavelengths less than 0.86 cm or, indeed, at small $\phi$ for any wavelength is in doubt.

In order to ascertain the range in which $\rho(1)$ and thus $E(1)$ most likely lie, we have assumed a number of plausible functional forms for $\sigma_0(\phi, \phi')$ consistent with the measured backscattering dependences $\sigma_0(\phi, \phi) \sim \cos^{3/2} \phi$ and $\sim \cos \phi$. These and the Lambert law are listed in Table 4. For each assumed form of $\sigma_0(\phi, \phi')$, the values of $G_M$, $\bar{\rho}$, and $\rho(1)$ are given as well as the formal error in $\rho(1)$ due to the assumed error in $\sigma$ alone. The resulting values for $E(1)$ assuming the generalized Kirchhoff's law (eq. [22b]) lie in the range 0.963–0.981. The “error” in $E(1)$ due to the range of uncertainty in the form of $\sigma_0(\phi, \phi')$ is thus twice that due to the uncertainty in the total measured cross-section. It seems reasonable to assume the mean value of $E(1)$ and estimate its uncertainty by assuming that the two sources of uncertainty add randomly. The resultant value for $E(1)$ is 0.972 ± 0.010. From the Evans and Hagfors (1964) analysis, we assume that the radar signal is totally diffuse and thus the present analysis valid for wavelengths less than 4 mm.

v) Best-Estimate Values for the Microwave Emissivity

As before in this paper, we must choose values for essential parameters given insufficient information. It seems clear that the long- and short-wave portions of the microwave spectrum are most reliably known and can serve as anchors for the middle portion of the spectrum. At 220 cm, the Brewster angle measurement of $\epsilon = 3.0 \pm 0.2$ implies $\rho(1) = 0.071 (± 0.009, -0.008)$ under the good assumption that the lunar
surface is plane-parallel and nonscattering. The long-wavelength limit of $\rho(l) = 0.075 \pm 0.013$ from terrestrial radar observations is consistent with and reinforces our acceptance of the Brewster angle determination of surface loss.

In the limit of short wavelengths where the radar returned signal is completely diffuse, we obtained a value of $\rho(l) = 0.028 \pm 0.010$. Gold et al.'s (1970) measurements of the dielectric constant of an Apollo 11 soil sample imply $\epsilon = 1.97 \pm 0.17$ for our assumed values of the surface soil density. If this material were smooth at all scales, then $\rho(l) = R(l) = 0.028 (\pm 0.004, -0.007)$, in excellent agreement with the above radar value. This agreement implies that in terms of its total internal reflectivity properties, the lunar surface behaves as if it were a continuous dielectric. The roughness at the surface serves only to reorient the direction of rays from below. This picture seems plausible in that the transition from medium to space at the surface occurs in much less than 1 millimeter and would thus appear to a millimeter wave to be a discontinuity. We adopt $\rho(l) = 0.028 \pm 0.010$ for $\lambda \leq 4$ mm.

Since the absorption length for photons is about $10 \lambda$, the radar data suggest that $\epsilon = 3.0$ material exists at depths of meters and $\epsilon = 2.0$ material in the first few or tens of centimeters of the regolith. From Gold et al.'s (1970) measurements of $\epsilon$ as a function of density, the density is roughly 1.9 g cm$^{-3}$ at meter depths corresponding to a porosity of 0.38, but near the surface the density is only 1.2 or less with the porosity 0.6 or greater. This increase in density with depth is consistent with Shoemaker et al.'s (1970) picture of the regolith being turned over by meteoritic impacts more rapidly at shallower depths. At the surface, the soil is very finely divided. According to the above picture, the soil should become coarser with depth. Thus, scattering should be important at millimeter wavelengths and become less important with increasing wavelength. At the same time, $\rho(l)$ should increase with wavelength due to the increase in $\epsilon$ with depth. This behavior is seen in Evans and Hagfor's (1964) analysis of radar time delay experiments where at 0.86, 3.6, and 68 cm they find the returned signal is 85, 30, and 20 percent diffuse, respectively, and $\rho(l)$ has values of 0.035, 0.060, and 0.065, respectively. The Surveyor 2.3-cm landing radar data, which were analyzed by Muhleman et al. (1969) assuming part quasi-specular and part diffuse scattering, yield $\rho(l) = 0.037$ for maria and 0.083 for the extreme highlands region near Tycho. These data imply an average Moon value of $\rho(l) \sim 0.05$ consistent with terrestrial-based radar. In figure 6 the long- and short-wavelength values of $E(1) = 1 - \rho(1)$ are plotted.

![Fig. 6.—Estimated normal emissivity of lunar soil as a function of wavelength. The designated lines refer to measurements of O (O'Leary and Briggs 1970), M (Murcray et al. 1970), A (Aronson and Emslie 1969), and T (Tyler 1968). Crosses, data of Aronson and Emslie (1969). Solid lines enclose estimated ranges of high certainty; dashed lines, those of lower certainty.](image)
as well as a schematic estimate of $E(1)$ at intermediate wavelengths guided by the stated error brackets and the above discussion.

In principle, equation (20) should be modified to take account of the diffuse scattering of upward-directed rays in the medium. We will not attempt this here because the nature of the scattering process is poorly known and because the scattered thermal component would be no more than 3 percent of the thermal signal. Since the effect of scattering would be to alter the depth of formation of at most 3 percent of the signal, the net effect of the scattering would be unmeasurable near the center of the disk.

vi) Some Speculations concerning Emissivity in the Far-Infrared

For completeness, we will try to estimate reasonable values for the emissivity between the middle infrared (10–20 μm) and the millimeter portions of the spectrum. Essentially, we will speculate on how and why the emissivity changes from \~0.89 in the range 10–20 μm to 0.972 at short millimeter wavelengths.

We have shown (Linsky 1972) that for an atmosphere with isotropic coherent scattering

$$E(1) = 1 - \rho(1) = 1 - \frac{\omega_0 H(1)}{2} \int_0^1 \frac{H(\mu')\mu' d\mu'}{(1 + \mu')},$$  \hspace{1cm} (29)

where $H(\mu')$ are the $H$ functions tabulated in Chandrasekhar (1960) and $\omega_0$ is the albedo for single scattering (the fractional part of the total opacity that is scattering, rather than absorption). If this description of scattering is valid, we can qualitatively account for this increase in $E(1)$ by noting that most soil particles are less than 10 μm in size (Gold et al. 1970). In general, scattering increases relative to absorption, and thus $\omega_0$ increases as the ratio of particle size to wavelength increases from zero (Aronson and Emslie 1969). Thus, $\omega_0$ should increase with decreasing wavelength. A decrease in $E(1)$ from 0.972 at, say, 4 mm to about 0.89 at, say, 10 μm could be explained by an increase in $\omega_0$ from 0.18 at 4 mm to 0.50 at 10 μm.

The behavior of $E(1)$ in the far-infrared will most likely not be smooth. In particular, resonances are likely especially in the vicinity of 30 μm (Campbell and Ulrichs 1969), where $\omega_0$ is likely to be small and $E(1)$ approaches unity. Such resonances due to lattice vibrations may not lead to pronounced structure in $E(1)$, however, as the effect of reducing a mineral powder to micron sizes is to wash out most structure in reflectivity and emissivity (Conel 1969; Aronson and Emslie 1969). It seems likely then that the submillimeter (100–1000 μm) emissivity is high and in the range 0.95–1.00. These error brackets are included in figure 6. Also included for comparison purposes are the observations of Murcray et al. (1970) at 7–13.5 μm, experimental value of $E(1)$ for 3-μm corundum powder at 7–20 μm of Aronson and Emslie (1969), and the near-infrared (1–1.8 μm) diffuse reflectance measurements of O'Leary and Briggs (1970) of an Apollo 11 dust sample from Mare Tranquilitatus.

b) Microwave Opacity

Wechsler and Simon (1966) derive an expression for the linear absorption coefficient (per centimeter of path length),

$$k(\nu) = \pi \varepsilon^{1/2} \vartheta \tan \delta.$$  \hspace{1cm} (30)

Here $\vartheta$ is the photon wavenumber, $\varepsilon$ is the real part of the dielectric constant, and the loss tangent $\tan \delta$ is the ratio of the imaginary to the real parts of the dielectric constant. This is valid for nonmagnetic material. Measurements of the paramagnetic susceptibility of lunar soil and rocks by Nagata et al. (1970) imply that this assumption
is valid. We consider in this section what physical processes might contribute to the loss and thus the opacity in the microwave and far-infrared. In particular, we consider how $k(v)$ might depend on density, frequency, and temperature as these quantities, especially the latter two, can influence the emergent thermal flux.

i) Dependence on Density

Campbell and Ulrichs (1969) have found that for dry terrestrial rocks and rock powders at microwave frequencies, there is no significant macroscopic conductivity, so that all relevant loss mechanisms are bulk ones. Furthermore, they find that of all expressions proposed relating the complex dielectric constant for the soil $\varepsilon^*_0$ of density $\rho_0$ to the complex dielectric constant $\varepsilon^*$ of the rock powder of density $\rho$, the classical Rayleigh formula

$$\frac{1}{\rho} \frac{\varepsilon^* - 1}{\varepsilon^* + 2} = \frac{1}{\rho_0} \frac{\varepsilon^*_0 - 1}{\varepsilon^*_0 + 2}$$

appears to best fit the data. Only porous rocks like pumice should not and do not follow the Rayleigh formula. If we write

$$\varepsilon^* = \varepsilon' + i\varepsilon''$$

and assume $\varepsilon'' \ll \varepsilon'$, which is reasonable for a dielectric, then the imaginary part of equation (31) is

$$\frac{\varepsilon''}{\rho(\varepsilon' + 2)} = \frac{\varepsilon''_0}{\rho_0(\varepsilon'_0 + 2)}.$$ 

Since $\tan \delta \equiv \varepsilon''/\varepsilon'$, the last expression is quite similar to

$$\frac{\tan \delta}{\rho} = \frac{\tan \delta_0}{\rho_0}.$$ 

In fact, for the various rocks tested by Campbell and Ulrichs (1969), the quantity $\tan \delta/\rho$ was essentially independent of $\rho$. Since equation (34) also appears to satisfy the data on terrestrial basalt and pumice powders at 9.2 and 25.4 $\times 10^9$ Hz (Wechsler and Simon 1966) and Apollo 11 soil at various densities and 0.45 $\times 10^9$ Hz (Gold et al. 1970), we assume this dependence where needed.

ii) Dependence on Frequency

Physically, the absorption of microwave energy is due to resonant interactions with permanent or induced dipoles in the material. Each loss mechanism is typically important only in a limited frequency range. Since we are interested in the wavelength range 30 cm–100 $\mu$m corresponding to 1 $\times 10^9$–3 $\times 10^{12}$ Hz or 0.03–100 cm$^{-1}$, we consider in detail those mechanisms which can be important at these frequencies.

1. Space charge polarization occurs when the applied electric field induces charge to build up on grain boundaries or grain imperfections. Strangway (1969) and Saint-Amant and Strangway (1970) attribute this mechanism as the cause for low-frequency ($10^9$–$10^{10}$ Hz) loss in most dry powdered terrestrial rocks. This absorption has a typical Debye relaxation character (cf. Fröhlich 1949) in that $\tan \delta$ has a peak in this range which is thermally activated and shifts to higher frequency with increasing temperature. Saint-Amant and Strangway (1970) show that this mechanism is very important in powders consisting of grains of different dielectric properties, the Maxwell-Wagner effect. Apollo 12 sample 12002 measured by Chung, Westphal, and Simmons (1971) and Apollo 11 sample 10020 measured by Chung et al. (1970) show this behavior in
the region $10^2$–$10^7$ Hz. These samples show a much larger $\tan \delta$ than terrestrial rocks, presumably due to the inclusion of particles of different dielectric properties. At frequencies above the resonance, that is, greater than $10^6$ Hz at lunar temperatures, $\tan \delta$ decreases with increasing frequency but increases with temperature. In the absence of other loss mechanisms, this behavior would persist in the region of our interest where the loss tangent would be very low. Such behavior is exhibited by fused silica (curve A) in figure 7.

2. Polar molecules like water in the crystal lattice can change their orientation in the applied electric field, a loss process referred to as orientation polarization. This process is also of the Debye type (Parkhomenko 1967) and can appreciably increase $\tan \delta$ at low frequencies even for small concentrations (Chung et al. 1970). In view of this process, it is very important to measure dry samples even at the high frequencies of interest.

3. At very high frequencies typically in the visible and near-infrared an incident electric field can displace electrons from their nuclei, producing electronic polarization. This loss process is resonant in character (cf. Von Hippel 1954a), which should have little if any contribution to $\tan \delta$ at the much lower frequencies of interest. A similar process, atomic polarization, occurs in molecular gases.

4. A very important process in the $3 \times 10^8$–$3 \times 10^{12}$ Hz region of interest is the transconduction mechanism wherein impurity ions trapped in the crystal lattice move with the applied electric field. Such ions as $\text{Na}^+$, $\text{K}^+$, $\text{Ca}^+$, $\text{Fe}^{++}$, and $\text{Fe}^{+++}$ are typically loosely bound and can greatly increase the loss even for small concentrations. Figure 7 summarizes loss tangent measurements for a range of materials and terrestrial rocks at $6.1$–$35 \times 10^9$ Hz and lunar soils at $0.45$–$1500 \times 10^9$ Hz. The materials included are described in table 5. Pure materials such as fused silica (curve A) are quite

![Figure 7](image-url)
<table>
<thead>
<tr>
<th>Sample</th>
<th>Density [g cm(^{-3})]</th>
<th>Temperature [(^{\circ}) K]</th>
<th>Wavenumber [cm(^{-1})]</th>
<th>Frequency ([\times 10^9])</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>A, fused silica (Corning 7940)</td>
<td>2.20</td>
<td>298–1673</td>
<td>0.23</td>
<td>6.1</td>
<td>Iglesias and Westphal (1967)</td>
</tr>
<tr>
<td>B, SiO(_2), sintered</td>
<td>1.92</td>
<td>298–1645</td>
<td>0.28</td>
<td>8.5</td>
<td>Westphal and Iglesias (1970)</td>
</tr>
<tr>
<td>C, quartz fiber</td>
<td></td>
<td>298–1268</td>
<td>0.28</td>
<td>8.5</td>
<td>Westphal and Iglesias (1970)</td>
</tr>
<tr>
<td>D, quartz powder (dry)</td>
<td>1.22</td>
<td>353–873</td>
<td>0.28</td>
<td>8.5</td>
<td>Iglesias and Westphal (1967)</td>
</tr>
<tr>
<td>E, alkali-silica glass (6.4% Na(_2)O, 6.4% K(_2)O)</td>
<td></td>
<td>298–673</td>
<td>0.33</td>
<td>10.0</td>
<td>Von Hippel (1954b)</td>
</tr>
<tr>
<td>F, magnesium orthosilicate (F-118)</td>
<td>3.09</td>
<td>298–823</td>
<td>0.28</td>
<td>8.5</td>
<td>Westphal and Iglesias (1970)</td>
</tr>
<tr>
<td>G, magnesium orthosilicate (F-202)</td>
<td>3.09</td>
<td>298–823</td>
<td>0.28</td>
<td>8.5</td>
<td>Westphal and Iglesias (1970)</td>
</tr>
<tr>
<td>H, magnesium aluminum silicate (cordierite ceramic)</td>
<td></td>
<td>298–1280</td>
<td>0.28</td>
<td>8.5</td>
<td>Iglesias and Westphal (1967)</td>
</tr>
<tr>
<td>I, magnesium metasilicate (F-66)</td>
<td></td>
<td>298–1273</td>
<td>0.47</td>
<td>14.0</td>
<td>Iglesias and Westphal (1967)</td>
</tr>
<tr>
<td>J, limonite</td>
<td></td>
<td>298–783</td>
<td>0.28</td>
<td>8.5</td>
<td>Iglesias and Westphal (1967)</td>
</tr>
<tr>
<td>K, Virginia greenstone (52% relative humidity)</td>
<td>2.94</td>
<td>301–613</td>
<td>0.33</td>
<td>10.0</td>
<td>Iglesias and Westphal (1967)</td>
</tr>
<tr>
<td>L, basalt powder</td>
<td>1.20</td>
<td>77–398</td>
<td>0.31</td>
<td>9.2</td>
<td>Wechsler and Simon (1966)</td>
</tr>
<tr>
<td>M, basalt powder</td>
<td>1.20</td>
<td>77–398</td>
<td>0.85</td>
<td>25.4</td>
<td>Wechsler and Simon (1966)</td>
</tr>
<tr>
<td>N, pumice powder</td>
<td>0.90</td>
<td>77–397</td>
<td>0.31</td>
<td>9.2</td>
<td>Wechsler and Simon (1966)</td>
</tr>
<tr>
<td>O, pumice powder</td>
<td>0.90</td>
<td>77–397</td>
<td>0.85</td>
<td>25.4</td>
<td>Wechsler and Simon (1966)</td>
</tr>
<tr>
<td>P, pumice powder with 0.7% Fe</td>
<td></td>
<td>298</td>
<td>0.85</td>
<td>25.4</td>
<td>Wechsler and Simon (1966)</td>
</tr>
<tr>
<td>Q, tholeiitic basalt</td>
<td></td>
<td>300–880</td>
<td>1.17</td>
<td>35.0</td>
<td>Campbell and Ulrichs (1969)</td>
</tr>
<tr>
<td>R, olivine basalt</td>
<td></td>
<td>300–880</td>
<td>1.17</td>
<td>35.0</td>
<td>Campbell and Ulrichs (1969)</td>
</tr>
<tr>
<td>S, Apollo 11 soil</td>
<td>1.00*</td>
<td>300</td>
<td>0.015</td>
<td>0.45</td>
<td>Gold et al. (1970)</td>
</tr>
<tr>
<td>T, Apollo 11 soil</td>
<td>1.00*</td>
<td>300</td>
<td>5.0</td>
<td>150.0</td>
<td>Bastin (1971)</td>
</tr>
<tr>
<td>U, Apollo 11 soil</td>
<td>1.00*</td>
<td>300</td>
<td>50.0</td>
<td>1500.0</td>
<td>Bastin (1971)</td>
</tr>
</tbody>
</table>

* Assumed
transparent in this frequency range. What loss there is could be due to rubbing at crystal boundaries (Bussey 1971), which could be enhanced by sintering the material (curve B). Quartz powder (curve D) may be lossier than quartz fiber (curve C) for the same reason. By comparison, an alkali-silica glass containing Na₂O and K₂O (curve E) is a factor of 100 lossier than fused silica, due to the transconduction process. Simple magnesium silicates (curves F, G, H, and I) also are less lossy than the measured terrestrial rocks and rock powders (curves J–O, Q, R) which contain impurity ions. The addition of 0.7 percent by weight of iron powder to pumice powder increases tan δ by an order of magnitude (point P) relative to the pumice powder itself (curve O).

There is no discussion in the literature to our knowledge concerning the frequency dependence of tan δ for the transconduction process.

5. Most terrestrial rocks exhibit a strong absorption peak in the region of 300 cm⁻¹ (10¹³ Hz) (Campbell and Ulrichs 1969), presumably due to resonances in the crystal lattice. Although this process is most important in the far-infrared, the low-frequency tail of these resonances might be important for some materials in the millimeter range (3 × 10¹⁰–3 × 10¹¹). This is especially likely for lunar soils and rocks, as the existence of impurities in the crystal lattice, which lead to the transconduction process, also broadens the infrared resonances (Westphal 1963; Iglesias and Westphal 1967) and thus increases their importance at lower frequencies.

The frequency dependence of tan δ for this process is given by the classical dispersion relation (Von Hippel 1954a)

$$\tan \delta = \frac{\alpha B}{A[(\bar{\nu} - \bar{\nu}_R)^2 + \alpha^2] + B(\bar{\nu} - \bar{\nu}_R)},$$

where $B/\alpha A$ is the maximum value of tan δ, near the resonant wavenumber $\bar{\nu}_R$, and $\alpha$ is related to the half-width. Bastin (1971) has measured the mass absorption coefficient $k(\bar{\nu})/\rho$ for Apollo 11 soil in the interval 5–50 cm⁻¹. These data do not precisely fit an expression of the form of equation (35) but do fit the simplified functional form

$$\frac{k(\bar{\nu})}{\rho} = a\bar{\nu} + \frac{b\bar{\nu}}{(\bar{\nu} - \bar{\nu}_R)^2} \text{cm}^2 \text{g}^{-1},$$

which arises when tan δ has contributions from the low-frequency tail of an infrared resonance and some other frequency-independent loss process. A least-squares fit to the data (see fig. 8) yields the parameters $a = 0.079$, $b = 868$, and $\bar{\nu}_R = 69.3$. The value of the resonant wavenumber (69.3 cm⁻¹) is only roughly determined and the half-width $a$ undetermined since there are no data close to the resonance. The resonant frequency is low compared with terrestrial rocks but not unreasonably so (Campbell 1971). In the limit of small wavenumbers (essentially $\bar{\nu} < 1$ cm⁻¹)

$$\frac{k(\bar{\nu})}{\rho} = 0.26\bar{\nu} \text{cm}^2 \text{g}^{-1}.$$  

More recent data on Apollo 11 soil at a density of 1.5 g cm⁻³ obtained by Clegg et al. (1972) are plotted in figure 8 and are not too different from the earlier data.

The few existing loss tangent measurements of terrestrial rocks near 1 cm⁻¹ (3 × 10¹⁰ Hz) generally show an increase in tan δ with frequency. Measurements of tan δ for pumice and basalt powders at 25.4 × 10⁹ Hz (curves M and O in fig. 7) are typically 50 percent higher than the data at 9.2 × 10⁹ Hz (curves L and N). Similarly, tan δ for a Virginia greenstone increases between 10⁹ and 10¹⁰ Hz (Iglesias and Westphal 1967). It thus seems reasonable to assume that tan δ for lunar soil will increase with $\nu$ for $\nu > 10^9$ Hz. In fact, Sinton (1962) and Weaver (1965) feel that the lunar thermal emission data imply this behavior.
Aside from Bastin's (1971) absorption coefficient measurements, the only other relevant data on lunar soil are those of Gold et al. (1970) at $0.015 \text{ cm}^{-1}$ ($0.45 \times 10^9 \text{ Hz}$). For Apollo 11 soil at a density of $1 \text{ g cm}^{-3}$, they measure an absorption length of 17.8 wavelengths and dielectric constant of 1.8 corresponding to a linear absorption coefficient of $8.4 \times 10^{-4} \text{ cm}^{-1}$ and a loss tangent of 0.0133 at this frequency. If the loss tangent is independent of frequency, then

$$k(\nu)/\rho = 0.056\nu \text{ cm}^2 \text{ g}^{-1}$$
$$k(\nu) = 0.056\nu \text{ cm}^{-1}. \quad (38)$$

This mass absorption coefficient (see fig. 8) is nearly a factor of 5 less than Bastin's data at low frequencies (eq. [37]), implying that the two samples are either different or that there is a true increase in opacity between $0.45 \times 10^9 \text{ Hz}$ and $60 \times 10^9 \text{ Hz}$ ($5 \text{ cm}^{-1}$). Clearly, more data are needed to answer this question.
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in § V we take as models for the frequency dependence of \( k(\bar{v})/\rho \) equations (36) and (38).

**iii) Dependence on Temperature**

Almost without exception, the loss tangents for the materials and terrestrial rock samples in figure 7 increase with temperature. This behavior is also typical of those inorganic materials measured at \( \nu > 10^9 \text{ Hz} \) by Von Hippel (1954b), Iglesias and Westphal (1967), and others. In materials for which transconductance of impurity ions is the predominant loss mechanism, Campbell (1971) suggests that

\[
\tan \delta \sim e^{-E/kT},
\]

where \( E \) is the activation for motion of the impurity ions and \( kT \) the thermal kinetic energy. This particular functional form does not fit the data. If the low-frequency tail of an infrared resonance is the primary contributor to the loss, then one might expect \( \tan \delta \) to increase with temperature when the half-width parameter \( \alpha \) in equation (35) increases with temperature. In particular, far from the resonance, one might expect \( \tan \delta \) to be proportional to \( \alpha \), since in the denominator of equation (35) \( (\bar{v} - v_b)^2 \) is \( \gg \alpha^2 \). Westphal (1971) finds that this broadening of the infrared resonance does not usually occur until the material softens or melts. For lunar soils and rocks these temperatures are apt to be much higher than those experienced on the lunar surface.

In the absence of an adequate theory for the temperature dependence of \( \tan \delta \), we will compute models in which \( \tan \delta \) is assumed to vary with temperature in the same manner as several of the measured samples. In figure 9 we plot \( \tan \delta(T)/\tan \delta(300^\circ \text{ K}) \) for the terrestrial rock and rock powder samples and one glass sample described above. The range is from essentially no temperature dependence to very

![Fig. 9](image.png)

**Fig. 9.**—The temperature dependence of the loss tangent \((\tan \delta)\) normalized at 300° K for several materials listed in table 5. Crosses, data of Clegg et al. (1972).
steep ones beginning at low temperatures or near $300^\circ$ K. Hopefully, this encompasses the range of temperature dependencies of lunar surface material. Included in figure 9 are very recent data from Clegg et al. (1972) on the relative opacity of an Apollo 11 soil sample at $77^\circ$, $194^\circ$, and $300^\circ$ K. Plotted are data which are the mean of their measurements between 300 and 700 $\mu$m. These data lie essentially on curve $R$.

Troitsky et al. (1968) have called attention to the significant enhancement in $\langle T_B(\lambda) \rangle$ that results from an increase in microwave opacity with temperature. They find that the enhancement is mainly at a few millimeters wavelength and diminishes rapidly to shorter wavelengths and less rapidly to longer wavelengths. Physically, the increase in $\langle T_B(\lambda) \rangle$ when $k(\nu, T)$ increases with temperature is due to higher opacity during the day when $T(x)$ decreases inward and lower opacity during the night when $T(x)$ increases inward. Naturally, there will be no enhancement in $\langle T_B(\lambda) \rangle$ when the opacity is so large that all the thermal radiation is effectively from the surface or so small that the effective emitting region is always isothermal. This accounts for no effect at wavelengths much longer or shorter than a few millimeters.

Clearly, the importance of this effect increases with the magnitude of the temperature variation of $k(\nu, T)$ over the range $80^\circ-400^\circ$ K and, to a lesser extent, the functional form of this variation.

V. A DETERMINATION OF THE MEAN BRIGHTNESS TEMPERATURE $\langle T_B(\lambda, t) \rangle$

a) Causes of a Wavelength Variation in the Mean Brightness Temperature

At a given instant of time, the brightness temperature $T_B(\lambda, t)$ is related to the surface temperature distribution $T(x, t)$ by equation (20). When the opacity is independent of temperature and thus time, the mean brightness temperature $\langle T_B(\lambda, t) \rangle$ at the center of the disk is related to the mean temperature distribution $\langle T(x, t) \rangle$ by

$$\langle T_B(\lambda, t) \rangle = [1 - R(1)] \int_0^\infty T(x, t)k(\lambda)(\exp [-k(\lambda)x]dx$$

(40)

in the Rayleigh-Jeans approximation. In general,

$$B_\lambda[T_B(\lambda, t)] = [1 - R(1)] \int_0^\infty B_\lambda[T(x, t)]k(\lambda)(\exp [-k(\lambda)x]dx,$$

(41)

where $B_\lambda(T)$ is the Planck function and $\langle T_B(\lambda, t) \rangle$ is computed by a time average of $T_B(\lambda, t)$. There are four processes which can each produce a wavelength dependence in $\langle T_B(\lambda, t) \rangle$, which we discuss below.

i) Radiative Conductivity

As described earlier, radiative conductivity can produce an increase in $\langle T(x, t) \rangle$ with depth. In figure 5 are plotted mean temperature distributions for the models under consideration. In all cases, the increase in mean temperature occurs in the first decimeter in the lunar surface below which essentially no diurnal temperature variation occurs. Mean brightness temperatures for these models are shown in figure 10 for the opacity given by equation (36) (solid lines) and by equation (38) (dashed lines). For these calculations, the surface losses given by $R(1)$ and the steady-state heat flux were set equal to zero and the opacity was assumed temperature independent. No increase in $\langle T_B(\lambda, t) \rangle$ occurs until the material begins to be somewhat transparent, say $k(\lambda) \leq 10$, at which time the mean depth of emission is about 1 mm. This occurs typically at a wavelength of 500 $\mu$m for equation (36) and 50 $\mu$m for equation (38). At longer wavelengths, $k(\lambda)$ decreases and $\langle T_B(\lambda, t) \rangle$ increases due to the increase in
Fig. 10.—Mean radiation temperatures \( \langle T(\lambda, t) \rangle \) assuming no surface losses and temperature-independent opacity for several lunar surface models. Solid lines, opacity given by eq. (36). Dashed lines, opacity given by eq. (38).

\( \langle T(x, t) \rangle \) with depth. Finally, when \( k(\lambda) \gg 0.1 \), \( \langle T_B(\lambda, t) \rangle \) essentially equals the asymptotic value of \( \langle T(x, t) \rangle \) reached at about 10 cm depth. The effect of decreasing the opacity is simply one of shifting the increase in \( \langle T_B(\lambda, t) \rangle \) to shorter wavelengths.

ii) Surface Losses

The term \([1 - R(1)]\) in equation (40) represents a surface loss which is simply multiplicative in its effect on the mean brightness temperature in the Rayleigh-Jeans approximation. In figure 11 are plotted mean brightness temperatures for Model 1000/250. The individual curves are labeled by letter according to the assumed temperature dependence of the loss tangent (see fig. 9) and by number for the opacity law used (1 for eq. [36] and 2 for eq. [38]). The designation \( NF \) means that no steady-state heat flux was assumed. For Model 1000/250, \( \langle T(x, t) \rangle \) is 220.1° K independent of depth for no steady-state flux. The difference between curve \( KINF \) and 220.1° K is due solely to the term \([1 - R(1)]\), that is,

\[
\langle T_B(\lambda, t) \rangle = [1 - R(1)]\langle T(0, t) \rangle.
\]  

(42)

This equation is valid, of course, only in the Rayleigh-Jeans limit which for the lunar range of temperatures is accurate to 0.5° K when \( \lambda > 300 \mu m \). At shorter wavelengths, one must use equation (41), and \( \langle T_B(\lambda, t) \rangle \) is larger than the value indicated by equation (42). An approximate expression accurate to 3° K for \( \lambda = 10 \mu m \) and \( R(1) = 0.11 \) and much more accurate for larger wavelengths and smaller \( R(1) \) is

\[
\exp \left[ \frac{1.4387 \text{ cm}^2 \text{ K}}{\lambda \langle T_B(\lambda, t) \rangle} \right] = 1 + \left\{ \exp \left[ \frac{1.4387 \text{ cm}^2 \text{ K}}{\lambda \langle T(0, t) \rangle} \right] - 1 \right\}/[1 - R(1)].
\]  

(43)

The effect of a relative maximum in the normal emissivity in the range 30 \( \mu m \)–1 mm is thus to produce a relative maximum in \( \langle T_B(\lambda, t) \rangle \) in this wavelength range.
Fig. 11.—Mean brightness temperatures \( \langle T_B(\lambda, t) \rangle \) for Model 1000/250. The curves are labeled by letter according to the temperature dependence of the loss tangent as in fig. 9 and by number for the opacity law used (1 for eq. [36] and 2 for eq. [38]). Two curves labeled NF are for calculations in which no steady-state flux was included.

iii) Steady-State Heat Flow

As noted above by equation (15), the steady-state heat flow from the hot interior increases \( \langle T(x, t) \rangle \) by a factor proportional to depth and the flux, and inversely proportional to the thermal conductivity. Clearly, this heat flow should manifest itself in an increase in \( \langle T_B(\lambda, t) \rangle \) with wavelength relative to the no-flux calculations as the opacity in general decreases with increasing wavelength. In figure 11 the difference between curves \( K1 \) and \( K1NF \) is due solely to the net heat flux and becomes appreciable for \( \lambda > 1 \) cm. The flux assumed is \( 7.9 \times 10^{-7} \) cal cm\(^{-2}\) sec\(^{-1}\). For the lower opacity used in producing curve \( K2 \), the increase in \( \langle T_B(\lambda, t) \rangle \) beyond 10 cm is greatly enhanced. Equation (38) is more likely to be valid beyond 10 cm, since it is based on the Gold et al. (1970) measurement at 67 cm rather than on data shorter than 2 mm. Thus, the increase in \( \langle T_B(\lambda, t) \rangle \) indicated by curve \( K2 \) is likely to be realistic.

In the absence of a high-conductivity subsurface layer, the effect of a steady-state heat flux will be an even steeper increase in \( \langle T_B(\lambda, t) \rangle \) with wavelength beyond 10 cm. At a wavelength of 1 m, the equivalent curve \( K2 \) for Model 1000 has \( \langle T_B(\lambda, t) \rangle \) equal to 455° K rather than 221° K for Model 1000/250. The other one-layer models show correspondingly high mean brightness temperatures at 1 m. Such models are inconsistent with the direct thermal conductivity measurements of Langseth et al. (1972) and the radio observations cited below. Thus, the one-layer models are probably not relevant in estimating realistic limits on \( \langle T_B(\lambda, t) \rangle \) beyond 10 cm wavelength.

iv) Temperature-dependent Opacity

For each lunar surface model under consideration, we have computed \( \langle T_B(\lambda, T) \rangle \) for the various temperature dependencies of the opacity given in figure 9. In general, the results for temperature dependence \( E \) and \( M \) were nearly identical with that for \( R \). As a result, we will discuss only curves \( K \), \( R \), and \( Q \). Curve \( K \) in figure 9 shows no variation with temperature, whereas curves \( R \) and \( Q \) show increasingly positive temperature dependence. Curves \( R1 \) and \( Q1 \) in figure 11 are considerably above
Fig. 12.—Mean brightness temperatures $\langle T_B(\lambda, t) \rangle$ for Models 1000 and 1000-1. Labeling of the curves is the same as in fig. 11.

The enhancement of $\langle T_B(\lambda, t) \rangle$ due to temperature-dependent opacity results from the difference in depth from which the emission occurs at night relative to daytime. This difference in depth increases with decreasing opacity and accounts for the enhancement of the peak in $\langle T_B(\lambda, t) \rangle$ with decreasing opacity. The mean depth of emission at any given time is roughly $k(\lambda, T)^{-1}$, and the difference in mean depths of emission between day and night is roughly $k(\lambda, T_d)^{-1} - k(\lambda, T_n)^{-1}$. This difference increases with $\lambda$. The enhancement of $\langle T_B(\lambda, t) \rangle$ is proportional to this difference and to the absolute value of the thermal gradient at the mean depth during the day or night. This gradient in turn decreases in magnitude with depth and therefore wavelength. As a result, there will be a peak in the enhancement of $\langle T_B(\lambda, t) \rangle$, and that peak will shift to shorter wavelengths with decreasing opacity.

In figures 11–14, the effect of temperature-dependent opacity on $\langle T_B(\lambda, t) \rangle$ is shown for the models under consideration. It is of interest that despite the diversity of these models, the difference at a given wavelength between curves $R1$ and $K1$ is only somewhat model dependent. The same is true for the difference between curves $Q1$ and $K1$. Thus, to a rough order of approximation the effect of temperature-dependent opacity on $\langle T_B(\lambda, t) \rangle$ is nearly independent of the other enhancement mechanisms cited above.

b) The Mean Brightness Temperature and Estimated Errors

i) Infrared

In the infrared, the opacity given by equation (36) is probably a realistic one indicating that the emission is essentially from the surface layer only as stated by Bastin (1971). In this case, the increase in $\langle T(x, t) \rangle$ with depth due to radiative conductivity and due to a steady-state heat flux are not yet important. The effect of radiative conductivity as shown in figure 10 becomes important at about 1 mm. The effect
Fig. 13.—Mean brightness temperatures $\langle T_B(\lambda, t) \rangle$ for Models 11(2) and 12. Labeling of the curves is the same as in fig. 10.

of temperature-dependent opacity similarly is unimportant out to 300 $\mu$m but begins to be appreciable at 1 mm (see fig. 14). Consequently, the major causes of uncertainty in $\langle T_B(\lambda, t) \rangle$ in the infrared are errors in the values of $\langle T(0, t) \rangle$ and the surface losses or emissivity.

Table 6 lists the resultant errors in $\langle T_B(\lambda, t) \rangle$ due to various causes at the wavelengths at which a careful analysis was made. At all wavelengths, the error in $\langle T(0, t) \rangle$ enters directly. In § IIe we estimated $\langle T(0, t) \rangle$ to be $220.5^\circ \pm 5.5^\circ$ K for the center of the lunar disk. In order to derive $\langle T(x, t) \rangle$, we must assume a model and then estimate the errors in its parameters. Clearly, the logical model to choose is 15(4), since it is

Fig. 14.—Mean brightness temperatures $\langle T_B(\lambda, t) \rangle$ for Model 15(4). Labeling of the curves is the same as in fig. 10.
### TABLE 6

**An Error Analysis for \( <T_0(\lambda, t)> \)**

<table>
<thead>
<tr>
<th>Source of Error</th>
<th>12 ( \mu )m</th>
<th>17.5 ( \mu )m</th>
<th>30 ( \mu )m</th>
<th>100 ( \mu )m</th>
<th>300 ( \mu )m</th>
<th>1 mm</th>
<th>3 mm</th>
<th>1 cm</th>
<th>3 cm</th>
<th>10 cm</th>
<th>30 cm</th>
<th>1 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>( &lt;T(0, t)&gt; ) [° K]</td>
<td>( \pm 5.5 )</td>
<td>( \pm 5.5 )</td>
<td>( \pm 5.5 )</td>
<td>( \pm 5.5 )</td>
<td>( \pm 5.5 )</td>
<td>( \pm 5.5 )</td>
<td>( \pm 5.5 )</td>
<td>( \pm 5.5 )</td>
<td>( \pm 5.5 )</td>
<td>( \pm 5.5 )</td>
<td>( \pm 5.5 )</td>
<td></td>
</tr>
<tr>
<td>Emissivity</td>
<td>( \pm 2.0 )</td>
<td>( \pm 2.9 )</td>
<td>( \pm 6.8 )</td>
<td>( \pm 4.2 )</td>
<td>( \pm 5.0 )</td>
<td>( \pm 2.2 )</td>
<td>( \pm 2.2 )</td>
<td>( \pm 2.6 )</td>
<td>( \pm 2.5 )</td>
<td>( \pm 2.3 )</td>
<td>( \pm 2.3 )</td>
<td>( \pm 2.2 )</td>
</tr>
<tr>
<td>Radiative conductivity</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>( \pm 0.7 )</td>
<td>( \pm 1.9 )</td>
<td>( \pm 4.1 )</td>
<td>( \pm 5.6 )</td>
<td>( \pm 6.6 )</td>
<td>( \pm 6.5 )</td>
<td>( \pm 6.7 )</td>
</tr>
<tr>
<td>Steady-state heat flow</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>( \pm 0.1 )</td>
<td>( \pm 0.3 )</td>
<td>( \pm 0.5 )</td>
<td>( \pm 0.8 )</td>
<td>( \pm 1.5 )</td>
</tr>
<tr>
<td>Temperature-dependent opacity</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>( \pm 2.0 )</td>
<td>( \pm 4.0 )</td>
<td>( \pm 2.8 )</td>
<td>( \pm 1.6 )</td>
</tr>
<tr>
<td>Absolute value of the opacity</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>( \pm 2.0 )</td>
<td>( \pm 0.5 )</td>
<td>( \pm 1.5 )</td>
</tr>
<tr>
<td>rms error</td>
<td>( \pm 6.0 )</td>
<td>( \pm 6.2 )</td>
<td>( \pm 8.7 )</td>
<td>( \pm 6.9 )</td>
<td>( \pm 8.1 )</td>
<td>( \pm 6.3 )</td>
<td>( \pm 8.9 )</td>
<td>( \pm 10.1 )</td>
<td>( \pm 9.2 )</td>
<td>( \pm 9.3 )</td>
<td>( \pm 9.1 )</td>
<td>( \pm 9.2 )</td>
</tr>
<tr>
<td>Best estimate of ( &lt;T_0(\lambda, t)&gt; )</td>
<td>213.9</td>
<td>212.7</td>
<td>213.7</td>
<td>216.1</td>
<td>217.6</td>
<td>219.1</td>
<td>227.7</td>
<td>239.5</td>
<td>238.5</td>
<td>240.5</td>
<td>244.8</td>
<td>259.0</td>
</tr>
</tbody>
</table>
based on the in situ measurements of Langseth et al. (1972). For this model, \(<T(0, t)>\) is 222.5° K. Model 15(4) is derived from data taken from one place on the Moon and thus likely differs from the average. As a result, we choose to lower \(<T(x, t)>\) at all depths by 2° K so that \(<T(0, t)>\) agrees with our estimated values.

Previously, we had estimated the normal emissivity to be 0.95–1.00 in the wavelength interval 100 μm–1 mm (§IVa[vi]) and 0.85–0.93 in the wavelength interval 10–20 μm (§ IIb[ii]). For the intermediate region 20–100 μm, the state of our ignorance is such that the range of normal emissivities to be assumed will be 0.85–1.00. As previously noted, the effect on \(<r_B(\lambda, t)>\) of a given percentage error in the emissivity decreases with shorter wavelength in the infrared.

### ii) Millimeter Wavelengths

At millimeter wavelengths, \(<T_B(\lambda, t)>\) varies with wavelength due to each of the mechanisms cited previously. Since the Rayleigh-Jeans approximation is valid, errors in normal emissivity (as shown in fig. 6) produce the same fractional errors in \(<T_B(\lambda, t)>\). Radiative conductivity produces an appreciable increase in \(<T_B(\lambda, t)>\) between 1 mm and 1 cm for Model 15(4). Perhaps a reasonable estimate in the uncertainty in this effect would be to assume that the temperature enhancement could be in error by ±20 percent at each wavelength. The error is our estimate of how much the Apollo 15 site may differ from the average Moon and is based on the agreement of \(<\Delta T>\) for Models 12 and 15(4). The Apollo 11 material is different, and it is unclear why. Similarly, a reasonable estimate of the error in the steady-state heat flux for the average Moon might be ±10 percent about the Langseth et al. (1972) value. This error has essentially no effect at millimeter wavelengths.

Finally, one must choose realistic bounds on the effects of temperature-dependent opacity. The result of Clegg et al.'s (1972) recent measurements (see fig. 9) is to make curve \(R\) the most plausible temperature variation for the lunar soil loss tangent. Their data were taken only in the far-infrared and may not be valid at longer wavelengths. We therefore take as outer bounds on opacity temperature-dependence curves \(K\) and \(Q\), which are the extremes for measured lunar-like material. The major error in \(<T_B(\lambda, t)>\) due to this effect occurs at millimeter wavelengths.

### iii) Centimeter Wavelengths

At centimeter wavelengths, an uncertainty in the wavelength dependence of the opacity can play an important role. Equation (38) is based on Gold et al.'s (1970) measurements of an Apollo 11 sample at 67 cm. It seems reasonable to assume it to be valid in the wavelength interval 30–100 cm provided one allows an uncertainty of, say, 20 percent in absolute value. Since a change in opacity alters the depths from which the emission occurs, this produces an uncertainty in \(<T_B(\lambda, t)>\). Between 3 and 30 cm, one might assume that the correct opacity law lies between equation (36) and equation (38) and that the range is given by the difference of curves \(R1\) and \(R2\) in figure 14. At 1 cm, equation (36) is probably more nearly correct. Table 6 summarizes these error estimates and includes the rms error estimate. The best estimate of \(<T_B(\lambda, t)>\) is deduced from curves \(R1\) and \(R2\) in figure 14, including a decrease in temperature of 2° K at all wavelengths.

### c) Conclusions

In figure 15, we summarize our best-estimate values of \(<T_B(\lambda, t)>\) and their errors. The central curve is the estimated value of \(<T_B(\lambda, t)>\), and the curves lying above and below correspond to one rms error. Our method of estimating the mean brightness temperature of the moon is semiempirical in nature but is not based on any radio
Fig. 15.—Central curve, the estimated best value of the mean brightness temperature $<T_b(\lambda, t)>$ at the center of the lunar disk. The curves above and below correspond to the estimated rms error in the value of $<T_b(\lambda, t)>$. The data cited are from Paper I.

observations. One of our aims, in fact, is to use the estimated mean temperature to recalibrate astronomical radio observations. We have included in figure 15 the by now old observations cited in Paper I. The aim is to show that for this reasonably homogeneous sample of observations, the data and theoretical predictions are not inconsistent. Clearly, the accuracy of our method is limited by our poor knowledge of the thermal and electromagnetic properties of the mean lunar surface. When such information becomes available, the Moon should be a more accurate calibration source for a wide range of astronomical data.

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