CORONAL EMISSION LINE POLARIZATION

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ABSTRACT

A discussion of a program for the computation of coronal emission line polarization is presented. The starting point is a general formulation of the scattering function for magnetic dipole transitions between any two total angular momentum levels, $J \rightarrow J, J \pm 1$. Illustration of the behavior of the scattering function for different transitions is given. The integration of the scattering function over the solar disk and along the line of sight accounting for arbitrary distribution of magnetic fields as well as an inhomogeneous temperature and density structure of the corona is considered next.

Sample results are presented for the numerical computation of the angle of maximum polarization and the degree of maximum polarization to be expected from idealized magnetic field configurations such as radial and dipole. A computation is included for a realistic field configuration predicted to exist at the time of the 1966 eclipse. The magnetic field input to the scattering calculation is based upon the potential field extension of photospheric magnetic fields. It is the purpose of the sample calculations to demonstrate how the measurement of emission polarization measurements can be interpreted in terms of the direction of coronal magnetic fields. Factors which lend ambiguity to such interpretations are clearly illustrated from the examples. These include the Hanle-effect depolarization and the depolarization at the Van Vleck angle.

1. Introduction

The structure of the inner solar corona is primarily controlled by the presence of local magnetic fields. In the past the direction of the magnetic
fields has been inferred from the shape of white light or emission line
structures (e.g. Saito and Billings 1964, Saito 1965). There is however
clearly a need for more direct measurements of the distribution of magnetic
fields and one such technique that avails itself to this problem is the
observation of coronal emission line polarization (Charvin 1965, Hyder 1965,
Hyder, Mauter, and Shutt 1968). The degree of polarization coupled with
the angle of maximum polarization in visible coronal emission lines yields
information on the direction of the magnetic field in the region where the
scattered radiation originates.

It is the purpose of the present paper to discuss the theoretical
computation of coronal emission line polarization with the goal being
to provide a basis for the interpretation of emission line polarization data.
Specifically, a discussion is given of a program that has been developed to
compute the emission line polarization expected from an inhomogeneous coronal
model with a realistic magnetic field structure. This program can generate
theoretical predictions of polarization expected from any magnetic dipole
transition and for any specified model of the corona. The coronal model can
be quite general: the program allows an inhomogeneous atmosphere in which
the temperature and density have arbitrary radial and lateral distributions;
moreover, structures simulating streamers can be included. Streamer regions
can have temperature and density gradients differing from the surrounding
corona both radially and laterally. In addition an arbitrary three dimensional
magnetic field can be treated. At present, the model utilizes hypothetical
magnetic fields generated by the current-free potential theory extension of
observed photospheric magnetic fields. The program for computing these fields
has been developed by Newkirk, Altschuler, and Harvey (1968), and Altschuler
Newkirk (1969). Once this information is available we next treat the integration of the incident photospheric radiation over the disk and as well as the integration along the line of sight. Using these results, we illustrate in Section 4 numerical calculations for various idealized magnetic field configurations. Sample calculations are also given for a more realistic magnetic field, namely the configuration predicted by the current-free extension of the photospheric magnetic fields for the solar eclipse of 1966. In the conclusions some of the difficulties inherent in the interpretation of emission line polarization data, as brought out by the calculations, are emphasized.

2. The Scattering Redistribution Function

A. General Expression for Coronal Transitions

The key atomic parameter needed to begin the problem is the polarized radiation scattering coefficient for magnetic dipole transitions. Charvin (1965) discusses the calculation of such a scattering coefficient and the present author has developed in a series of papers* (House, 1970a, b, 1971) a formalism for computing the frequency dependent scattering coefficients for more general conditions that is needed for the coronal calculations. In the general calculation the magnetic field strength may range from "effectively weak", where degeneracy of the magnetic sublevels produces coherency effects, to "effectively strong" where the degeneracy is removed so that a standard Zeeman pattern is obtained.

The requirement for an "effectively strong" field as discussed in Paper I is that the ratio of the Larmor frequency to the inverse lifetime,

*These papers hereafter will be referred to as I, II and III and equation references will be prefixed according to the paper in which they appear.
or damping width, of the excited state level is much greater than unity,

\[ \frac{\omega_L}{\gamma} = \frac{eB}{m_ec} \approx 2 \times 10^7 \frac{B}{A} >> 1 \]  

(1)

where \( B \) is the magnetic field strength in Gauss, \( e, m_e \), and \( c \) have their usual meaning. For the coronal problem we may take the damping width as the natural width expressed in terms of the Einstein transition probability, \( A \). Since probabilities for magnetic dipole transitions in coronal ions are so low, \( 10^{-100} \text{ sec}^{-1} \), from the above inequality we conclude that coronal fields are "effectively strong". This conclusion greatly simplifies the equations for resonance fluorescence.

A second point of simplification over the more general scattering functions, lies in the fact that for the coronal case one need not include frequency dependence since measurements to date average over the entire line; thus we may use a frequency averaged scattering coefficient. This averaging of course rules out the possibility of measuring field strengths from the coronal emission lines.

From the above discussion we conclude that the appropriate scattering redistribution function for coronal emission lines obtained from Equation I-40a, is

\[ W_{\alpha', \alpha} (\theta') = c \sum_M \left[ G_1(M) \varepsilon_0^2 \varepsilon_1^2 + G_2(M) \frac{\varepsilon_0^2 (1 - \varepsilon_0^2)}{2} \right] \]

(2)
where the scattering probability \( W_{\alpha\alpha',(\theta,\theta')} \) is a function of the linear polarization angles \( \alpha \) and \( \alpha' \) and the photon propagation directions \( \theta \) and \( \theta' \). The unprimed quantities denote the incident photon while the primed quantities are for the scattered photon. The constant \( c \) insures the normalization

\[
\sum_{\alpha} \sum_{\alpha'} \int_{0}^{4\pi} \int_{0}^{4\pi} W_{\alpha\alpha',(\theta\theta')} \, d\Omega \, d\Omega' = 1 ,
\]

which involves for both incoming and outgoing states summations over two orthogonal states of polarization and integration over all solid angles \((\Omega,\Omega')\). The summation in equation (2) is over magnetic quantum numbers, \( M \), for the initial level.

In equation (2), the \( \varepsilon_o^2 \)'s contain the geometric dependence of scattering while the \( G \)'s define the strengths of the various component transitions between the magnetic sublevels of two different \( J \) levels. The geometric parameters are

\[
\varepsilon_o^2 = \sin^2 \theta \, \sin^2 \alpha \\
\varepsilon_o'^2 = \sin^2 \theta' \, \sin^2 \alpha'
\]

where \( \theta \) is the angle between the direction of the incident radiation and the direction of the magnetic field at the scattering point and \( \theta' \) is the angle between the direction of the magnetic field at the scattering point and the observer. The linear polarization angles, \( \alpha \) and \( \alpha' \), are measured from unit vectors in the direction of increasing \( \theta \) and \( \theta' \).
respectively. The polarization angles appear in sine functions rather than in cosine functions as in Paper I, because we consider magnetic dipole transitions here rather than the electric dipole transitions. For the strong field case the scattering function is independent of the polar angles $\phi$ and $\phi'$. The strength quantities are defined in the form:

\[ G_1(M) = (g_{M,M}^M)^2 \]  
\[ G_2(M) = (g_{M,M}^{M+1})^2 + (g_{M,M}^{M-1})^2 \]  
\[ G_3(M) = (g_{M+1,M}^{M+1})^2 + (g_{M-1,M}^{M-1})^2 \]  
\[ G_4(M) = (g_{M,M+1}^{M+1})^2 + (g_{M,M+1}^{M+2})^2 + (g_{M,M-1}^{M-1})^2 + (g_{M,M-1}^{M-2})^2 \]

where the lower case g's are expressed as products of squares of Wigner 3-j symbols (see equation I-18),

\[ (g_{M,M}^{M,n})^2 = \begin{pmatrix} J & 1 & J \\ -M & (M^n-M') & M \end{pmatrix}^2 \begin{pmatrix} J & 1 & J \\ -M & (M'-M) & M^n \end{pmatrix} \]

The total angular momentum quantum numbers, the J's, and the magnetic quantum numbers, the M's, for the initial, intermediate, and final atomic levels are respectively $J,M$; $J^n,M^n$; and $J,M'$. Equation 2 applies to the situation where the incident radiation is totally polarized. In the corona where we treat single scatterings it is
appropriate to assume that the incident radiation is unpolarized. To modify the scattering function for this condition we simply add scattering functions, weighting them equally, for two orthogonal incident states of polarization $\alpha$ and $\alpha + \pi/2$, i.e.

$$W_{\alpha, \alpha'}(\theta, \theta') = W_{\alpha, \alpha'}(\theta, \theta') + W_{\alpha, \alpha' + \pi/2}(\theta, \theta')$$ (8)

If we use equations (2) and (4) in equation (7), we may write the frequency averaged scattering function for unpolarized incident light in the convenient form

$$W_{\alpha, \alpha'}(\theta, \theta') = c(a' + b' \cos^2 \theta)$$ (8)

where

$$a' = a_1 + a_2 \sin^2 \theta' \sin^2 \alpha'$$

$$b' = b_1 + b_2 \sin^2 \theta' \sin^2 \alpha'$$ (9)

with

$$a_1 = \frac{G_2}{2} + \frac{G_4}{4}, \quad a_2 = G_1 + \frac{G_3}{2} - a_1$$

$$b_1 = \frac{-G_2}{2} + \frac{G_4}{4}, \quad b_2 = -G_1 + \frac{G_3}{2} - b_1$$ (10)

For the case of incident unpolarized light from a point source the polarization resulting from a single scattering will be

$$p_{\alpha'} = \frac{W_{\alpha, \alpha' + \pi/2}}{W_{\alpha, \alpha' + \pi/2}}$$ (11)
When we use equation (8) in this expression, we find
\[
P_{\alpha'} = \frac{-\cos 2\alpha' \sin^2 \theta' (a_2 + b_2 \cos^2 \theta)}{2a_1 + a_2 \sin^2 \theta' + (2b_1 + b_2 \sin^2 \theta') \cos^2 \theta}.
\] (12)

Note that, if we view the scattered radiation along the direction of the magnetic field, \(\theta' = 0\), the radiation is unpolarized. This is the depolarization for "effectively strong" fields due to the Hanle effect.

B. Particular Transitions

The scattering function, equation (9), as well as the polarization for a single scattering, equation (12), are completely defined for a specific transition in terms of the coefficients \(a_1, a_2, b_1,\) and \(b_2\). Table 1 summarizes the values for these coefficients for most transitions of interest in the solar corona.

From this table we may draw some conclusions on the properties of the polarization of scattered radiation: (a) some scatterings will not produce linear polarization \((J = 3/2 \rightarrow J'' = 1/2 \text{ and } J = 1/2 \rightarrow J'' = 1/2)\), (b) the transition \(J = 2 \rightarrow J'' = 2\), produces identical results with the \(J = 1 \rightarrow J'' = 2\) transition, and (c) all transitions yielding non-zero polarizations have the \(a_2\) and \(b_2\) coefficients in the ratio \(a_2/b_2 = 1/(-3)\). This latter result is most interesting since it can profoundly affect the interpretation of coronal polarization measurements.

When conclusion (c) above is introduced into Equation (12), one can see that the polarization goes through zero and changes sign as \(\theta'\) passes through the angle \(\cos^{-1} \theta = 1/\sqrt{3}\) or \(\theta \approx 54.6^0\). This angle is known as the depolarizing angle or the Van Vleck angle. If the incident radiation field makes the Van Vleck angle with the direction of the magnetic field, the
scattered radiation is unpolarized independent of the transition and independent of the direction of view. Since the calculations described so far pertain to a point source, we will see later that if the incident radiation subtends a finite solid angle the polarization will be a minimum, rather than exactly zero. Clearly this effect will make the interpretation of the coronal scattering in terms of the direction of magnetic fields difficult. We shall discuss this problem in more detail in Section 4.

In Figure 1 we present plots of the polarization as a function of the angle between the observer and the direction of the magnetic field for various transitions (Equation (12)). These results strictly apply to a single scattering from a point source. Two sets of curves are given for each transition, one for the unpolarized radiation incident perpendicular to the direction of the magnetic field and the other for the radiation incident along the direction of the magnetic field. One point to note in the figure is that the polarization change sign in going from $\theta = 0$ to $\theta = 90$ because the direction of the incident radiation has passed through the Van Vleck angle.

3. Integration over the disk and along the line of sight

The polarization of scattered radiation in the corona depends upon the anisotropy of the incident photospheric radiation field. The degree of anisotropy is governed for a single scattering by the solid angle subtended by the disk as seen from the scattering point and by limb darkening. Furthermore, the polarization measured by an observer at the earth is influenced by variations in the density of scattering ions along the line of sight. We must account for both of these effects.
A. Integration Over the Disk

First we write the expression for the integration of the scattering redistribution function over the solid angle subtended by the solar disk and then define the various parameters in the expression. According to equation (II-19a), we have

$$W_\alpha(A,B,E) = \int_{D=0}^{2\pi} \int_{C=\pi-\Omega}^{\pi} W_\alpha(\theta,\theta') \left[ 1 - U + \frac{U}{\sin \Omega} \sqrt{\sin^2 \Omega - \sin^2 C} \right] \sin C \, dC \, dD$$

(13)

This equation is defined relative to the coordinate system illustrated in Figure 2. The direction of $\hat{R}'$ is specified by the polar angle $E$ in the $Y'-Z'$ plane and the azimuthal angle $F=90^\circ$ measured from the $X'$ axis in the $X'-Y'$ plane. The propagation vector $\hat{R}'$ from an arbitrary point on the surface of the sun to the scattering point is defined by the polar angle $C$ lying in the plane of the radius vector and $\hat{R}$. The azimuthal angle $D$ is measured in the $X'-Y'$ plane. The latter two angles provide the variables for the integration of the increment in solid angle. The direction of the magnetic field at the scattering point is specified by the polar angle $A$ and the azimuthal angle $B$ in the $X'$, $Y'$, $Z'$ coordinate system.

The solid angle of the disk, $\Omega$, is given by the expression

$$\sin \Omega = \frac{1}{1+\rho} = \frac{\cos x}{1+\rho_0}$$

(14)

with $\rho = h/R_0$, $\rho_0 = h_0/R_0$. 
The angle $\chi$ between the perpendicular to the line of sight and the radius vector to the scattering point is given by

$$\cos \chi = \frac{1 + \rho}{1 + \rho_0} .$$

(note $E = 90 - \chi$).

The limb darkening coefficient $U$ is introduced in equation (13) through the use of the expression

$$I = I_0 \left[ 1 - U + U \cos \psi \right] \tag{15}$$

where $\psi$ is defined through the equation

$$\cos \psi = \sqrt{\frac{\sin^2 \Omega - \sin^2 \zeta}{\sin^2 \Omega}} \tag{16}$$

The scattering function, Equation (8), is expressed in terms of the angles $\theta$ and $\theta'$ which are polar angles measured relative to the direction of the local magnetic field. A sequence of Euler rotations can be found that allows one to transform from the local system of the atom referenced to the direction of the field to a system relative to the observer. The expressions relating angles in the two systems are found to be (equations (II-18a and c))

$$\cos \theta = \sin A \sin C \cos (B-D) + \cos A \cos C \tag{17a}$$

$$\cos \theta' = \sin A \sin B \sin E + \cos A \cos E \tag{17b}$$

The polarization angle $\alpha'$ is also measured in the frame of reference of the scattering ion; hence to obtain the angle of polarization referred
to the plane of the sky, which we shall denote as $\alpha''$, we write

$$ \alpha'' = a + \alpha' $$

(18a)

where

$$ \tan(a/2) = \sqrt{\frac{\sin(S-e') \sin(S-E)}{\sin(S) \sin(S-A)}} $$

(18b)

and where

$$ S = \frac{1}{2} (\theta' + E + A) $$

(18c)

The angle $\alpha''$ is now measured counterclockwise, perpendicular to the line of sight from the plane containing the local radius vector and the line of sight.

With all the above definitions we may now introduce the scattering function, equation (8), into the integral, equation (13), and perform the integration in closed form. The results of this integration are similar to those given by equation II-28 although, in the present paper we can generalize the coefficients so that the integration is applicable to a magnetic field having an arbitrary orientation specified by the angles $A$ and $B$ rather than for a radial magnetic field configuration ($A=B=0$) as given in Paper II. To generalize the results we must introduce equation (17a) into equation (8). If we then substitute this result into equation (13), and carry out the integral over the azimuthal angle, $D$, the result is

$$ c \int_{D=0}^{2} (a'+b' \cos^2 \theta) \, dD = a'' + b'' \cos^2 \theta $$
where \[ a'' = \pi c(2a' + b'sin^2 A) \text{ and } b'' = \pi c b'(3cos^2 A - 1) \]

In the coefficient \( b'' \) as given by the equation above, we note that the Van Vleck angle again presents itself.

We may now carry out the integration over the polar angle, \( C \). The form of the expression after this second integration is identical to that given by equation II-28, with the exception that the constants are generalized. We find then for the integration over the solid angle of the disk the final expression

\[
W_{\alpha}(A,B,E;\Omega) = \left\{ (1-U) \left[ a''(1-cos\Omega) + \frac{b''}{3} (1-cos^3\Omega) \right] + \frac{U}{8} \left[ 4a''+b''(1+sin^2\Omega) - \frac{cos^3\Omega}{sin\Omega} (4a''+b''cos^2\Omega) \ln \left( \frac{1+sin\Omega}{cos\Omega} \right) \right] \right\}
\]

This expression is a generalization of that given by Minnaert (1930) since it is applicable to any magnetic dipole transition.

B. Integration over the Line of Sight

The integration along the line of sight is easily formulated, but it cannot be integrated analytically for the inhomogeneous corona we wish to consider. The integration along the line of sight may be expressed in the form

\[
W_{\alpha''}(\rho_o) = \int_{-\pi/2}^{\pi/2} W_{\alpha''} \left[ A(\rho,\chi), B(\rho,\chi), E(\chi) \right] \frac{N(\rho,\chi)(1+\rho_o)}{cos^2\chi} \, d\chi \quad (22)
\]
where we have converted the distance along the line of sight measured in units of the solar radius to the position angle $\chi$ by the relation

$$\xi = \frac{L}{R_0} = (1 + \rho_0) \tan \chi$$  \hspace{1cm} (23)

The function $N(\rho, \chi)$ is the distribution of scattering ions along the line of sight.

This integral can now be used to evaluate the polarization of emission lines provided one specifies a model of the coronal magnetic fields, density, and temperature as a function of $\rho$ and $\chi$. From a numerical evaluation of equation (22) for $\alpha''$ and $\alpha'' + 90$ the degree of polarization can be determined. This is accomplished if we use an equation of the type similar to equation (11), but introduce the integrated scattering probabilities for a fixed height. From a sequence of such calculations, the angle of maximum polarization and the degree of maximum polarization can be fixed by interpolation.

4. Computations for Various Field Configurations

The purpose of the calculations presented in this section is to illustrate how the direction of the magnetic field distribution in the corona controls the polarization of emission lines in different atomic transitions, and in particular to point out the difficulties involved in the inverse problem, the deduction of magnetic field directions from polarization measurements.
A. Assumptions and Coronal Parameters

A computer program has been developed* that can be used to evaluate the theoretical emission line polarization resulting from an arbitrary three dimensional distribution of magnetic fields, electron density, and electron temperature in the solar corona.

The coronal magnetic field directions are generated in the computer program discussed by Altschuler and Newkirk (1969). This program yields the current-free magnetic field distribution in spherical geometry by means of a spherical harmonic expansion of observed photospheric magnetic fields. The program gives the direction of the magnetic field at any specified point in the corona. Hence, through the transformations discussed in the previous section, one may determine the scattering function at any point in the corona relative to the quantization axis prescribed by this local magnetic field. This must be done at each point along the line of sight as the integration of equation (22) is carried out.

The inhomogeneous coronal atmosphere is treated by introducing regions simulating streamers of elliptical cross section with density and temperature structures different from those of the ambient medium. These densities and temperatures can vary laterally as a function of height. The ambient medium temperature and density vary with height alone.

For any specific coronal ion, one must have the ionization equilibrium data to specify the density of scattering ions as a function of density and temperature. Curves for ionization such as presented by Jordan (1970) are fitted by a polynomial and introduced into the program.

*The computations have been carried out on the NCAR CDC6600 through the programming assistance of Russell Rew.
A correction for collisional depolarization is especially important near the limb and this has been incorporated into the program. If one assumes that collisional excitations are isotropic, then they produce no net polarization. A correction for the fraction of photons generated in the spectral line by scattering compared to the total generation rate, scattering plus collisions, would therefore represent the collisional depolarization correction. This ratio is expressed as

\[
\text{Coll. Depol.} = \frac{\text{Rad. Rate}}{\text{Rad. Rate} + \text{Coll. Rate}}
\]  

(24)

where the intensity of the radiation field is a function of height in the atmosphere and is diluted by the expression

\[
\text{Dilution} = (1-U)(1-\cos\Omega) + \frac{U}{2} \left[ 1 - \frac{\cos^3\Omega}{\sin\Omega} \ln \left( \frac{1 + \sin\Omega}{\cos\Omega} \right) \right]
\]  

(25)

The program incorporates this correction. Corrections for collisional de-excitation are assumed insignificant at coronal densities and hence neglected.

For the purposes of illustration, in the present section we consider mainly the effect of the field configuration and therefore neglect the inhomogeneous structure of a model corona. The temperature will be assumed uniform throughout and the electron density as a function of height is expressed by either the equation

\[
N_e \propto k_1(1+\rho)^{-\beta_1} + k_2(1+\rho)^{-\beta_2} + k_3(1+\rho)^{-\beta_3}
\]  

(26a)
or

\[ N_e \propto k_4 e^{-\beta_4 \rho} \]  

(26b)

We consider five different density variations which we label as Cases 1-5. First we note that when we compute the polarization this is a ratio of two quantities and since we consider only an isothermal atmosphere in this section, the absolute value of the density cancels in the ratio. Only relative values are important. The density variation for Case 1 is that given by Newkirk, Dupree and Schmahl (1970): \( k_1 = 7.55, \beta_1 = 5.35, \)

\( k_2 = -1.68 \times 10^2, \beta_2 = -14.74, k_3 = 1.04 \times 10^3 \) and \( \beta_3 = -20.45. \)

For comparison, less complicated gradients are selected for Case 2 \((k_3 = 1, \beta_3 = 7)\) and Case 3 \((k_3 = 1, \beta_3 = 8)\) where the rest of the parameters, \( k_1, \beta_1, k_2, \beta_3 \) are zero. Case 4 is an isothermal corona with a temperature of \( 2 \times 10^6 \) k and a mean molecular weight based upon a 10 percent abundance of helium by number \( k_4 = 1, \beta_4 = 7. \) Case 5 is a constant density corona, \( k_4 = 1, \beta_4 = 0. \)

Because the calculations in the following section are meant only to illustrate general results we will not consider lateral variations in electron temperature or density. Also since we treat various transitions, but do not assign them to specific ions we do not at present include corrections for collisional depolarization. When these calculations are applied to a specific problem, then the additional details will be treated.

B. Results

Typical calculations are presented below for a variety of magnetic field configurations.
1) Radial Field

For the radial field every line of sight around the limb will be identical provided we assume the corona is homogeneous and thus we need only present the polarization as a function of height at one position. This is illustrated in Table 2 where we give the polarization as a function of height above the solar limb for 6 different transitions and in each case for no limb darkening and for complete limb darkening. The asymptotic values corresponding to a point source, or equivalently the maximum possible polarizations are also listed.*

The extent to which the polarization is modified due to different density gradients can be seen by comparing the results for the \( J=0 \) to \( J''=1 \) transition shown in Table 2 with those given in Table 3.

2) Dipole Field

In Figure 3 we illustrate the polarization in the various transitions for a dipole magnetic field. The projection of the 3-dimensional magnetic field distribution is given in part A of the figure. Polarization vectors for the various transitions are plotted in parts B-F of the figure. These polarization vectors are plotted at position angles, \( \omega \), 0 through 350\(^\circ\) at increments of 10\(^\circ\). At a given position angle a polarization vector is plotted at heights \( \rho_0 = 0.1 - 0.9 \) in steps of 0.2. The length of each polarization vector is proportional to the degree of polarization while the angle of the polarization vector relative to the radius as drawn through the centers of the polarization vectors is the angle of maximum polarization. In all but

*These numbers may be used to correct the errors appearing in Tables 4 and 5 of the paper by Hyder et al. (1968).
one case (J=2 → J''=1) a polarization vector whose length is equal to the
radius of the disk in the plot would correspond to 100 percent polarization.
For J=2 → J'' = 1, the polarization vectors have been multiplied by a factor
of 100.

For this calculation the corona has been assumed to be homogeneous and
isothermal with density variation given by Case 1.

We note some important features of these results. First, as seen
in Figure 3, all transitions produce identical maximum polarization
angles at any given position angle. For the uniform corona and for the
highly symmetric dipole field this polarization angle varies with the position
angle, but not significantly with height. Second, one can recognize a
gap in the plots at a position angle ~ 70°. The value of polarization
at this angle is too small to appear on the plots and it results from
the incident radiation being at Van Vleck angle relative to the field.
Recall we stressed previously this angle was identical for all transitions.

For the dipole field we can easily establish why the effects of the
Van Vleck angle appear at ~ 70°. In a uniform corona, because of the
steep density gradients the major contribution to the scattering comes
from the point along the line of sight at L=0 or equivalently χ = 0.
Thus we can estimate the results by considering only the direction of the
field in the plane of the sky. For a dipole field, the ratio of radial
to tangential field strength at an angle ω would be

$$\tan A = \frac{B_\omega}{B_r} = \left( \frac{\sin \omega}{r^3} \right) / \left( \frac{2 \cos \omega}{r^3} \right) = \frac{\tan \omega}{2}. \quad (27)$$
The angle $A$, the angle between the field and local radius, when equal to 
the Van Vleck angle is $A = \cos^{-1}(1/\sqrt{3})$. If we solve equation (27) for $\omega$, 
we find that the Van Vleck angle corresponds to $\omega = \tan^{-1}(2\tan A) \approx 70^\circ$. 
The polarization is not precisely zero because the solid angle is finite 
rather than infinitesimal.

It is further to be emphasized that as one passes through the 
Van Vleck angle, the direction of the polarization vector rotates by $90^\circ$ 
relative to the direction of the field. It is evident in Figure 3 that 
in the range $\omega = 0$ to $60^\circ$ the polarization vectors provide an accurate 
map of the field since they lie along the direction of the field. However, 
after the Van Vleck angle is passed the polarization vectors are at 
$90^\circ$ to the direction of the field. This effect can introduce obvious 
difficulties in the interpretation of observational data since the 
Van Vleck angle can occur for any field distribution whenever $A \approx 54.6^\circ$.

Computations for other idealized field configurations have been 
made, but are not presented here.

Variations in the density gradient make little change in the 
polarizations and angles of maximum polarization for non-radial fields. 
Calculations for density Cases 2, 3, 4, and 5 show that variations of the 
polarization are of the same order as that noted for the radial case, 
namely, 1 or 2 percent in polarization and at most 1 or 2 degrees.

3) 1966 Eclipse

In Figure 4a we show the potential field extension of the photospheric 
magnetic fields for the sun at the time of the 1966 eclipse (Altschuler 
and Newkirk 1969). Next to the magnetic field map are the plots of
polarization vectors for the \( J = 0 \rightarrow J'' = 1 \) transition. In Figure 4b the corona is assumed homogeneous and isothermal while in Figure 4c we have introduced two streamers. The polarization vectors are plotted as thicker lines in the regions of the streamers. The models used are those given by equations (11), (12), and (13) in the paper by Newkirk et al. (1970). The streamer in the southeast quadrant (lower left hand portion of Figure 4a) is a radial fan with its plane in the line of sight and with an e-folding width of about 10°. The streamer is centered at a longitude of 140° (center of disk in figure is at 270°, thus streamer is 40° behind east limb) and at a latitude 23°S. The streamer in the southwest (lower right hand portion of Figure 4a) is at a longitude of 20° and a latitude of 28°S. In both streamers the radial density gradients are enhanced according to the expressions given by Newkirk et al. Lateral gradients, treated as exponentials, also differ between the two streamers. Clearly, the magnetic field is sampled in regions of highest density which would appear in the streamers.

5. Conclusions

In the present paper we have presented some results from a computation that is capable of giving theoretical emission line polarizations for any magnetic dipole transition under conditions simulating realistic coronal models. It has been demonstrated that the general scattering function could be integrated analytically to account for the incident limb darkened radiation field, but the integration over the line of sight for any arbitrary distribution of magnetic fields required a numerical solution. With the capabilities of treating inhomogeneous coronal models it is hoped

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that when data are available, this program will prove useful in their interpretation.

It is clear from the few examples shown, that the interpretation of observational data will not be lacking in difficulties. One principal problem will be the ambiguity introduced at the Van Vleck depolarization angle. If one observed unusually low polarizations in regions where one would rather expect high polarizations, i.e., beyond the region of collisional depolarization, the result could be attributed to one of two sources. In the region where the main contribution to the scattering arises, either the field is in the direction of the observer, where the Hanle depolarization acts, or the region is at the Van Vleck angle. Also if a Van Vleck angle does occur for some line of sight, as one traces a magnetic field line from the polarization vectors through this region, vectors on one side may be at 90° to the field. It appears that it will be most important to have polarization measurements over a large region of the corona so that through the continuity of the field one may attempt to resolve some of the ambiguities.

It has been suggested by Hyder, Mauter, and Shutt (1968) that the measurement of emission line polarization in many lines could help fix the identification of coronal lines. Unless measurements could be carried out over an extensive region of the corona, it appears that this procedure might be quite difficult. We can easily see from the results shown here that no distinction is made in the angle of maximum polarization between the various transitions. The actual degree of polarization is controlled by many factors in addition to the particular transition, including,
density of the region and direction of the local magnetic field in the region giving rise to the emission.

Perhaps one of the most useful results that may be obtained from the use of the present theoretical tool in conjunction with observational data would be the verification of the field directions computed by the potential field extension of photospheric magnetic fields. The current free fields calculated from the photospheric data have found wide use in coronal research and it appears that it would be of great interest to attempt to confirm the validity of this technique.

Acknowledgments

The author gratefully acknowledges the programming efforts of Russell Rew and the many helpful comments on the manuscript provided by Drs. J. A. Eddy and E. Tandberg-Hanssen.
References


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*Multiply polarization for ρ = 0 to ρ = 2 by 10^-2.
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TABLE 3: % Polarization for Radial Field (J = 0, J'' = 1, U = 0)
Fig. 2  Scattering geometry relative to sun and observer. (See detailed explanation in text.)
Fig. 1 Plots of polarization versus angle between the observer and the direction of the magnetic field for various atomic transitions. The curves in the upper half of the figure apply for unpolarized radiation incident along the direction of the magnetic field while curves in the lower half apply for radiation incident perpendicular to the direction of the magnetic field. The degree of polarization for the dotted curves must be multiplied by a factor of $10^{-2}$. 

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Fig. 3 Illustration of the relationship between the direction of magnetic field and degree of polarization at the angle of maximum polarization for a dipole magnetic field. Part A is the projection of the three-dimensional representation of a dipole magnetic field. Parts B through F represent calculated polarizations at various position angles for the specified transitions. Length of the vector is proportional to the degree of polarization where a vector having a length corresponding to the radius of the disk would represent 100 percent polarization. The orientation of the vector shows the direction of maximum polarization.
Fig. 4 Illustration of relationship between potential field extension of photospheric magnetic fields at the time of the 1966 eclipse and polarization vectors for the $J=0 \rightarrow J"=1$ transition. The rotational axis is vertical. In part B the results are plotted for a uniform isothermal corona while in part C two major streamers are included and plots of the polarization vectors for these regions are shown by heavier lines in regions labeled S.