THE FORMATION OF Mg I 4571 Å IN THE SOLAR ATMOSPHERE

I: A Model Analysis of a One-Dimensional Static Atmosphere

RICHARD C. ALTROCK* and C. J. CANNON

Depart. of Applied Mathematics, The University of Sydney, Sydney, Australia

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Abstract. A one-dimensional analysis of the 4571 Å line of neutral magnesium is presented. The Harvard-Smithsonian Reference Atmosphere (HSRA) and the Bilderberg Continuum Atmosphere (BCA) are used to compute the emergent line profiles at various positions on the solar disc. The resultant profiles, when compared to the observations, indicate that the HSRA electron temperature distribution is a more satisfactory representation of the solar atmosphere in the region of the temperature minimum than is the BCA. A slight modification to the HSRA is suggested which reduces the minimum temperature to 4140K and enables an even more satisfactory ‘fit’ to the available data.

1. Introduction

The Mg I 4571 Å intercombination line has been observed by White et al. (1972) to have a line-centre brightness temperature of 4190K ± 50K. This value closely approximates the minimum electron temperature of the Harvard-Smithsonian Reference Atmosphere (HSRA – Gingerich et al., 1971), and thus suggests that the 4571 Å line may be formed in the region of the temperature minimum. Further, Athay and House (1962) and Athay and Canfield (1969) have shown the 4571 Å line to be well-represented by an LTE source function. Thus the line should be very sensitive to the electron temperature and should, therefore, provide a powerful means of studying both the minimum temperature in the solar atmosphere and the multi-dimensional structure of the atmosphere in the region of the temperature minimum.

There have been, in recent years, a number of discussions appearing in the literature involving the value of the minimum electron temperature. The HSRA, for example, prefers a minimum temperature of 4170K whereas the Bilderberg Continuum Atmosphere (BCA – Gingerich and de Jager, 1968) chooses a value of 4600K. In this paper, therefore, we present a one-dimensional synthetic analysis of the 4571 Å line in a static atmosphere (i.e., no macroturbulence) in an attempt to help resolve this problem.

In the future we plan to undertake a comparison of empirical and synthetic analyses of the 4571 Å line. Further papers will be aimed at the multi-dimensional structure of the atmosphere in the region of the temperature minimum and will incorporate an analysis of the line with and without the inclusion of macroscopic velocity fields.

2. Method of Analysis

The emergent intensity from a one-dimensional atmosphere in a line having an

* On leave from Sacramento Peak Observatory, Air Force Cambridge Research Laboratories.

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LTE source function is given by

$$I_v(0, \mu) = \int_0^\infty B_v(\tau_v) e^{-\tau_v/\mu} \frac{d\tau_v}{\mu},$$

(1)

where \( \mu \) is the cosine of the heliocentric position angle, \( \tau_v \) is the total (line plus continuum) optical depth, and \( B_v(\tau_v) \) is the Planck function at the local electron temperature.

A. THE LINE OPACITY

The line optical depth as a function of physical depth \( z \) is given by

$$\tau_v'(z) = \int_0^z N_1(z') \alpha_0(z') H(a, v) \, dz',$$

(2)

where \( N_1 \) is the number of atoms per unit volume in the lower state of the transition, \( \alpha_0 \) is the absorption coefficient at line centre in the absence of damping, and \( H(a, v) \) is the Voigt function with damping coefficient \( a \). The Doppler width \( \Delta \lambda_D \) is a function of the vertical and horizontal microturbulent velocities, \( \xi_v \) and \( \xi_H \), and appears in \( \alpha_0 \) and \( H(a, v) \), where \( v = \Delta \lambda / \Delta \lambda_D \).

We may rewrite \( N_1 \) as

$$N_1 = \frac{N_1}{N_{\text{Mg}}} N_H A_{\text{Mg}},$$

(3)

where \( A_{\text{Mg}} \) is the relative abundance of magnesium with respect to hydrogen in the solar atmosphere, and \( N_H \) is the number of hydrogen atoms and ions per unit volume. The relative population of the lower state is a function of the excitation and ionisation of the atom. If we assume all of the magnesium in the region of line formation is either neutral or singly-ionised, we have

$$\frac{N_1}{N_{\text{Mg}}} = \left[ \frac{N_{\text{MgI}}}{N_1} + \frac{N_{\text{MgII}}}{N_1} \right]^{-1}.$$

(4)

If we further assume that all \( \text{MgI} \) atoms are in the first (\( 3^1S \)) or second (\( 3^3P \)) state, and if we take into account the fact that the non-LTE departure parameters \( b \) obey the relationship (cf. Athay and House, 1962; Athay and Canfield, 1969)

$$b_1 = b_2,$$

we find

$$\frac{N_{\text{MgI}}}{N_1} = \frac{U_I}{g_1},$$

(5)

where \( U_I \) is the (LTE) partition function for \( \text{MgI} \) and \( g_1 \) is the statistical weight for the lower (\( 3^1S \)) state.
The relative ionisation is specified by
\[ \frac{N_1}{N_{\text{Mg II}}} = \left( \frac{N_1}{N_{\text{Mg II}}} \right)^* b_1, \]  
where the asterisk denotes the LTE value. Previous analyses (Athay and House, 1962; Athay and Canfield, 1969) have shown that the ionisation of Mg I is controlled by radiative transitions between the second level and the continuum. We adopt this result and, assuming an ionisation cross-section proportional to \( \nu^{-3} \), obtain
\[ b_1 \approx \frac{\nu_{2\nu} e^{h\nu_{2\nu}/kT_r}}{T_r^{(2)}} \frac{T_e}{e^{h\nu_{2\nu}/kT_e}}, \]
where \( \nu_{2\nu} \) is the ionisation potential from the second level of Mg I. The corresponding radiation temperature is denoted by \( T_r^{(2)} \). Following Athay and Canfield (1969) we let \( b_1 \) be unity below the level in the photosphere where \( T_e > T_r^{(2)} \). Above that level we take \( T_r^{(2)} \) to be a constant.

B. THE CONTINUUM OPACITY

The continuum opacity is added to the line opacity to produce the total opacity in the line. It is given by the sum of the contributions from H\(^-\), H, Rayleigh scattering from hydrogen and electron scattering. The FORTRAN routines to compute these were taken from Carbon and Gingerich (1969). Above the temperature minimum we have set the H contribution to zero due to non-LTE effects, and we have ignored the decrease of H\(^-\) and Rayleigh scattering due to the ionisation of hydrogen. In this region, however, electron scattering dominates, and these assumptions have a negligible effect.

By comparison with the HSRA opacities, the opacities obtained from the above sources provide over 90% of the continuum opacity throughout the atmosphere. The continuum absorption is of little importance throughout most of the line \( (\tau_0 = \kappa_c/\kappa_0 \approx 6 \times 10^{-3}) \), but we have attempted to account for other sources by arbitrarily increasing the above opacity by 6.5%.

C. OBSERVATIONAL DATA

The data published by White et al. (1972) are in the form \( I_v(0, \mu)/I_c(0, \mu) \), where \( I_c(0, \mu) \) is the emergent intensity in the neighbouring continuum. We convert absolute computed intensities to this form by dividing by the continuum limb darkening curve from Pierce and Waddell (1961) and by the absolute intensity at disc centre from Labs and Neckel (1970).

The observed line profiles exhibit red-blue asymmetries due to differential macroscopic velocity fields in the solar atmosphere. In this paper we ignore the asymmetries and attempt to reproduce only the line profiles averaged about the minimum intensity. It is important to realise, however, that the velocity fields appear in the equation of radiative transfer as a highly non-linear term and, therefore, a simple averaging of
the line profiles introduces a further approximation in the model analysis. This will be discussed in detail in future papers.

D. MODEL SPECIFICATION

We adopt a microturbulence model similar to that of Athay and Canfield (1969). Specifically,

\[
\xi_v = 0.5 \text{ km s}^{-1}, \quad J \leq 29 \\
= 0.02381 (J - 29) + 0.5 \text{ km s}^{-1}, \quad 30 \leq J \leq 49 \\
= 1.0 \text{ km s}^{-1}, \quad J \geq 50
\]

\[
\xi_H = 0.5 \text{ km s}^{-1}, \quad J \leq 29 \\
= 0.05238 (J - 29) + 0.5 \text{ km s}^{-1}, \quad 30 \leq J \leq 49 \\
= 1.6 \text{ km s}^{-1}, \quad J \geq 50,
\]

where \( J \) is the step number of the HSRA atmosphere, running from 1 at the top to 95 at the bottom. We vary the oscillator strength \( f \) from \( 1.88 \times 10^{-6} \) (Allen, 1963, p.69) to \( 6.00 \times 10^{-6} \) (Wiese et al., 1969) and the relative abundance of magnesium from \( 2.50 \times 10^{-5} \) (Goldberg et al., 1960) to \( 3.02 \times 10^{-5} \) (Lambert and Warner, 1968).

We take the radiation temperature to be the black-body temperature which gives an intensity equal to one-half the emergent intensity averaged over the solar disc at the ionisation limit near 2500 \( \text{Å} \), and diluted by another factor of one-half to account for the presence of lines (cf. Allen, 1963, p. 172). Using this definition we obtain values ranging from 4500K (Kodaira, 1965) to 4800K (Goldberg, 1967). (See also Allen, 1963, p. 172; Labs and Neckel, 1970.) The value of 5400K referred to by Athay and Canfield (1969) and attributed to Goldberg (1967) was chosen to provide the best fit to their line profiles (R. C. Canfield, private communication). It may be obtained from the value of Goldberg (1967) by ignoring the presence of lines and applying a reasonable observational uncertainty.

Emergent line profiles were computed using both the HSRA and BCA distributions of \( T_e, N_e \) and \( N_H \), where \( N_e \) is the electron density. Model situations, in which the \( T_e \) distributions were slightly modified, were also studied. Similar variations in \( N_e \) and \( N_H \) produce a rather secondary effect when compared to the \( T_e \) variations and were not considered. The damping factor \( a \) was allowed to vary as a free broadening parameter.

E. COMPUTATIONS

The value of \( N_1 \) at each physical depth point is determined by combining Equations (3) to (7) with the model atmosphere chosen. This then specifies the opacity which, together with the Planck function, enables Equation (1) to be solved for the emergent intensity. This was done to an accuracy of three significant figures utilising Simpson’s integration rule. The synthetic analysis approach then requires the computed emergent line profiles to be compared with those observed, and the model atmosphere altered until a reasonable ‘fit’ is obtained.
3. Results and Discussion

Figure 1 compares the observations with the emergent line profiles for models A and B calculated at four positions on the solar disc. Model A represents the emergent intensities determined using the HSRA while model B shows the fit obtained by slightly modifying the HSRA electron temperatures. These temperature distributions, along with the BCA values, are shown in Figure 2. Also shown are the physical

Fig. 1. The computed line profiles for models A and B are shown at four positions on the solar disc. The circles represent the average of the observed red-blue intensities.

depths at which the line is formed using the various models. The changes to the HSRA temperature distributions used to obtain model B are given by

$$\Delta T = T_B - T_A = -160 + 16|J - 50|, \quad 41 \leq J \leq 55$$
$$= 24J - 1400, \quad 56 \leq J \leq 59$$
$$= 80 - 8|J - 65|, \quad 60 \leq J \leq 74,$$

where $J$ is the step number of the HSRA. This modification reduces the minimum
temperature to 4140 K at \( J = 44 \). Table I lists the other parameters used in the calculation.

**TABLE I**
The parameters used in the computation of models A, B and C

<table>
<thead>
<tr>
<th>Model</th>
<th>Parameter</th>
<th>( T_e )</th>
<th>( f )</th>
<th>( A_{\text{Mg}} )</th>
<th>( a )</th>
<th>( T_r^{(2)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model A</td>
<td>HSRA</td>
<td>1.88 ( \times 10^{-6} )</td>
<td>2.5 ( \times 10^{-5} )</td>
<td>0.2</td>
<td>4700 K</td>
<td></td>
</tr>
<tr>
<td>Model B</td>
<td>Modified HSRA</td>
<td>1.88 ( \times 10^{-6} )</td>
<td>2.5 ( \times 10^{-5} )</td>
<td>0.2</td>
<td>4700 K</td>
<td></td>
</tr>
<tr>
<td>Model C</td>
<td>BCA</td>
<td>1.88 ( \times 10^{-6} )</td>
<td>2.5 ( \times 10^{-5} )</td>
<td>0.15</td>
<td>4700 K</td>
<td></td>
</tr>
</tbody>
</table>

Model B, although reproducing the observations quite satisfactorily, could be further improved. This was deemed unnecessary for two reasons. First, a one-dimensional analysis has inherent limitations in determining the temperature distribution in an inhomogeneous radiating atmosphere (see, for example, Wilson and Williams, 1972). Secondly, the uncertainties in the physical parameters, such as

oscillator strength, abundance, etc., do not warrant further computations and improvements.

Models A and B were computed using a radiation temperature \( T_r^{(2)} \) of 4700 K. This value was varied within the range 4500 K to 5000 K (see Section 2A), and was found to have a noticeable effect on the computed line profiles, particularly near the limb. However, it is felt that a reasonable fit, such as that obtained with model B, could be reproduced for any radiation temperature in the above range.

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Fig. 2. The electron temperature distributions for models A, B and C are shown as a function of physical depth and optical depth in the continuum using both the HSRA and BCA scales. The arrows indicate the approximate position in the model atmospheres at which the optical depth at line centre is unity.
A depth independent damping factor of 0.2 was used in the computation of models A and B. It was found, however, that the effect of a change in the damping factor could be offset by a suitable change in the microturbulent velocities alone. This situation is brought about by the lack of any significant damping wings in the observed profile. Thus, for example, if the microturbulent velocities used in model B were altered, a satisfactory fit to the data could be obtained just by a corresponding change in the damping factor.

![Graphs showing intensity versus wavelength with different damping factors (\(\mu\))](image)

**Fig. 3.** Same as Figure 1 but with 'model C' replacing 'models A and B'.

The calculated intensities are dependent on the product of the oscillator strength and the abundance. These parameters enter the equations as factors of proportionality in the optical depth. Changes in the optical depth result in changes in the mapping of the electron temperature (as a function of physical depth) to intensity (as a function of wavelength). For example, an increase in the product \(f \times A_{\text{Mg}}\) results in the line being formed higher in the atmosphere, and this results, in most parts of the line, in a lower emergent intensity. The same effect can be achieved by lowering
the temperature or by increasing the broadening functions. Hence, model B is not necessarily unique.

Figure 3 shows the emergent intensities for model C. This model is identical to model A except that the HSRA distributions for $T_e$, $N_H$ and $N_e$ have been replaced by the BCA values. It is obvious that although the disc centre profiles are in reasonable agreement with those observed, those at the limb are far too shallow. In particular, the computed intensities are a factor of two too large at the limb. Variations in the oscillator strength, abundance, damping factors, etc., even with the line formed in LTE (i.e. $b_1 = 1$), could produce results no better than those for model C. In effect, the source function would have to be at least a factor of two less than the LTE value in order for a reasonable fit to be obtained. This is ruled out by the previous studies to which we have referred. Further, the 'flatter' emergent profile in the core of the line illustrated in Figure 3 ($\mu = 0.125$) reflects the 'flatter' BCA electron temperature distribution in the region of the temperature minimum (see Figure 2). The conclusion to be drawn from these calculations, therefore, is that the BCA electron temperature distribution in the region of the temperature minimum is quite unsatisfactory.

4. Conclusion

Both the Harvard-Smithsonian Reference Atmosphere and the Bilderberg Continuum Atmosphere were used in computing emergent profiles for the 4571 Å line of MgI. It was found that a satisfactory reproduction of the observations was obtained using only a slight modification to the HSRA (i.e., model B) while the BCA model gave intensities (i.e., model C) of the order of 100% greater than those observed near the limb.

This suggests that the HSRA represents a more satisfactory distribution in $N_H$, $N_e$ and $T_e$ in the region of the temperature minimum than does the BCA. In particular, given the assumptions outlined in the preceding sections, it appears more likely that the minimum temperature is of the order of 4100 K – 4200 K rather than 4600 K.

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