WAVELENGTH DEPENDENCE OF THE INTERPLANETARY SCINTILLATION INDEX

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ABSTRACT
Published observations of the interplanetary scintillation index $m_z$ are shown to vary with wavelength in a manner consistent with a smooth, power law spectrum of plasma fluctuations. This is in contrast to recent work arguing that the data require a spectrum with two separate regimes. It is concluded that published observations of $m_z$ are consistent with either type of density spectrum.

A problem of considerable interest in the physics of the solar wind is the relation between interplanetary scintillation of radio sources and the structure of the solar wind turbulence. In particular, it is hoped that interplanetary scintillations can be used to help determine the structure of the solar wind in regions not accessible to direct measurement.

A reasonable first approximation to the scintillation problem is given by the “thin-screen” model in which the fluctuating solar wind plasma is replaced by a thin, phase-changing screen perpendicular to the direction of propagation of the wave. The intensity fluctuations are then built up by interference as the wave propagates to the observer. The observer is situated at a distance $z$ from the plane of the screen. In the solar wind, the equivalent screen is assumed to be placed at the point of nearest approach of the ray path to the sun, since this is where the effect of the solar wind is greatest, and then $z$ is of the order of 1 AU. As the fluctuations are carried out from the sun at the solar wind velocity $V_w$, the intensity fluctuations in the plane of the observer are also convected at the wind velocity. Hence, the spatial variations in intensity with wavelength $\lambda$ are seen as temporal fluctuations with time $\sim \lambda / V_w$.

Of interest, then, is the wave number spectrum of intensity fluctuations in the plane of the observer, at a distance $z$ from the phase-changing screen, and the relation of this spectrum to the power spectrum of density fluctuations in the solar wind. A thorough discussion of this problem is given by Salpeter [1967].

Let $\delta \rho(r)$ be the fluctuation in solar wind plasma density about its mean. Then the power spectrum of density fluctuations is defined as

$$P_\rho(q) = \int d^3 \zeta \langle \delta \rho(r) \delta \rho(r + \zeta) \rangle e^{iq \cdot \zeta} \tag{1}$$

Similarly, if $\delta I(r) / \langle I \rangle$ is the relative fluctuation in radio intensity in the plane of the observer, as discussed above, we may define the power spectrum of intensity fluctuations as

$$m_z^2(q) = \frac{1}{\langle I \rangle^2} \int d^2 \zeta \langle \delta I(r) \delta I(r + \zeta) \rangle e^{iq \cdot \zeta} \tag{2}$$

where the integration is carried out over the entire plane. It may be shown [Jokipii, 1970] that if the scintillation index $m = (\langle \delta I^2 \rangle / \langle I \rangle)^{1/2} \ll 1$, the two spectra are related by

$$m_z^2(q_x, q_y) = 4 \sin^2 \left[ \frac{(q_x^2 + q_y^2)}{2k^2} z \right] \frac{C}{k^2 P_\rho} \tag{3}$$

$$q_x q_y q_z = 0$$

where $k$ is the wave number of the electromagnetic wave, $z$ is the distance from the screen to the observer.

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and \( C \) is a constant.

Note that the dependence of the \( \sin^2 \) factor on \((a_x^2 + q_y^2)\) states quite generally that plasma fluctuations much larger than the Fresnel scale ~\( \sqrt{\kappa/k} \) are not effective in causing scintillations. Using typical values for the parameters, \( z \approx 1 \) AU and \( k \) corresponding to 100-MHz radio waves, one finds that the Fresnel scale is of the order of 200 km. Using this fact, Jokipii and Hollweg [1970] pointed out that the observed scintillation scales of a few hundred km or so are quite consistent with the observed dominant solar wind scales of the order of \( 10^5 \) km [Intriligator and Wolfe, 1970].

Here we briefly consider the wavelength dependence of the scintillation index \( m \) to see whether or not observations of \( m \) can be used to rule out certain forms of the spectrum \( P_\rho(q) \). Hewish [1971] has argued that the data rule out a smooth variation of \( P_\rho(q) \) from the small values of \( q \) corresponding to the dominant density scales to the higher values relevant to scintillation. We shall argue that the available data do not force such a conclusion. From equation (2), one easily derives

\[
m^2 = \frac{\langle \delta f \rangle^2}{\langle f \rangle^2} = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} dq_x dq_y \ m^2_z(q_x, q_y) \tag{4}
\]

Now assume \( P_\rho(q) \) is isotropic, so that \( m_z^2(q_x, q_y) = m^2_z(q) \), with \( q = \sqrt{q_x^2 + q_y^2} \). Then equation (4) becomes

\[
m^2 = \frac{2}{\pi} \frac{C}{k^2} \int_0^{\infty} q \sin^2 \left( \frac{q^2 z}{2k} \right) P_\rho(q) dq \tag{5}
\]

We consider a simple power law for \( P_\rho(q) \). Let

\[
P_\rho(q) = A q^{-\alpha_3} \tag{6}
\]

where the subscript 3 is used to emphasize that this is a three-dimensional spectrum. It is possible that a better representation of the actual situation in the solar wind would be given by a power law spectrum with a cutoff at some wave number \( q_0 \). This interesting case is not considered further here. For a given value of \( \alpha_3 \), the temporal spectrum observed on a spacecraft [Intriligator and Wolfe, 1970] would be

\[
P_\rho(f) = B f^{-(\alpha_3 - 2)} \tag{7}
\]

[Hollweg, 1970], where \( f \) is frequency.

If \( P_\rho(q) \) has the form given in equation (6), it is then a simple matter to substitute this into equation (5) to obtain, for \( 2 < \alpha_3 < 6 \),

\[
m^2 = - \frac{A C}{\pi k^2} \left( \frac{\alpha_3}{2} - 2 \right) \left( \frac{2}{2k} \right) \Gamma \left( 1 - \frac{\alpha_3}{2} \right) \tag{8}
\]

The wavelength dependence of \( m_z \) follows immediately as

\[
m \propto k^{-\left(\frac{1}{2} + \alpha_3/4\right)} \propto \lambda^{\left(\frac{1}{2} + \alpha_3/4\right)} \tag{9}
\]

Thus with \( \alpha_3 \approx 3 \), as observed [Intriligator and Wolfe, 1970], we expect \( m \propto \lambda^{1.25} \). This is in contrast to a gaussian density fluctuation spectrum, which leads to \( m \propto \lambda \).

Hewish [1971] has argued forcibly that the available published data required \( m \propto \lambda^{1.95} \). He uses this to infer that there are two regimes in the density power spectrum, a long-wavelength regime that contains most of the power and a separate short wavelength regime that causes the observed scintillations. Between these regimes he postulates little or no spectral power. This view, of course, is consistent with the arguments of Jokipii and Hollweg [1970] concerning the dominant scale of the density fluctuations.

Such a spectrum, if true, would be of considerable interest physically. Hence we decided to check whether, indeed, the data used by Hewish actually rule out a dependence such as that given in equation (9) with \( \alpha_3 = 3 \). We find that they do not. Following the procedure outlined by Hewish [1971], we utilized the data reported by Bourgeois [1969], Harris and Hardebeck [1969], and Hewish and Symonds [1969].

The idea is that if \( m \propto \lambda^{\alpha_3} \), then \( m v^\alpha \) (where \( \nu = c/\lambda \)) should be independent of frequency. Unfortunately, reliable simultaneous measurements of \( m \) at different frequencies do not exist. We must instead compare measurements obtained at different times and at different elongations. Since the characteristics of the solar wind vary from day to day, there is considerable spread in the data. Nevertheless, if \( m v^\alpha \) is plotted as a function of source elongation, data obtained for different observing frequencies should fall, within the aforementioned spread in the data, on a smooth curve. Figure 1(b) shows a plot of \( m v \) versus elongation. The data fall on a smooth curve, and one might be tempted to conclude that, in fact, \( m \propto \lambda^{1.0} \) [Hewish, 1971]. But figure 1(a) shows the same data, with \( m v^{1.25} \) plotted against elongation. The data points again lie on a smooth curve. We conclude that within the uncertainties of the data, \( m \propto \lambda^{1.25} \) is as good as \( m \propto \lambda \). To put this more quantitatively, the mean square deviations of the points
from the smooth curves for \( mv \) and \( mv^{1.25} \) versus elongation, are 20 and 23, respectively, in arbitrary units.

We therefore conclude that the published scintillation indices at various frequencies do not force the conclusion that \( m \) is proportional to \( \lambda \). Hence smooth, power law, density spectra are consistent with the published scintillation index measurements, in contrast to the conclusions published by Hewish [1971]. Hopefully, improved measurements will make it possible to resolve this question in the near future.

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REFERENCES


