MAGNETICALLY TRAPPED PARTICLES
IN THE LOWER SOLAR ATMOSPHERE

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Abstract. The trapping of energetic electrons and protons in a simple, arched magnetic field imbedded in the lower solar atmosphere was considered. The lifetime of electrons with kinetic energies up to about 1.5 MeV was found to be completely determined by the motion of the mirror points, provided the gyro-synchrotron loss can be neglected. The same motion also influences the lifetimes of more energetic electrons, up to 10 MeV. This was not found to be the case for protons in the range from 1 MeV to 100 GeV. Some fluid and streaming instabilities were also considered; they pull the particles upward, raise their mirror points, and increase their lifetime. The emission of gyro-synchrotron radiation and bremsstrahlung in this model has been related to observations. Using the duration of non-thermal X-ray peaks given by Kane (1969), the altitude of injection of energetic particles was estimated.

1. Introduction

The structure of the magnetic field in the chromosphere and inner corona is not well known. We do know, however, that there must be a strong vertical field above the umbra of spots. Since the magnetic field lines diffuse relatively little through the photosphere during the lifetime of a spot, most of them have to return to the photosphere close to the spot. Arch-type structures must be very common in the lower solar atmosphere in the vicinity of sunspots. Very large arches of this type are frequently observed optically at solar eclipses. Recent soft X-ray photographs by Van Speybroeck et al. (1970) show a large number of such coronal arches in thermal emission (Figure 1). Kais (1970) observed some evidence that Type I and stationary Type IV bursts are caused by plasma oscillations due to thermal electrons trapped in coronal magnetic arches.

Arched tubes of force stretching from the photosphere up into the chromosphere and corona from strongly magnetic regions of the solar surface must be largely force free (Gold, 1964). This is clear from the fact that the gas pressures, both static and dynamic, must be quite small compared with $B^2/8\pi$. On the other hand, the high conductivity of the gas makes clear that the field need not have the shape of the potential field, built upon the same surface.

The force free field will differ from the potential field chiefly in two respects. Arches of tubes of force may connect over much wider distances than they generally would in the potential configuration. The feet of lines of force may have been dragged further apart, and in the presence of the conducting gas will not reconnect to the potential configuration. This effect is clearly seen on some prominence pictures, and especially on Lyman-$\alpha$ photographs of the sun, which show the general drawing...
out of arches due to the differential rotation between equator and middle latitudes.

The other departure from the potential field will be the possibility and perhaps general tendency for tubes of force to have twists. Pitch angles of twisted bundles will in general be steeper than $45^\circ$ (stable force free fields demand this), but nevertheless the effect of the twist on trapped particle orbits may on occasions be significant.

![X-ray exposure of the corona in the nominal pass bands 3–30 Å, 44–55 Å. Note the bright arches due to thermal bremsstrahlung of trapped electrons (after Van Speybroeck et al., 1970).](image)

We are not at present in a position to discuss any special effects due to the departure of the actual fields from the potential ones.

We assume the magnetic field to be strongest at the feet of these arches. If the field changes slowly with altitude and time, the orbit of a charged particle with a velocity $v$ is described by Alfvén's mirror equation, which is equivalent to the conservation of the magnetic moment:

$$\cos^2 \phi (h) = \frac{v_\perp (h)^2}{v} = \frac{B(h)}{B_m}$$  \hspace{1cm} (1)
and the drifts perpendicular to the magnetic field $B$; where $v_L(h)$ is the gyration velocity perpendicular to the magnetic field as a function of the altitude $h$ above the photosphere and does not include any drift velocity, $\phi$ denotes the pitch angle between the velocity vector and the perpendicular plane to the field line, and $B_m$ the field where $\phi = 0$, i.e. at the mirror point.

Considering an injection of particles at an altitude $h_0$, all mirror points are initially at the same altitude or lower. The particle number density along the magnetic field line is constant for altitudes $h > h_0$ after a short time, as Welch and Whitaker (1959) showed for the magnetosphere of the Earth. This is indeed what one observes for some of the coronal arches (probably the younger ones).

Since energetic particles appear to be generated chiefly in flares, we will concern ourselves with these. De Jager (1968) estimates the total number of energetic electrons in a high energy flare between $10^{33}$ and $10^{39}$. Flares generally occur near spots. We expect therefore that many of the energetic particles are injected and trapped in arched magnetic fields. There is indeed good evidence of such trapping high in the corona in Type V bursts (Weiss and Steward, 1965). There is general agreement that a part of the moving Type IV bursts is also produced by electrons trapped in arches (see e.g. Wild, 1970).

The properties of trapped particles will be discussed in detail in the following sections, assuming the simplest model, an untwisted, arched magnetic field in a plane perpendicular to the solar surface. The drifts perpendicular to this plane will be discussed later. We will consider arches in the chromosphere and inner corona, at lower altitudes therefore than sources of the radio bursts mentioned earlier. This is the region where the microwave and hard X-ray bursts originate. For simplicity we will not consider electron energies above 10 MeV, since no bremsstrahlung above 1 MeV has ever been observed. Direct observations near the Earth suggest taking 100 GeV as a maximum energy for the protons in our consideration. Section 2 gives a summary of the most important energy loss mechanisms; Section 3 deals with the motion of the mirror points and the lifetimes of energetic particles and the stability of arches, and Section 4 develops the connection with the observed effects.

2. Energy Loss Mechanisms

Our goal is to calculate the lifetime of energetic particles until they reach thermal velocities. We will compare the time taken by a particle to lose its energy along a path bounded by fixed mirror points with the time taken by its mirror points to move to a lower altitude, e.g. the photosphere, where the particle's energy is lost immediately due to collisions in the higher density material. This comparison will show for various particle species and energies, whether or not the motion of the mirror points plays an important role. In this section we will consider the most important energy loss mechanism for electrons and protons.

For the energy range we are considering gyro-synchrotron radiation is only important for electrons. This energy loss proceeds according to the well-known formula
for $E_k > 0.01$ MeV:

$$\frac{dE_k}{dt} = 3.8 \times 10^{-9} B^2 E_t^2 \beta^2 \text{ MeV/s}$$

(2)

where $E_k$ is the kinetic energy, $E_t$ the total energy of the electron, both in MeV, $\beta$ the ratio of its velocity to the velocity of light and $B$ the magnetic field strength in Gauss.

The total energy loss due to excitation of plasma waves, ionization and collisions of a particle traveling in a plasma with ionization $\varepsilon$ was calculated by Hayakawa and Kitao (1956). For our purpose and energy range we can roughly approximate their results by:

$$\left. \frac{dE_k}{dt} \right|_{\varepsilon} \approx (2\varepsilon + 1) \left. \frac{dE_k}{dt} \right|_{\varepsilon = 0}.$$  

(3)

Taking the numerical results of Berger and Seltzer (1964) for neutral hydrogen and using Equation (3), the loss of electrons for $100$ keV $\leq E_k \leq 10$ MeV can be roughly estimated

$$\frac{dE_k}{dt} = k_1 N \text{ MeV/s}$$

(4)

where $k_1$ is about $3 \times 10^{-13}$ for the practically non-ionized lowest part of the chromosphere, $(6.3 \pm 1.0) \times 10^{-13}$ for the completely ionized solar material and $N$ the number of ions and neutral atoms per cm$^3$.

Gyro-synchrotron radiation is therefore more efficient than the collisional loss, if:

$$\frac{B^2 E_t^2 \beta^2}{N} \geq 2 \times 10^{-4}.$$  

(5)

This situation probably obtains in the immediate flare region and discriminates against highly relativistic electrons (Švestka, 1970). Outside the flare region and below 20000 km we will assume that collisional losses predominate. With increasing altitude and decreasing density the energy above which synchrotron losses predominate diminishes. In moving Type IV bursts we are probably observing trapped, synchrotron radiating electrons at very high altitudes.

Protons follow a law similar to (3). Their collisional loss for $1$ MeV $\leq E_k \leq 0.5$ GeV is, according to Hayakawa and Kitao (1956):

$$\frac{dE_k}{dt} = k_2 N \frac{\beta (\ln E_k + 22.3)}{E_k} \text{ MeV/s}$$

(6)

where $k_2$ is about $3 \times 10^{-12}$ for only slightly ionized and $7.2 \times 10^{-12}$ for the completely ionized solar material. If $0.5$ GeV $< E_k < 100$ GeV, we take formula (4) as a
rough approximation, where now \( k_1 \approx 2 \times 10^{-13} \) for the only slightly ionized and 
\( 5.3(\pm 0.6) \times 10^{-13} \) for the completely ionized solar material.

For the slightly relativistic particles we are considering, the energy loss by brems-
strahlung can be neglected. We will also neglect that the excitation of plasma waves
becomes slightly more important at high altitudes (where the number of particles
within a Debye sphere becomes large) than Equation (3) indicates.

3. Motion of the Mirror Point and Lifetime of a Particle

We consider now a particle with constant energy in a time independent magnetic field.
When it suffers a collision, its pitch angle \( \phi \) changes by \( \varepsilon \) which is most likely to be a
small angle. If the electron and ion densities are about equal and the velocity of the
particle is much larger than the thermal electron velocity in the medium, a good
approximation of the average deflection after the time \( \Delta t \) is:

\[
\langle \varepsilon^2 \rangle \approx \frac{kN\Delta t}{E_i^2\beta^3} \tag{7}
\]

where \( K \approx 4.7 \times 10^{-13} \) for completely ionized and about \( 1.8 \times 10^{-13} \) for slightly ionized
solar material.

The deflection causes a change in the mirror point altitude \( h_m \). If the collision occurs
near the mirror point, any change in the pitch angle moves the mirror point down-
ward, since the velocity parallel to the field is enhanced at the expense of the perpen-
dicular velocity. Using (7) an average mirror point velocity \( v_m \) can be defined as a
function of \( h_m \) and the altitude and the velocity of the particle. We are only interested
in the average of \( v_m \) over the whole path and proceed similarly to the calculations of
Welch and Whitaker (1959). Assuming that the path is not changed dramatically
during one period this leads to:

\[
\langle v_m(h_m, E_i) \rangle = \left( -\frac{KB_m^2}{E_i^2\beta^3} \right) \times \frac{dB}{d\beta} T(v, h_m) \times \left\{ \int_{h_m}^{h_{\text{max}}} \int_{h_m}^{h_{\text{max}}} \frac{N(h)}{B(h)} \frac{ds}{\sqrt{1 - \frac{B(h)}{Bm}}} \right. \int_{h_m}^{h_{\text{max}}} \frac{N(h)}{Bm} \frac{ds}{\sqrt{1 - \frac{B(h)}{Bm}}} \right\}. \tag{8}
\]

To simplify the procedure we have assumed a symmetrical arch. \( T(v, h_m) \) is the time
required by a particle with velocity \( v \) to go from its mirror point at \( h_m \) to the top of
the arch at \( h_{\text{max}} \).

\[
T(v, h_m) = \int_{h_m}^{h_{\text{max}}} \frac{ds}{v_{\|}} = \frac{1}{v} \int_{h_m}^{h_{\text{max}}} \frac{ds}{\sqrt{1 - B(h(s))/Bm}}. \tag{9}
\]

d\( s \) is the element of path length along the magnetic field line.
At this point we have to specify the behavior of $B$ with altitude. From observations of coronal arches we have found as a first approximation for low altitudes:

$$\frac{B(h_1)}{B(h_2)} \approx \left(\frac{h_2}{h_2}\right)^\kappa \quad \text{where} \quad 1 \leq \kappa \leq 2. \quad (10)$$

This is compatible with a sinusoidal arch, if it is not abnormally tall and narrow.

A. MIRROR POINT IN THE LOWER CHROMOSPHERE ($300 \text{ km} \leq h_m \leq 3000 \text{ km}$)

The main contribution to both integrals in (8) comes from $h \approx h_m$, because $N(h)$ diminishes rapidly with increasing altitude. For $h \approx h_m$, we can use the approximation:

$$N(h) \approx N(h_m) \exp\left((h_m - h)/a(h_m)\right) \quad (11)$$

where $a(h)$ is the scale height. In most models of the lower chromosphere $a(h) \ll h$. Assuming that $h_{\text{max}} - h_m \gg a(h)$, we can expand the integrals in (8) to infinity.

$$\langle v_m(h_m, E_i) \rangle \approx -\sqrt{\frac{\pi K}{E_i^2 \beta^3 \nu}} \frac{a(h_m) N(h_m)}{\cos \alpha_m T(v, h_m)} \left(\frac{h_m}{\kappa}\right)^{3/2} \quad (12)$$

where $\alpha_m$ is the angle at $h_m$ between the arch and a line perpendicular to the solar surface.

Let $\tau(h_m, E_i)$ denote the average time until a particle’s mirror point reaches an altitude where the particle’s kinetic energy is lost immediately. The result is not influenced appreciably, if we arbitrarily take this altitude to be the top of the photosphere.

$$\tau(h_m, E_i) \approx -\int_{h_m}^{0} \frac{dh}{\langle v_m(h, E_i) \rangle} - \frac{a(h_m)}{\langle v_m(h_m, E_i) \rangle} \quad (13)$$

This expression can be evaluated using Equations (9) and (12). The results for a simple, sinusoidal arch with a length of $\pi$ times the maximum height and a powerlaw (10) for the magnetic field strength with exponent $\kappa = 1.5$, are given in Table I. The Bilderberg Model (Gingerich and de Jager, 1968) was used for the densities below 2000 km, and the model of Athay (1959) at greater altitudes.

<table>
<thead>
<tr>
<th>Initial mirror point altitude in km</th>
<th>$h_{\text{max}}/h_m = 10$</th>
<th>$h_{\text{max}}/h_m = 5$</th>
<th>$h_{\text{max}}/h_m = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>$7.6 \times 10^{-1} E_i^2 \beta^3$</td>
<td>$4.0 \times 10^{-1} E_i^2 \beta^3$</td>
<td>$1.5 \times 10^{-1} E_i^2 \beta^3$</td>
</tr>
<tr>
<td>1500</td>
<td>$2.4 \times 10 E_i^2 \beta^3$</td>
<td>$1.3 \times 10 E_i^2 \beta^3$</td>
<td>$4.9 E_i^2 \beta^3$</td>
</tr>
<tr>
<td>2000</td>
<td>$4.1 \times 10^2 E_i^2 \beta^3$</td>
<td>$2.1 \times 10^2 E_i^2 \beta^3$</td>
<td>$8.2 \times 10 E_i^2 \beta^3$</td>
</tr>
<tr>
<td>2500</td>
<td>$2.5 \times 10^3 E_i^2 \beta^3$</td>
<td>$1.3 \times 10^3 E_i^2 \beta^3$</td>
<td>$5.1 \times 10^2 E_i^2 \beta^3$</td>
</tr>
<tr>
<td>3000</td>
<td>$2.2 \times 10^4 E_i^2 \beta^3$</td>
<td>$1.2 \times 10^4 E_i^2 \beta^3$</td>
<td>$4.4 \times 10^3 E_i^2 \beta^3$</td>
</tr>
</tbody>
</table>
We compare these results with the time $\tau_{\text{coll}}$ taken by a particle to lose its energy by collisions. We can again use approximation (11) for the density and average the energy loss (4) over the unperturbed path. For electrons, we obtain:

$$
\tau_{\text{coll}}(h_m, E_k^0) \approx \frac{E_k^0}{\langle \frac{dE_k}{dt} \rangle_{\text{coll}}} \approx \frac{E_k^0}{k_1 N(h_m)} \int_0^T \exp \left( -\frac{\kappa \cos^2 \alpha_m v_0^2}{4h_m a(h_m)} t^2 \right) dt
$$

where the superscript 0 refers to the initial values.

To a good approximation, we can extend the integral to infinity. The comparison with (13) leads to:

$$
\frac{\tau_{\text{coll}}}{\tau} \approx \frac{K h_m}{k_1 \kappa a(h_m)} \frac{E_k^0}{(E_i)^2 \beta^3}
$$

This shows that the lifetimes for electrons with kinetic energies up to 10 MeV will be considerably shortened by the downward motion of the mirror points. The mirror point velocity fully determines the lifetime for $E_k \leq 1.5$ MeV, if (5) is not fulfilled, i.e. if the gyro-synchrotron loss can be neglected. For $1.5$ MeV $\leq E_k \leq 10$ MeV, Table 1 gives only upper limits to the lifetime, because the term $E_i^2 \beta^3$ decreases considerably during the lifetime.

For protons the ratio $\tau_{\text{coll}}/\tau$ is of the order of $10^{-2}$. The motion of the mirror points, which becomes significant at the time when only little kinetic energy is left, does not considerably shorten their lifetimes.

These statements are fairly general and appear to hold for a broad range of shapes of arches.

B. MIRROR POINT IN THE UPPER CHROMOSPHERE AND INNER CORONA

Since recent models of the upper chromosphere differ substantially, we cannot say with any certainty where the basic assumption of case A, namely that $a(h) \ll h$, breaks down. According to Beckers' (1968) model this breakdown does not occur above 2000 km at all. The derived expression for the mirror point velocity (12) still holds provided $h_{\text{max}} - h_m \gg a(h_m)$. In the case of other models, where $a \gg h$, the particle density in (8) varies more slowly than the other integrands. Consequently the deflections have to be considered over the entire path. Near the maximum altitude they perturb the pitch angle from $\pi/2$ (for $h_{\text{max}} \gg h_m$) to $\pi/2 - \varepsilon$ and therefore increase the mirror point altitude. This is not balanced out by the downward motion due to deflections near the mirror point, if the density is slowly varying and $h_{\text{max}} \gg h_m$. The average upward mirror point velocity is of the order of $1/E_i^2 \beta^3$ km/s ($E_i =$ total energy in MeV). If, on the other hand, $h_{\text{max}} - h_m$ is small, the pitch angle is generally less than $\pi/4$ and we again get a downward mirror point motion. Calculations of the average mirror point velocity for a sinusoidal arch show that it vanishes at a ratio of $h_{\text{max}}/h_m$ between 2 and 4 depending on the shape of the arch.

There are, however, serious objections to these steady-state considerations. We
must, in fact, expect our calculations to be altered by the inclusions of various time dependent perturbations, such as motion of the feet of the arched field lines, whistlers and hydromagnetic waves. Whistlers, for example, have the same effect as collisions and could shorten the lifetimes given in Table I. When the particle lifetimes become longer these transient perturbations can be expected to play an important role. They not only control the lifetime, but diffuse the particles throughout a large volume rather than only along the sheet defined by the drift motions, and make it possible that in a large number of cases some particles reach a line of force that leads them to the vicinity of the earth (Gold, 1963).

If $h_{\text{max}} \gg h_m$, the pitch angles near $h_{\text{max}}$ will be nearly $\pi/2$. If moreover the thermal velocity of the trapped particles is sufficiently small, streaming instabilities could occur that would cause the mirror points to rise.

Another type of magnetohydrostatic instability is likely to occur in the solar atmosphere. We shall refer to it as the ‘hernia’ instability. To understand it let us consider an arched tube of force into which the supply of fast particles or hot ionized gas is increasing. The extra pressure will cause the tube to become distended in cross-section and raised in height. We believe that for a certain range of excess pressure this will be a gradual process in which the relationship to neighboring tubes of force is not changed. If, however, the gas pressure in the particular tube rises above some critical value, the tube will cleave the overlying fieldlines and rise through between them into a region of much lower fieldstrength. The feet are of course firmly anchored in the dense atmosphere and their position cannot be much changed. The instability will therefore have created a region somewhere above the feet where one bundle of lines of force runs at some angle to its neighbors and arches up much higher. (Such an effect has been referred to as ‘ballooning’ in the plasma physics literature.)

In observations of prominences one often sees effects that suggest such a behavior. Also on the occasions that limb flares have been recorded cinematographically there is a suggestion of this kind of instability.

This gives rise to a third possibility for an upward motion of the mirror points. The rising tube of force will expand, and the magnetic fieldstrength will diminish. If this decrease has a time scale much larger than the time of one particle bounce period, which is of the order of seconds, the longitudinal invariant $L$ is conserved:

$$L = \int_{h_m}^{h_{\text{max}}} (B_m - B(s))^{1/2} \, ds = \text{invariant}.$$  

The fieldstrength $B_m$ at the mirror point has therefore to decrease. However, the field at the initial mirror point might not decrease enough, and the mirror point has to rise to an altitude with an appropriate fieldstrength.

4. Possible Relations to Observed Phenomena

Kane (1969) found for nonthermal X-ray spikes at 100 keV lifetimes of 30 s and less.
If all the energetic electrons are produced at the same time and have a steep powerlaw spectrum, this is the lifetime of 100 keV electrons. In the simple model used for Table I this would lead to an average mirror point altitude of about 3000 km and a slightly higher injection height. The model consisting of trapped particles in arched magnetic fields can explain the hard X-ray lifetimes very well without the introduction of abnormally high target densities. The model predicts a flattening of the nonthermal spectrum in time, if all particles are injected at the same time.

Pintér (1969) observed a spatial distribution of X-ray flares on the solar disk suggesting that the electrons radiating bremsstrahlung have predominantly horizontal orbits. This follows immediately from our model, since the X-ray emission comes from near the mirror points where the electrons are moving in horizontal circles. Vaiana (1970) observed a soft X-ray flare in the flash phase as two bright points. These points may well correspond to the locations of the intersection of the arched magnetic field with a layer of a certain density.

We would also like to point out that both the low and high frequency cutoff mechanisms of microwave radio bursts proposed by Holt and Cline (1968), and Holt and Ramaty (1969) respectively, are readily applicable to our model of particle trapping. If we are dealing with gyro-synchrotron radiation, it has to come from near the mirror points where the magnetic field is strongest. At low frequencies we might expect gyro-synchrotron reabsorption since the magnetic field decreases with altitude and so does the gyrofrequency. The emission from below is then absorbed by higher harmonics. The more the line of sight passes through the magnetic arch, the higher the resulting low frequency cutoff frequency and the smaller the maximum of the observed radiation intensity, as it is indeed observed. The high frequency cutoff, on the other hand, can be explained by anisotropies in the pitch angle distribution in the emitting region, i.e. near the mirror points.

The lifetime of protons trapped in constant magnetic fields is determined by collision losses and is considerably shorter than for electrons with the same velocities. The magnetic gradient and centrifugal drift velocities, however, are larger by the ratio \( m_e/m_p \) for the same particle velocities. For 1 GeV protons in an arcade with a sinusoidal cross-section and a rectangular plan, they amount to the order of \( 10^3/B \) km/s. Protons can therefore be widely distributed and transport energy downwards to the photosphere, causing a non-equilibrium condition suitable for the emission of spectral lines. This proton mechanism, most recently discussed by Švestka (1970), may thus be able to explain some of the Hα radiation observed in flares.

Acknowledgements

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