THE USE OF THE GOLDBERG-UNNO METHOD FOR THE INVESTIGATION OF SMALL-SCALE PHOTOSPHERE MOTIONS

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Abstract. The Goldberg-Unno method is analysed. Accounting for the instrumental profile correction reduces the derived microturbulent velocities only slightly. A similar effect may be caused by an unresolved macroturbulence. The method of accounting for the damping effect is considered. The correction for the influence of the damping effect does not change substantially the general trend of the variation of \( \xi_t \) with \( \tau_0 \). The microturbulent velocity \( \xi_t \) is reduced appreciably.

An attempt to analyse the microturbulent velocities by the Goldberg-Unno method under deviation from LTE is made.

The main conclusion is that the Goldberg-Unno method, especially in its modified form, is valid and useful.

1. Introduction

One of the main problems of the solar photosphere is that of its small-scale motion field. Relatively many investigations on this subject have been published in recent years. Nevertheless, their results are uncertain and often contradictory.

The small-scale motion field is due to the microturbulence and ‘unresolved’ macroturbulence (convection and wave motions). The microturbulence affects the absorption line coefficient, while the macroturbulence influences the line profiles. How to separate these two effects is a rather complicated problem, and to study it, the main characteristics of the macroturbulence must be known. For the present we shall consider the small-scale motion field to consist mainly of the microturbulence, but it will be implied that the microturbulent velocities derived from the line profiles are spoiled by an unresolved macroturbulence.

There are two more or less accurate methods for studying turbulent velocities. In the first method the theoretical computation of the equivalent widths or the line profiles with an assumed microturbulent velocity \( \xi_t(\tau) \) should be carried out. When varying \( \xi_t(\tau) \) one should achieve fairly good agreement between the computations and observations. This method we call the ‘absolute’ one. In this method all inexactnesses (errors of abundance, oscillator strengths, photospheric model etc.) influence the result directly. Besides, the method itself is complicated and labor-consuming.

The second is the well known Goldberg-Unno method. Its attraction is due to its simplicity. In addition it does not depend on the photospheric model and allows one to obtain in principle the \( \xi_t(\tau) \) by measuring various parts of line profiles of the multiplet. But the main advantage of the method is its differential character in which the various kind of errors affect the results only to a small degree.

It is known that the Goldberg-Unno method for optical depths \( \tau > 0.05 \) gives an increase of the radial component of the microturbulence with depth in the photo-
sphere (Unno, 1959). Roddier (1965) criticized Goldberg's method and the results by Unno. Afterwards Olson (1966) verified the validity of the Goldberg-Unno method. Recently Banos (1968) restricted the use of the method in some cases and questioned the results obtained with this method. At the same time the authors (Gurtovenko and Troyan, 1969) obtained nearly the same results as Unno.*

Such contradictory results lead to the conclusion that the method is not trustworthy. But we believe that the use of heterogeneous observational material and maybe different approaches of various authors may account for these discrepancies. Besides, results obtained by the 'absolute' method (Waddell, 1958; Nissen, 1965) do not contradict the data of Unno, while Schmalberger's investigation (1963) confirms the microturbulence model of Unno. Therefore, the Goldberg-Unno method needs to be tested carefully.

2. The Analysis of Possible Errors in the Goldberg-Unno Method

We use the following parameters: \( \tau \) = the optical depth in the near-by continuum in the line-of-sight; \( \tau_0 \) = the radial optical depth in the near-by continuum; \( \tau_{0(5000)} \) = = optical depth in the continuum for \( \lambda = 5000 \, \text{Å} \); \( S_\lambda \) = the total source function; \( t_\lambda \) = the optical depth inside the line in the line-of-sight; \( t = \tau + t_\lambda \); \( t_0 \lambda \) = the radial optical depth inside the line; \( \xi \lambda \) = microturbulent velocity component; \( v = \Delta \lambda / \Delta \lambda_B \); \( I_\lambda \) = the emergent intensity inside the line in the line-of-sight; \( r_\lambda \) = the residual intensity; \( K_\lambda \) = the absorption coefficient inside the line; \( K \) = the absorption coefficient in the near-by continuum.

As the Goldberg-Unno method is very simple it is useful to employ also a simple method for the evaluation of the optical depth \( \tau \) for any part of the line profile. The most suitable method is that suggested by de Jager and Neven (1963). The principle of the method is the following. The total source function \( S_\lambda (\tau_0) \) can always be expressed by the function \( E_\lambda (t) \). In this case

\[
I_\lambda = \int_0^\infty E_\lambda (t) e^{-t} \, dt \approx E_\lambda \quad (t = 1).
\]  

(1)

Equation (1) is sufficiently valid for the conditions existing in the solar photosphere.

In using Goldberg's method one measures the multiplet line profiles at the points \( \lambda_A \) and \( \lambda_B \) where the intensities

\[
I_\lambda = \int_0^\infty S_\lambda (\tau_0) e^{-\tau_0 / \cos \theta} \, d\tau_0 / \cos \theta
\]

are equal. The equation

\[
K_{\lambda_A} = K_{\lambda_B}
\]

(2)

is satisfied if \( S_{\lambda_A} = S_{\lambda_B} \).

* The recent paper by Dunn and Olson (1971) verifies the Goldberg method.
This last equality is valid if the excitation temperatures in the two lines are equal and the source function is independent of $\lambda$ (the first and second of Goldberg's assumptions (Goldberg, 1958). If deviations from LTE occur it is appropriate to ask whether the excitation temperature for two lines of a multiplet will be equal at the same optical depth. If not, then $S_{\lambda_A} \neq S_{\lambda_B}$ and Equation (2) is not valid. It is also not valid if the source functions depend on $\lambda$. It is obvious that for this case one must find two such values $I_{\lambda_A} \neq I_{\lambda_B}$ for which $K_{\lambda_A} = K_{\lambda_B}$, and only then Goldberg's method may be used.

We are in a position now to list the causes of errors in the Goldberg-Unno method.

1. The source function for two lines of the multiplet depends on $\lambda$ or differs at the same optical depth in the photosphere.
2. Observational errors in the line profiles.
3. The occurrence of macroturbulent (or convective) motions in the photosphere.
4. The line broadening is assumed to be only due to Doppler broadening.

2.1. Instrumental errors

At first an attempt was made to apply Unno's initial treatment (1959) to eight of the same lines (Table I) but this time observed with the double-pass spectrograph at the Kiev-Golosejevo Observatory and interpreted assuming LTE and BCA model (Gin-

<table>
<thead>
<tr>
<th>No. of pairs</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element</td>
<td>V1</td>
<td>V1</td>
<td>NaI</td>
<td>ScII</td>
<td>Cr I</td>
<td>Ti I</td>
<td>Cr I</td>
<td>Fe I</td>
</tr>
<tr>
<td></td>
<td>6081.458</td>
<td>6199.195</td>
<td>6154.235</td>
<td>5657.883</td>
<td>5296.704</td>
<td>4617.280</td>
<td>5300.755</td>
<td>5217.398</td>
</tr>
<tr>
<td></td>
<td>6090.222</td>
<td>6243.120</td>
<td>6160.759</td>
<td>5684.201</td>
<td>5300.755</td>
<td>4639.948</td>
<td>5348.327</td>
<td>5253.470</td>
</tr>
<tr>
<td>Damping</td>
<td>a</td>
<td>0.05</td>
<td>0.04</td>
<td>0.08</td>
<td>0.03</td>
<td>0.05</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>parameter</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

gerich and De Jager, 1968). Our attempt (Figure 1) led to the conclusion that the use in Goldberg's method of observational materials referring to the same lines but from different instruments, and the evaluation of the optical depth by various methods, led to almost the same results.

The general trend of the change of $\xi_{r}$ with $\tau_{0}$ is not obeyed by pair No. 2, which also displays an anomalous behaviour in the results of Unno.

Next, we tried to derive the microturbulent velocities using corrected line profiles. Unfortunately, we had to restrict ourselves to the investigation in pairs Nos. 1, 2 and 3 only, as the instrumental profile was not known for the fifth order. The correction was made by the method of 'repeated blurring' (de Jager and Neven, 1963; Gurtovenko, 1966).

The values of microturbulent velocity are presented in graphical form in Figure 2. The main conclusion is: after the instrumental correction of special line profiles, the general character of $\xi_{r}(\tau)$ does not change substantially. Only a small reduction of its value is apparent.
2.2. MACROTURBULENT VELOCITIES

The study of macroturbulent velocities interpreted by convection have been made by De Jager and Neven (1967, 1968). They used the observed line asymmetry and obtained $\xi_{\text{mac}}$-values of the order of 2–3 km/s.

Here we shall consider a simplified macroturbulence model: (a) the motion elements extend over the whole photosphere; (b) they are small enough to record the line profile as a statistic average from the profiles of individual elements. Next we neglect the asymmetry that seems to occur as a second order effect in comparison with the blurring itself. Under these assumptions it is easy to show that the results are what one could expect if the line profile is blurred by the instrumental profile $\varphi(\xi_{\text{mac}})$, where $\varphi$ is the distribution function of the macroturbulent velocities $\xi_{\text{mac}}$. The effect will be larger for faint unsaturated lines and in this case a simple convolution of the absorption line coefficient with the $\varphi$-function occurs. For example, if $\varphi$ is the Doppler function then the total turbulent velocity equals $\sqrt{(\xi_t^2 + \xi_{\text{mac}}^2)}$, i.e. both kinds of velocities can not be separated from the study of the line profile.

The influence of this macroturbulence model on the real solar line profiles was estimated by Gurtovenko and Ratnikova (1970) using $\xi_{\text{mac}} \approx 2$ km/s. For some narrow lines the blurring can reach 8% in central dip and 13 mÅ in halfwidth. For other lines
these corrections are smaller. Generally the effect is of the same order as the instrumental profile correction. Thus, we conclude that the macroturbulence considered above can increase the $\xi_r$-values, but not to a large extent.

If the moving elements are smaller in comparison with the whole depth of the photosphere and their velocities are increasing with depth, the problem becomes more complicated and a depth-dependence effect should be expected.

2.3. THE INFLUENCE OF THE DAMPING EFFECT

In the general case the line absorption coefficient $K_\lambda \sim (\lambda^2 f/\Delta \lambda_p) H(a, v)$. For our task it is enough to estimate approximately the parameter $a = \lambda^2 y/4\pi c \Delta \lambda_p$. This estimation was made under the condition that the damping constant $\gamma$ is mainly due to collision of the absorption particles with hydrogen atoms (Mustel, 1960). The $\Delta \lambda_p$ found earlier without accounting for the damping effect were used in calculations. It turned out that the parameter changed slightly with depth. Therefore the values $a$ were computed only
for optical depths corresponding to the half of the residual intensity of the fainter line in every pair. They are given in the last line of Table I.

The damping effect was taken into account in the following way: for two lines $A$ and $B$ of the multiplet in the measured points of line profiles with equal $r_\lambda$ we have $K_{\lambda A} = K_{\lambda B}$; hence,

$$ (g f)_A H(a_A, v_A) = (g f)_B H(a_B, v_B). $$

(3)

For the multiplet $a_A = a_B$ and (3) can be written as:

$$ \frac{H(a, v_A)}{H(a, v_B)} = \frac{(g f)_B}{(g f)_A} = \frac{\int_{-\infty}^{\infty} \frac{e^{-y^2}}{a^2 + (v_A - y)^2} \, dy}{\int_{-\infty}^{\infty} \frac{e^{-y^2}}{a^2 + (v_B - y)^2} \, dy} = \Phi(\Delta \lambda_D). $$

(4)

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3}
\caption{Illustration for finding the ratios $H(a, v_A)/H(a, v_B)$ and $v_A/v_B$ according to relevant values $(gf)_A/(gf)_B$ and $\Delta \lambda_A, \Delta \lambda_B$.}
\end{figure}
The solution of Equation (4) appears to be quite a difficult task. But we use another way. In the measured points of the line profiles the ratio

$$\frac{v_A}{v_B} = \frac{\Delta \lambda_A}{\Delta \lambda_B}$$  \hspace{1cm} (5)

is known. In a graph of $H(a, v)$ vs $v$, only one pair of $H(a, v)$ values simultaneously satisfies Equations (4) and (5). See Figure 3.

We used all eight uncorrected pairs. In Figure 4 the derived Doppler half-widths are compared with those obtained without accounting for the damping effect. Pairs Nos. 1, 2 and 3 were investigated more carefully once more. The damping effect was taken into account by using the corrected line profiles. Derived turbulent velocities are presented in Figure 2. The results lead to the following conclusions:

1. The general trend of change of $\Delta \lambda_D$ with depth does not break.

![Figure 4](image-url)
2. Accounting for damping makes the turbulent velocities systematically smaller. For faint lines (pairs Nos. 1 and 2) the $\Delta \lambda_D$-correction ($\delta \Delta \lambda_D$) is comparatively small: 2–3 mÅ. It increases for the stronger lines.

3. The $\delta \Delta \lambda_D$-values increase when going from the core to the wing of the lines. It is clearly seen from Table II, where the average $\delta \Delta \lambda_D$ for the stronger lines (pairs Nos. 3–8) are tabulated.

<table>
<thead>
<tr>
<th>$r_\lambda$</th>
<th>$\delta \Delta \lambda_D$ (mÅ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\geq 0.9$</td>
<td>5.6</td>
</tr>
<tr>
<td>0.8–0.9</td>
<td>3.5</td>
</tr>
<tr>
<td>0.7–0.8</td>
<td>2.9</td>
</tr>
<tr>
<td>0.6–0.7</td>
<td>2.1</td>
</tr>
<tr>
<td>&lt; 0.6</td>
<td>1.6</td>
</tr>
</tbody>
</table>

The conclusions 2 and 3 are not unexpected. But it was not obvious that even in the core of the faint lines the correction for damping effect is required.

The correction for damping effect, as seen in Figures 2 and 4, can not always be neglected in practice.

The method itself may be a useful supplement to the Goldberg-Unno method. Its disadvantages are following: (1) the parameter $a$ must be known even if only approximately; (2) in the far wings of the strong lines the values $v_A$, $v_B$ are determined with relatively large errors because of the slow change of $H(a, v)$. Consequently $\Delta \lambda_D$ is derived less accurately.

The study of the turbulence by the improved Goldberg-Unno method and using the centre-to-limb observations is now in progress at our observatory.

2.4. THE DEVIATIONS FROM L.T.E.

If in two lines $A$ and $B$ of the multiplet, $S_A \neq S_B$, then the inaccuracy in Goldberg's method may depend on the $\lambda$ and may change with depth. It is possible to assume a more simple case of deviations from LTE: $S_A = S_B \neq B(\tau)$. If the $S(\tau_0)$ for the two lines of the multiplet is known, the $\xi_\ell(\tau_0)$ can be also found. But it is very likely that under non-LTE the circumstances are more complicated, and the method of determination of $\xi_\ell(\tau_0)$ by measuring the line profiles will demand an additional development. Next, we shall consider a possible version of an attempt in that direction.

Briefly it consists of the following successive steps:

1. According to (1)

   \[ I_\lambda \approx S_\lambda(\tau_0) = B_\lambda(T^*) \]  

   where $\tau_0$, $T^*$ are the optical depth and the excitation temperatures of the effective layer. From (6) we find $T^*$ using $I_\lambda$. The derived values of $T^*$ will be equal for both lines, because $r_{\lambda_A} = r_{\lambda_B}$. But it is not known which optical depth $\tau_0$ the temperatures $T^*$ refer to.
2. For both lines of the pair find $\tau_0$ from the equation

$$t \cos \theta = \int_0^{\tau_0} (1 + \eta_\lambda) \, d\tau_0.$$  \hspace{1cm} (7)$$

Here we put $t = 1$; calculate $\eta_\lambda = K_\lambda / K$ using the Boltzmann-Saha equations, assuming the ionization temperature $T_i = T$ and excitation temperature $T^* = \beta T$. As soon as $\tau_0$ is found, we derive $T$ using a photospheric model. If $T^* \neq \beta T$ the same procedure with another $\beta$ must be repeated. As a result we have a series of $\tau_0$ and $\beta$ corresponding to the measured values of $r_\lambda$ (Figure 5).

Fig. 5. The values $\beta$ and $r_\lambda$ versus $\tau_0$ found with accounting for the deviations from LTE.
3. From the graph (Figure 5) find two values of \( r_{\lambda A}, r_{\lambda B} \) (and consequently \( \Delta \lambda_A, \Delta \lambda_B \)) corresponding to the same \( \tau_0 \). Under equality of \( \tau_0 \) in accordance with \( t_A = t_B = 1 \) the equality \( t_{\lambda A} = t_{\lambda B} \) is valid. The calculation of \( \xi_r \) is known.

The pair No. 3 has been subjected to such an analysis, using the BCA model, the Na abundance and the oscillator strength from the paper by Lambert and Warner (1968), and the \( \Delta \lambda_{D} \)-values found above in this paper. Parameter \( a \) was assumed to be constant. The results are presented in Figure 5 and Figure 2 (crosses).

We summarise briefly.
1. The general trend of the variation of \( \xi_r \) with \( \tau_0 \) does not change appreciably, as is clearly seen in Figure 2.
2. The microturbulent velocities are essentially less in comparison with the previous results.
3. The difference between parameter \( \beta \) and unity is appreciable. It is smaller for the fainter line, that is believable.
4. The source function \( (S_r \sim r_{\lambda}) \) of both lines differs at the same \( \tau_0 \), to a small extent. The decrease with depth of the difference between source functions is clearly visible. That is also believable.

For the present we are not inclined to attach much weight to the conclusions 2, 3 and 4. It is not because of some assumptions made, but because of the use of the ‘absolute’ method of the calculations, for which some quantities must be known precisely. The error in computed \( \eta_{\lambda} \) influences directly the derived \( \tau_0 \) and consequently the source function. The errors in source functions may affect considerably the derived \( \xi_r \)-values.

But the method employed here seems to be attractive because it can be used with any pair of lines, not only with the multiplet one.

3. Conclusion

The inaccuracies of Goldberg’s method that may be due to the instrumental errors, macroturbulence, damping and feasible deviations from LTE, can not be a cause of discrepancies between the results obtained by various authors. The Goldberg-Unno method is valid, and it reveals the real effect of the influence of small-scale motion field on the shape of spectral line profiles.

Damping influences the results obtained even from the central part of the core of line profile. Taking it into account reduces slightly the derived turbulent velocities and makes steeper the increase of \( \xi_r \) with depth. If one does not suppose the damping constant unreasonably high, the increase of \( \xi_r \) with depth does exist. The ‘unresolved’ macroturbulence (convection) velocities increasing with depth may account for this effect. But there is another effect. The \( \xi_r \)-values obtained by Goldberg’s method and averaged over the line profile show the increase from the centre to limb. The increase of the halfwidths of faint lines toward the limb, revealed in many investigations, is, as a matter of fact, the same phenomenon. The convection can not explain this effect. At least one must suppose an anisotropic microturbulence with a large tangen-
tial component, or admit the existence of large macroturbulent velocities in the tangential direction. This question is waiting for its solution.

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References