A METHOD FOR CONSTRUCTING STREAMLINES FOR THE SUN'S LARGE SCALE FLOW FROM DOPPLER VELOCITIES

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Abstract. We show that, if the large scale departures from the mean differential rotation, measured by Howard and Harvey, represent nearly horizontal flow, we may under certain assumptions deduce a pattern of streamlines for these motions from the doppler line of sight velocities. This can be done with data from a single day, without having to construct the total flow from different projections of the (assumed) same velocity vectors seen on different days. Mathematically the method involves integrating a single first order inhomogeneous partial differential equation along a set of characteristic curves which are circles concentric with the center of the solar disk.

The structure of the resulting streamline pattern could be compared to large scale magnetic patterns, and latitudinal transports of magnetic flux and momentum could be estimated.

1. Introduction

Howard and Harvey (1970) and Howard (1971) have presented evidence from systematic full-disk Doppler velocity measurements made at the Hale Observatories that the differential rotation is a function of time and contains imbedded in it a system of 'eddy' or 'cellular' motions whose horizontal scale is much larger than that of supergranules. In this note I wish to put forward a relatively simple method by which we may deduce a pattern of streamlines for these motions from the Doppler line-of-sight velocities. An important advantage of the method is that given certain assumptions, streamlines can be constructed from one day's measurements, rather than having to take different projections of the (assumed) same velocity vectors on different days to reconstruct the total vector.

2. The Method

The principal assumption for the method is that the deviations from the Sun's average differential rotation of largest horizontal scale represent motions which, like the differential rotation itself, are primarily horizontal, i.e., they lie in spherical shells. We also assume that this horizontal motion is incompressible, meaning really that we ignore horizontally propagating sound waves and large scale horizontal density gradients. This latter assumption seems quite easily satisfied, considering the relatively long time scales likely to be associated with such motions.

It follows from the above assumptions that the horizontal divergence of the hori-

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zontal flow vanishes. If we denote solar longitude by $\phi$, latitude by $\theta$, and the corresponding velocity components by $V_\phi$ and $V_\theta$, this means that

$$\frac{\partial V_\phi}{\partial \phi} + \frac{\partial}{\partial \theta} (V_\theta \cos \theta) = 0. \quad (1)$$

It follows, then, from Equation (1) that we may define a streamfunction $\psi$ such that

$$V_\phi = -\frac{\partial \psi}{\partial \theta}; \quad V_\theta = \frac{1}{\cos \theta} \frac{\partial \psi}{\partial \phi}. \quad (2)$$

Our object, then, is to deduce the single function $\psi$ from the line of sight velocity, $V_l$.

Consider first the geometrically simplest case when the Sun's rotation axis is perpendicular to the Earth-Sun line. Then the line of sight velocity $V_l$ of a velocity horizontal on the Sun is related to $V_\phi, V_\theta$, by

$$V_l = -\sin \phi V_\phi - \sin \theta \cos \phi V_\theta. \quad (3)$$

in which $-90^\circ \leq \theta, \phi \leq 90^\circ$, i.e. $\theta$ is measured from the equator and $\phi$ from the central meridian. Then substitution of the relations (2) into (3) gives a linear, inhomogeneous first order equation for $\psi$ in terms of $V_l$, given by

$$\sin \phi \frac{\partial \psi}{\partial \theta} - \tan \theta \cos \phi \frac{\partial \psi}{\partial \phi} = V_l. \quad (4)$$

Equation (4) may be written in the form

$$\frac{d\psi}{ds} = V_l \quad (5)$$

with the solution

$$\psi = \psi_0 + \int_{s_0}^{s} V_l \, ds \quad (6)$$

in which $s$ is a coordinate measured along a particular member of the single family of characteristics $C(\theta, \phi)$ for the homogeneous part of (4). If we note that from (4)

$$ds = -\frac{d\phi}{\tan \theta \cos \phi} = \frac{d\theta}{\sin \phi} \quad (7)$$

it follows that, on a characteristic

$$-\tan \phi \, d\phi = \tan \theta \, d\theta. \quad (8)$$

Integrating functionally and taking the exponential of both sides, we find the form of the characteristics is

$$C(\theta, \phi) = \cos \phi \cos \theta = \text{const.} \quad (9)$$

From (9), if for example we introduce solar disk coordinates $x = \sin \phi \cos \theta, y = \sin \theta$, we may easily prove that the characteristics defined by (9) are circles concentric
with the center of the disk. This is clearly also true when the Sun's axis is not perpendicular to the Earth-Sun line, because we can always define $\theta$ and $\phi$ in the same manner even if they do not represent true solar longitude and latitude. Therefore the integration would always be done on the concentric circles.

We can never determine a streamfunction to better than an additive constant, and we have no need to since the velocities always involve its derivatives. However, in (6) there is an as yet undetermined $\psi_0$ which may be different on different characteristic curves. In order to find a unique solution for $\psi$ (except for the additive constant) everywhere on the solar disk, we must be able to specify $\psi_0$ on some curve which cuts all the characteristics and begin the integration from that curve. The simplest procedure would seem to be to simply assume $\psi_0=0$ on the solar equator. This is acceptable for the mean differential rotation, and we would essentially be assuming that the disturbances have no flow which crosses the equator. It remains to be seen how reasonable this is, but we note in its support that sunspots never and larger scale magnetic regions almost never are seen to migrate across the equator, which presumably some would do if there were a flow to carry them.

A second, somewhat similar procedure would be to assume the streamfunction on the central meridian is antisymmetric about the equator. In either case, we could carry out the integration along the circular characteristics in either a clockwise or counterclockwise sense in each of two hemispheres separated by the curve (either the equator or the central meridian) on which $\psi_0$ is specified. Since we may not allow discontinuities in $\psi$ (they would imply infinite velocity), we should therefore 'match' the two integrations by some smoothing technique. One test of our assumptions would be to see, if when we set $\psi_0=0$ and integrate starting from $\theta=0$, $-90^\circ<\phi<0$ (the eastern half of the equator) around to the western half of the equator, we get $\psi$ close to 0 again.

The fact that the integration is carried out along circles concentric with the disk allows us to avoid inaccuracies near the limb due to foreshortening, and also very close to the center of the disk, where horizontal velocities are measured least accurately.

3. Comments

In practical terms, the flow need not be entirely horizontal, but only horizontal enough that its local vorticity about a radial axis is large compared to its horizontal divergence. The mean differential rotation and meridional circulation appear to satisfy this condition. It also holds fairly well for the planetary scale flow in the Earth's atmosphere. The degree to which it will prove to be true on the Sun will be determined mostly by how important for the large scale motions the Coriolis forces are compared with turbulent viscous and magnetic stresses and the pressure gradients. That is, the stronger the Coriolis forces, the more radial vorticity the flow will have.

Among the properties to be looked for in the results are

(a) the degree of persistence of the stream patterns from one rotation to the next;
(b) resemblance between the patterns and the bipolar magnetic regions, in partic-
ular the scale in longitude and the 'tilt' with latitude (see Starr and Gilman, 1965, for a discussion of the dynamical implications of the tilt), and correlation with the larger scale sector structure;

(c) computation of the horizontal Reynolds stress to see whether momentum is transported toward the equator as Ward (1965) has inferred from sunspot motions;

(d) estimation of the transport in latitude of vertical magnetic field by the motions as occurs in the Rossby wave dynamo model of Gilman (1969a, b), with comparison to the observed evolution of the photospheric fields.

While the method is simple in principle, working the Hale Observatory data into a form suitable to use with the method appears to be a difficult task, so that we have not yet tested the method with solar data. Basically what needs to be done is to obtain $V_t$ smoothed over fairly large areas on the disk (large enough to include several supergranules), with an appropriate uniform rotation subtracted out, rather than using the residuals left after fitting the data to a differential rotation profile, as was done by Howard and Harvey (1970) and Howard (1971).

H. Fischer has successfully tested the method described above on data for the planetary scale flow in the Earth's atmosphere, for which the assumptions we have made hold rather well and for which we already know the form of the streamlines. This test is reported briefly in the research note which follows.

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References