Abundances of the Elements in Gaseous Nebulae*

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INTRODUCTION

Though most published research on the abundances of the elements has resulted from stellar spectroscopy, a considerable amount of work has also been done on gaseous nebulae. In nebulae the abundances are derived from the relative strengths of emission lines, rather than from absorption lines as in stars. The individual spectral lines in nebulae are optically thin, and there are no saturation or curve-of-growth effects. Also many light elements are observable, including H, He, N, O, and Ne, although unfortunately C is not. Furthermore, besides observations of objects in our own Galaxy, it is also possible to measure the abundances of elements in other galaxies, both in their H II regions and in their central nuclear regions. On the other hand, a disadvantage of using nebulae for abundance determinations is that the reduction depends on a model which is necessarily an approximation, more or less true, to the real nebula. Also, the strength of collisionally excited lines depends strongly on the temperature $T$. Furthermore, not all stages of ionization of an element are observable in the optical spectral region; for instance, though [O II] and [O III] are observable in nebulae, O IV and O V are not. Another difficulty is that in comparing lines of different wavelengths to find relative abundances, the correction for interstellar reddening is needed, and this is often rather difficult to determine. Finally, elements of lower abundance (for example Fe) are difficult to observe in gaseous nebulae.

EMISSION COEFFICIENTS

In general the observed strength $I$ of an emission line is proportional to the integral

$$I \propto \int N_e N_i \epsilon(T) dl,$$

where $N_e$ and $N_i$ are the electron density and the density of the ion responsible for the emission, respectively. We shall first discuss the emission coefficient $\epsilon(T)$. The recombination lines of H I, He I and He II give the abundances of H+, He++, and He+ respectively. For these ions $\epsilon(T)$ comes from the well-known recombination cascade theory originally due to Aller, Baker, Goldberg, Menzel and others.

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In more recent calculations the effects of electron collisions are also taken into account. The cross-sections are largest for those collisions in which the principal quantum number \( n \) is unchanged but the angular momentum quantum number \( l \) is shifted to \( l \pm 1 \), and the next largest cross-sections are for collisions in which \( n \) is changed by \( \pm 1 \), then by \( \pm 2 \), etc. The best available calculations at the present time for \( \text{H I} \) and \( \text{He II} \) are those of Pengelly (1964), in which all collisions are neglected, so that the results apply to low-density \( \text{H II} \) regions; Burgess (1958) in which collisions are taken to be so effective that at every \( n \) the various angular momentum levels \( l \) are in statistical equilibrium and are therefore populated proportionally to \( 2l+1 \), so the results apply at the relatively higher densities of planetary nebulae; and Clarke (Aller & Liller 1968); and for \( \text{He I} \) those of Robbins (1968). It may be noted that these recent results of Robbins have the effect of reducing the calculated helium abundance for a given line strength by about 20 per cent in comparison with previous calculations except those of Pengelly (Seaton 1968b), and for this reason many of the relative abundances of helium quoted in the present review are smaller than the values given by the authors of the original papers.

The calculations all show that for recombination lines \( \epsilon(T) \propto T^{-m} \) over a limited range of temperature to a good approximation, with \( m \approx 1 \); for instance \( m = 0.90 \) for \( \text{H} \beta \), and \( m = 1.13 \) for \( \text{H I} \lambda 5876 \). Thus the recombination emission coefficients are not particularly temperature sensitive.

Less abundant ions such as \( \text{C II}, \text{O IV}, \) and \( \text{O V} \) have weak permitted emission lines in planetary nebulae, which have often been interpreted as resulting from recombination and have been used to derive abundances of the ions in question. However, Seaton (1968a) has shown that a good deal of the excitation of these lines is actually due to resonance fluorescence. Thus for them \( \epsilon \) is not a function of \( T \) alone, but instead depends on the local radiation field, and therefore these lines cannot be used to derive abundances.

In addition to the recombination lines, there is also the \( \text{H I} \) Balmer recombination continuum which has \( \epsilon(T) \propto T^{-3/2} \). Since this temperature dependence is different from that of the \( \text{H I} \) recombination lines it can in principle be used to determine \( T \) observationally. The Balmer continuum is contaminated by the two-photon continuum and by the scattered continuum from dust, so it can therefore be determined observationally only by measuring the Balmer discontinuity.

It is also possible to measure relative abundances of \( \text{He} \) in \( \text{H II} \) regions from relative strengths of the radio recombination lines of \( \text{H I} \) and \( \text{He I} \) such as \( 137\alpha \), etc. Though it is possible to give a formula for \( \epsilon(T) \) for these lines, it is not necessary because at these high \( n \), \( \text{H} \)
and He are both nearly identical one-electron systems except for their masses, so that the relative strengths of their lines (separated by the isotope shift) are directly proportional to their relative abundances.

For elements other than H and He, only collisionally excited lines are available for abundance determinations, and for these lines, in contrast to recombination lines, the emission coefficient depends more strongly on the temperature

$$\epsilon(T) = \frac{8.63 \times 10^{-6}}{T} \frac{\Omega(i,j)}{\omega_i} e^{-\chi_{ij}/kT} h v b$$

(2)

In this formula $\Omega(i,j)$, the mean collision strength, is derived from quantum mechanical calculations, and $b$ is the branching ratio, to be explained below. The available numerical results for $\Omega(i,j)$ are chiefly due to Saraph, Seaton & Shemming (1969), Czyzak et al. (1970), and others. It is interesting to see how calculated values of the collision strengths have changed in the past eleven years as shown in Table I. An increase in the calculated value of $\Omega(i,j)$ causes a proportional decrease in the calculated abundance of the ion in question.

**Table I**

<table>
<thead>
<tr>
<th>Comparison of published collision strengths</th>
<th>Seaton (1958)</th>
<th>Saraph, Seaton &amp; Shemming (1969)</th>
</tr>
</thead>
<tbody>
<tr>
<td>N$^+$ ($^3P, ^1D$)</td>
<td>2.17</td>
<td>2.99</td>
</tr>
<tr>
<td>O$^+$ ($^3S, ^2D$)</td>
<td>1.28</td>
<td>1.47</td>
</tr>
<tr>
<td>O$^{+2}$ ($^3P, ^1D$)</td>
<td>1.59</td>
<td>2.48</td>
</tr>
<tr>
<td>O$^{+2}$ ($^3P, ^1S$)</td>
<td>0.22</td>
<td>0.35</td>
</tr>
<tr>
<td>Ne$^{+2}$ ($^3P, ^1D$)</td>
<td>0.76</td>
<td>1.29</td>
</tr>
<tr>
<td>Ne$^{+4}$ ($^3P, ^1D$)</td>
<td>0.84</td>
<td>1.46</td>
</tr>
</tbody>
</table>

It must further be noted that the effects of resonances are included only in an average way in these calculated collision strengths. For a few ions the detailed resonance structure of $\Omega(i,j)$ has been calculated, and it can play an important role in the abundance derived from a particular set of observations. An example is the case of [O III] studied by Eissner et al. (1970) in which $\Omega(^3P, ^1D)$ varies by about 20 per cent from 8000$^\circ$ to 12 500$^\circ$.

The branching ratio $b$ in the equation (2) is easily derived. For the simple case in which a single upper level can decay into either of two lower levels, it is fixed by transition probabilities, $b = A_1/(A_1 + A_2)$. Another simple case is that of collisional de-excitation in which $b = A/(A + N_e < \sigma_{\text{de-ex}} v >)$, where the brackets denote an average over the Maxwellian velocity distribution. In general, whatever the energy-level structure, the branching ratio can be determined by solving simple determinantal statistical-equilibrium equations.
ABUNDANCES FROM LINE STRENGTHS

Next we shall discuss the reduction procedure to calculate relative abundances from measured relative line intensities. The correction for interstellar extinction can be derived from the relative intensities of H I recombination lines, since the calculations of $\epsilon(T)$ show that these ratios are nearly independent of $T$. Therefore, the calculated Balmer decrement or Paschen/Balmer ratios can be assumed for the intrinsic ‘colour index’, and by comparison with the measured decrement or line ratios the extinction is determined. It should be noticed that often some dust is contained in the nebula; in general it is not all distributed along the light path from the nebula to the observer. Under these circumstances scattering does not cause extinction as it would if the dust were more distant from the nebula. However, some calculations by Mathis (1970) show that for ‘normal’ interstellar particles the reddening lines due to internal dust or to external dust are quite similar to one another, so that in practice the location of the dust does not cause large uncertainties. It is conceivable that the dust particles might be quite abnormal (for instance, in the nucleus of another galaxy), and this method would then break down. However, it would still be possible to estimate the extinction by comparison of the observed and calculated recombination line strengths.

Observations of Hα/Hβ/Hγ ratios in planetary nebulae show slight discrepancies with respect to the calculated emission coefficients and the assumed reddening line, but these discrepancies are quite small for the best observed nebulae (Osterbrock, Capriotti & Bautz 1963; O'Dell 1963a). This provides a valuable check on the theory from which the mission coefficients $\epsilon(T)$ are calculated. However, there are larger discrepancies (up to a factor of 2 or 3) for the higher Balmer lines ($n = 20$ to $30$), and there seems to be a correlation between the amount of the discrepancy and the line width (Kaler 1966; Kaler & Lee 1967). Either the recombination theory is incorrect, or else there is some as yet unknown observational error in the relative Balmer-line strengths. I believe that it would be quite valuable for someone to make a new accurate photoelectric study of this problem, as the theory should be well understood. A highly sophisticated theoretical treatment, taking all collisional effects into account, will soon be published by Brocklehurst, and should be compared with new observational results.

From the observed strengths of the lines and the information on $\epsilon(T)$ described above the abundances can be determined from equation (1) on the basis of a model. The simplest model treats the nebula as homogeneous, with constant $N_e$ and $T$, and is thus more or less equivalent to the one-layer or curve-of-growth methods of stellar atmospheres. The numerical value of $N_e$ is derived from line ratios
such as [O II] $\lambda 3729/\lambda 3726$ or [S II] $\lambda 6717/\lambda 6731$, while $T$ is determined from line ratios such as [O III] $(\lambda 4959+\lambda 5007)/\lambda 4363$ or [N II] $(\lambda 6548+\lambda 6583)/\lambda 5755$. Then from each observed line strength the abundance of the ion which emits it can be determined. In some cases two successive stages of ionization of the same element are observed, such as [O II] and [O III], and their relative abundances can be used to plot an empirical ionization curve $N(A^{+m+1})/N(A^{+m})$ as a function of ionization potential. Thus finally the abundance of every element with at least one observed line can be determined. Discrepancies, for instance in $N_e$ and $T$ as determined from different line ratios, indicate that this model is too simplified to give highly accurate results, though the abundances determined from it are usually believed to be correct to within a factor of 2 or 3 (Aller & Liller 1968).

Another possibility is to assume more sophisticated models, a procedure more or less equivalent to the use of model stellar atmospheres. The aim is to use all available information—properties of the central star, equilibrium equations, etc.—and calculated a complete photoionization model of the nebula, including the equilibrium temperature at each point. In principle, all parameters can be varied until all observed data are matched. Static models of the planetary nebulae NGC 7662 and IC 418 have been calculated in this way by Flower (1969a) and by Harrington (1969). However, complications are caused by the structure of the actual nebula, which is partly unknown, as it is seen only in projection. Furthermore, the real structure is too complicated to handle because of patchiness, filamentary structure, deviations from spherical symmetry, etc. Dynamical effects may be important, as is particularly shown by the observed strengths of the [O I] $\lambda 6300$, $6364$ lines which have almost zero calculated intensity in all models, but which arise in the outermost parts of the nebulae, in the region presumably partly ionized and heated by an expanding shock wave (Flower 1969b).

An intermediate reduction scheme due to Peimbert (1967), which has also been applied by Rubin (1969), is based on the idea of taking account of the variations of $T$ to the second order only, and using the observations as much as possible. The emission coefficient is expanded in a power series.

$$
\epsilon(T) = \epsilon(T_o) + \left( \frac{d\epsilon}{dT} \right)_{T_o} (T-T_o) + \frac{1}{2} \left( \frac{d^2\epsilon}{dT^2} \right)_{T_o} (T-T_o)^2
$$

(3)

and integrated

$$
N_e N_l \int \epsilon(T) dl = \epsilon(T_o) \int N_e N_l dl + \frac{1}{2} \left( \frac{d^2\epsilon}{dT^2} \right)_{T_o} \int N_e N_l (T-T_o)^2 dl,
$$

(4)
with
\[ T_o = \frac{\int N_e N_i T \, dl}{\int N_e N_i \, dl}. \tag{5} \]

If all ions have the same distribution \( N_i \), then from two line ratios (such as \([\text{O III]} \) and \([\text{N II}] \)) both \( T_o \) and
\[ t^2 = \frac{\int N_e N_i (T - T_o)^2 \, dl}{\int N_e N_i \, dl \cdot T_o^2} \tag{6} \]
can be determined instead of the one constant, \( T_o \), from one line ratio as in the single-layer model. Then \( T_o \) and \( t^2 \) can be used to determine the abundances of all the ions with measured lines.

**OBSERVED RESULTS FOR HELIUM**

Next let us discuss the observed relative abundances of the elements in the nebulae, considering helium first of all. The most studied object is NGC 1976, the Orion Nebula, with published results due to Mathis (1957), Aller & Liller (1959), and Méndez (1968). The most recent measurements are those of Peimbert & Costero (1969), who observed four different points and derived the ratios \( \text{He}^+/\text{H}^+ = 0.097, 0.095, 0.077, \) and \( 0.009 \), the last measurement being in the nearby companion nebula NGC 1982. It definitely shows that this fourth point is in an \( \text{H II} \), \( \text{He I} \) zone (where \( \text{He} \) is neutral). The exciting star of NGC 1982 is a \( \text{B} \) \( \text{V} \) star, cool enough so that the helium is theoretically expected to be neutral, in agreement with this observation. On the other hand, the amount of ionized \( \text{H} \) is considerably greater than expected from this calculation, so the nebula cannot be said to be completely understood theoretically. However, it is clear that some correction to the abundance of \( \text{He} \) for the amount of \( \text{He}^+ \) is necessary in all parts of the Orion Nebula. This correction is based on the observed strength of the \([\text{S II}] \) lines \( \lambda6717, \lambda6731 \), because the emitting ion \( \text{S}^+ \) has an ionization potential \( 23.4 \) eV, just a little less than the ionization potential of \( \text{He} \), \( 24.6 \) eV. The observed presence of \([\text{S II}] \) lines in \( \text{H II} \) regions therefore shows that \( \text{He}^+ \) must also exist in them. \( \text{O}^+ \) has a somewhat higher ionization potential, \( 35.1 \) eV, and Peimbert and Costero therefore interpolate between the relative abundances of \( \text{S}^+ \) and \( \text{O}^+ \) to find the abundance of \( \text{He}^+ \). Their final result for all three points in NGC 1976 is \( N(\text{He})/N(\text{H}) = 0.11 \). Two other \( \text{H II} \) regions observed optically, M8 and M17, have essentially the same relative helium abundances as NGC 1976. It may be noted that the correction for \( \text{He}^{++} \) is negligible, as \( \lambda4686 \) is not observed in any \( \text{H II} \) region.

Radio determinations of \( \text{He}^+/\text{H}^+ \) abundance ratios are available from the work of Palmer \textit{et al.} (1969), Reifenstein \textit{et al.} (1970), and others. These determinations are in fairly good agreement with the
optical measurements for nebulae common to both sets of observations, as shown in Table II. The average from seven H II regions observed in the radio-frequency region is N(He+)/N(H+) in the range 0.07 to 0.09. There is, however, at present no known way in which the correction for He can be obtained from radio measurements alone. Two nebulae with N(He+)/N(H+) ≈ 0, NGC 2024 and NGC 1982, show that this correction very probably exists.

**Table II**

<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 1976</td>
<td>0.090</td>
<td>0.078</td>
<td>0.083</td>
</tr>
<tr>
<td>M17</td>
<td>0.097</td>
<td>0.082</td>
<td>0.090</td>
</tr>
</tbody>
</table>

Most of the H II regions that have been observed for helium abundance are close to us in the Galaxy, at distances within 2 kpc, except for W 49A, which is about 14 kpc distant but at approximately the same distance from the centre of the Galaxy as the Sun. However, it should be possible in time to make radio-frequency measurements of the relative helium abundance in different regions of the Galaxy.

Next let us discuss the abundance of helium in planetary nebulae, many of which have been observed for the relative strength of the He lines. Helium may occur both as He+ and He++ in these objects, and it is strongly established theoretically that if the central star is hot enough to produce He++, then the outer edges of the H+ and He+ zones coincide and there should be no correction necessary for unobserved He. The most complete discussion of He abundances in planetary nebulae is that of Harman & Seaton (1966), who conclude that N(He)/N(H) = 0.16 with only observational scatter. A few apparently very deviant cases occur, such as IC 351, with indicated N(He)/N(H) = 0.65, and NGC 6778, with N(He)/N(H) = 0.33, but each of them depends on a weak observation of a single faint He I line. They are almost certainly incorrect, but should be re-observed to check this conclusion.

If instead of all available observations of planetary nebulae, only the most accurate photoelectric measurements are used, a somewhat smaller helium abundance is found. The average for the eight best observed planetaries is N(He)/N(H) = 0.13, with values for individual nebulae ranging from 0.11 to 0.15, but in some cases the accuracy as judged from relative intensities of He I λ4471, λ5876 is ±0.02. Therefore, the difference in N(He)/N(H) between planetary nebulae and H II regions is marginal in any case, and may not exist at all.
Some special planetaries with well-determined helium abundances are K 648 in the globular cluster M15 with $N(\text{He})/N(\text{H}) = 0.13$ (O'Dell, Peimbert & Kinman 1964) and 49 $+$88°, near the galactic pole and approximately 20 kpc distant, with $N(\text{He})/N(\text{H}) = 0.13$ (Miller 1969). Thus these extreme Population II objects have completely normal helium abundances.

In connection with the helium abundance it should be noted that there is strong observational evidence that the central star of a planetary becomes a white dwarf within a few times $10^5$ years after the first appearance of the nebular shell (O'Dell 1963b, 1968). This shows that the central star has almost no H in its interior, and, therefore, that whatever process leads to the formation of a planetary nebula results in a fairly clean separation between the H-rich shell and the H-poor interior (Osterbrock 1966).

The planetary nebulae apparently make a barely significant contribution to the mass exchange between interstellar matter and stars. Near the Sun the best available data on space density, lifetimes and masses of planetaries show that the rate of mass return to interstellar space (integrated over a cylinder perpendicular to the galactic plane) is about $10^{-10} M_\odot/\text{pc}^2\text{yr}$, (O'Dell 1968). On the other hand, according to studies of stellar evolution the present rate of consumption of interstellar gas by star formation is about $1.4 \times 10^{-9} M_\odot/\text{pc}^2\text{yr}$, and the present rate of ejection of gas to interstellar matter by evolving stars is about $0.5 \times 10^{-9} M_\odot/\text{pc}^2\text{yr}$ (Schmidt 1959). The planetaries thus contribute about 20 per cent of this rate of mass return.

The helium abundance is quite anomalous in the Crab Nebula, NGC 1952, for which Woltjer's (1958) analysis of Mayall's photographic spectrograms gives $N(\text{He})/N(\text{H}) = 0.4 - 0.6$, while Williams' (1967) model gives $N(\text{He})/N(\text{H}) = 0.45$. There is no doubt that this supernova remnant has an abnormally high helium abundance, and it would definitely repay further photometric observation and discussion.

NGC 6302, an odd southern planetary nebula, has been observed by Oliver & Aller (1969), who find $N(\text{He})/N(\text{H}) = 0.25$ approximately, but the nebula is so far south and the observations therefore necessarily have such large probable errors that its apparent deviation from the normal helium abundance may not be real. Other peculiar nebulae such as the Cygnus Loop, IC 443, and the Nova Persei 1901 shell, all should be observed for helium abundance, but there are no published results for any of them.

In other galaxies H II regions are observable and photometric observations of them can be used to determine the helium abundances. In these extragalactic H II regions the correction for He° has been estimated from the $[\text{O II}]/[\text{O III}]$ ratio by analogy with the Orion
Nebula, and a list of results is given in Table III. High accuracy, including the correction factor in particular, is necessary to be sure whether or not there are real differences in the helium abundances between different galaxies.

**Table III**

*Helium abundances in other galaxies*

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>Object</th>
<th>He$^+$/H$^+$ Factor</th>
<th>He/H</th>
</tr>
</thead>
<tbody>
<tr>
<td>M33</td>
<td>NGC 604</td>
<td>0.094</td>
<td>1.37</td>
</tr>
<tr>
<td></td>
<td>NGC 604</td>
<td>0.134</td>
<td>1.30</td>
</tr>
<tr>
<td>M101</td>
<td>NGC 5461</td>
<td>0.082</td>
<td>1.20</td>
</tr>
<tr>
<td></td>
<td>NGC 5471</td>
<td>0.092</td>
<td>1.06</td>
</tr>
<tr>
<td>NGC 4449</td>
<td>H II Region</td>
<td>0.078</td>
<td>1.25</td>
</tr>
<tr>
<td>LMC</td>
<td>NGC 2070</td>
<td>0.105</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>NGC 2070</td>
<td>0.083</td>
<td>1.08</td>
</tr>
<tr>
<td>SMC</td>
<td>NGC 346</td>
<td>0.086</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Variations of the helium abundance with distance from the centre of a galaxy can probably best be observed in external galaxies. Schmidt (1962) attempted to observe this effect photographically some years ago in M31, by measuring not only the relative He I/H I line ratios but also the relative sizes of the H II and He II zones in order to determine the correction for He$^3$. The measurements were subject to large uncertainties because of the very small angular sizes of the H II regions in M31, but Schmidt seemed to find a decrease in helium abundance outward from $N$(He)/$N$(H) = 0.14 at 5 kpc distance from the centre to 0.08 at 18 kpc. More recently Rubin & Ford (1968) have used image-tube spectrograms to study the same effect in M31. Their result is a decrease from $N$(He$^+$/N(H$^+$) = 0.14 at 3 kpc to 0.035 at 12 kpc. They also find that the [O III]/H$\alpha$ ratio has large variations at all distances, that is, that there is no systematic change of ionization with distance from the centre of M31. On the other hand Aller (1942) found that in M33 the [O III] lines are stronger in the H II regions farther from the centre.

**Heavy Elements**

Next we shall discuss the observational results on the abundances of elements other than helium. It should be remembered that these abundances are much more sensitive to the adopted value of $T$ than the helium abundances described above. Again, NGC 1976 is the best observed H II region in our Galaxy. The results of Peimbert & Costero (1969) show that it has fairly normal abundances of N, O, Ne, and S as detailed in Table IV. There is practically no difference in the abundances of these elements in NGC 1976, M8, or M17, although the dust contents of these nebulae (as determined from their reflection continua) are quite different. This seems to indicate that only a small fraction of the heavy elements are locked up in dust in any H II region.
### Table IV

*Summary of Logarithmic abundances*

<table>
<thead>
<tr>
<th>Element</th>
<th>Planetary Nebulae</th>
<th>Orion Nebula</th>
</tr>
</thead>
<tbody>
<tr>
<td>H</td>
<td>12.00</td>
<td>12.00</td>
</tr>
<tr>
<td>He</td>
<td>11.73</td>
<td>11.20</td>
</tr>
<tr>
<td>N</td>
<td>8.17</td>
<td>7.74</td>
</tr>
<tr>
<td>O</td>
<td>8.9</td>
<td>8.40</td>
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<tr>
<td>F</td>
<td>4.9</td>
<td>—</td>
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<tr>
<td>Ne</td>
<td>7.9</td>
<td>7.80</td>
</tr>
<tr>
<td>Na</td>
<td>6.6</td>
<td>—</td>
</tr>
<tr>
<td>S</td>
<td>7.9</td>
<td>—</td>
</tr>
<tr>
<td>Cl</td>
<td>6.9</td>
<td>—</td>
</tr>
<tr>
<td>A</td>
<td>7.0</td>
<td>—</td>
</tr>
<tr>
<td>K</td>
<td>5.7</td>
<td>—</td>
</tr>
<tr>
<td>Ca</td>
<td>6.4</td>
<td>—</td>
</tr>
</tbody>
</table>

The [N II] lines $\lambda 6548, 6853$ are close to H$\alpha$, and therefore their intensities may easily be compared with it. Observations show that the ratio $\lambda 6583/H\alpha$ often varies from point to point within an H II region, generally being largest at ionization fronts. This behaviour results from a combination of the relatively low ionization of N$^+$ in the outer parts of nebulae together with the higher temperature there, which increases $\varepsilon(T)$. The average N$^+$ abundance in nebulae has been studied by Courtes, Louise & Monnet (1969), who made Fabry-Perot measurements of H$\alpha$ and [N II] from which line intensities and line widths can both be measured. As the two ions involved have different molecular weights, both $T$ and the turbulent velocity can be determined from their line widths, and then from $T$ and the relative intensities the abundance of N$^+$ can be found. These authors find considerable ranges in $T$ and in the N$^+$ abundance, with a strong correlation between the two as exhibited in Table V. This correlation might actually exist, because a low heavy element content in a particular nebula would tend to make it relatively inefficient in cooling, and it therefore would tend to be heated to a relatively high temperature by photoionization. However, the abundance of N$^+$ derived in the coolest nebula, IC 434, is considerably higher than the ‘cosmic’ abundance of N, and it would, therefore, be worth checking to be sure that some error has not crept into the temperature determination. For instance, in the reduction, the line profiles and the turbulent spectrum are both assumed to be gaussian, while other assumed forms might lead to different calculated temperatures.
Relative abundances of the heavy elements have been determined for numerous planetary nebulae on the basis of the homogeneous model. As the planetaries are relatively bright objects, many weak emission lines can be measured on long-exposure high-dispersion spectrograms, and as a result the abundances of a fairly large number of elements can be determined. A good summary of the results has been given by Aller & Czyzak (1968), and their final list of mean abundances in planetary nebulae is reproduced in Table IV.

More detailed models have been calculated by Flower (1969b) and Harrington (1969) for the planetaries NGC 7662 and IC 418 and used to determine abundances, with the results also included in Table IV. It is particularly interesting to note that the Ne abundance derived by Flower depends partly on the observed strength of the [Ne II] λ12·8μ line in IC 418. As further observations are made in the infra-red and also in the satellite ultra-violet, improved determinations of nebular abundances can be expected.

The most peculiar planetary nebulae observed are K 648 in the globular cluster M15, in which both Ne and O have abundances smaller by a factor of about 30 than the mean planetary composition listed in Table IV (O’Dell, Peimbert & Kinman 1964) and the Coma planetary 49 +88°1, with Ne abundance smaller by a factor of about 20, and O smaller by a factor of about 3 than the mean (Miller 1969). It may be noted that the observations show that in K 648 $T = 14500^\circ$ in contrast to $T \approx 10000^\circ$ of typical planetary nebulae, an effect of the decreased abundance of cooling ions. There is a considerable spread in the abundances of Ne and O among the individual planetaries, and though it is probably partly observational there is no doubt that at least some of the effect represents a real variation in abundances.

We next turn our attention to the abundances of heavy elements in H II regions in other galaxies. Very little work has been published on this subject, but there are measurements of NGC 604 in M33 (Aller, Czyzak & Walker 1968) and also of 30 Dor in the Large Magellanic Cloud (Faulkner & Aller 1965; Wares & Aller 1968). Both of these objects have approximately normal abundances of O, S, and Ne. It
would be valuable to press this problem observationally, particularly if the He/H variations mentioned earlier are confirmed.

Emission lines are observed in the nuclei of some galaxies, the best known example being the Seyfert galaxies with their high level of ionization and wide emission lines. Both He I and He II recombination lines are observed in Seyfert galaxies and the derived helium abundances shown in Table VI show considerable variation from one object to another. There is, however, some uncertainty in these results because of the very great width of the H and He lines which may indicate that they are formed under conditions of relatively high density or large optical depth and that the conventional nebular recombination theory might not apply. Interstellar reddening is important in several of these objects, as indicated by the Balmer decrement and by [S II] line ratios (Wampler 1968), but even after correction for this effect there are some discrepancies between the Balmer-line strengths and the theoretical calculations.

| NGC 1068 | 0.08 | 0.04 | 0.12 |
| NGC 1275 | 0.02 | 0.01 | 0.03 |
| NGC 4151 | 0.03 | 0.03 | 0.06 |
| NGC 7469 | 0.26 | 0.03 | 0.29 |
| 3C 120   | 0.17 | 0.02 | 0.19 |

Turning to other elements in Seyfert galaxies, the photoionization models calculated by Williams & Weymann (1968) approximately match the observed line spectra of the Seyfert galaxies NGC 1068 and 4151. A noticeable exception is the great observed strength of the [O I] emission lines $\lambda\lambda$6300, 6364, and to a lesser extent [N I] $\lambda$5200. These lines are calculated to be quite weak on the photoionization model because they are emitted only in the transition region containing both neutral atoms and free electrons (from the ionization of H). Probably the strength of these lines indicates that some of the gas is ionized by shock heating, because neutral-atom lines can be expected to be excited in the recombining region. Williams and Weymann find that the best match to the observed line spectrum is obtained with abundance ratios for $\text{Ne/H} = 5 \times 10^{-5}$ and $\text{O/H} = 1 \times 10^{-4}$ (both down from the normal by a factor of about 0.1). From the relative strength of the [Fe VII] and [Ne V] emission lines Nussbaumer & Osterbrock (1970) have derived $\text{Fe/H} = 1 \times 10^{-5}$, an approximately normal abundance ratio. However, all these calculations are based on comparisons with observed line strengths and will have to be done over taking into account the corrections for interstellar extinction.

In addition to the Seyfert galaxies, many normal galaxies also have emission lines in their central regions. They are typically weaker and
sharper than the emission lines of the Seyfert galaxies. Though this nuclear emission tends to be more common in spiral galaxies, about 25 per cent of elliptical galaxies are known to show at least [O II] \( \lambda 3727 \) emission. In many galaxies with nuclear emission \([\text{N II}] \lambda 6583/\text{H}\alpha > 1\), contrary to the situation in H II regions except in condensations or fronts near their outer edges (Burbidge & Burbidge 1962). Burbidge, Gould & Pottasch (1963) suggested that in the nuclei of galaxies the strength of the [N II] line might be due to an unusually high \( T \). They indicate that the high \( T \) might be due to ionization and heating by fast particles rather than the ordinary nebular situation of heating by photoionization. With the normal abundance of N and with all N in the stage N\(^+\), \( \lambda 6583/\text{H}\alpha = 2 \) at \( T = 10^4 \), and \( 3.5 \) at \( T = 1.2 \times 10^4 \) while if some of the N is ionized to higher stages then even higher \( T \) is required to match the observed ratios. The higher temperatures can occur where the heating is due to fast particles because in this situation the energy is transferred directly to free electrons. Thus the heating rate is proportional to \( N_e \) while the cooling rate is proportional to \( N_e^2 \), so that for fairly low densities \( T \) can rise to high values.

Recent observations by Peimbert (1968) of the emission lines in the nuclei of M51 and M81 have clarified this problem considerably. He made photometric observations of [N II] and [O II] \( \lambda 3727 \) and found \( \lambda 3727 \) too weak by a factor of 5–20 at the \( T \) required to explain the [N II] line strength with normal abundances. Therefore, the conclusion is that \( T \) is approximately normal and that N is overabundant in the nuclei of these galaxies by a factor of 3–8 compared with normal abundances. Another possibility is that O is less abundant by a factor of 5–20, but Peimbert argues that this would require too high an electron temperature and also that the enhancement of N is more likely to have occurred as a result of stellar evolution followed by return of processed material to interstellar space.

It should be noted, however, that again the observations show that [O I] \( \lambda 6300 \) is quite strong in the galaxies studied by Peimbert. Its presence is good evidence that the ionization is not completely radiative as Peimbert has supposed. Thus it is quite likely that \( T \) varies with position in the nucleus, and that the degree of ionization also varies. But it is quite difficult to see any way in which the [N II] emission could occur in a different region from the [O II] emission, so the conclusion that N is overabundant probably is correct. A crucial test of this idea would be to try to observe the [N I] line \( \lambda 5200 \). If N is overabundant by a factor of the order of 5 as Peimbert concludes, then [N I] \( \lambda 5200 \) should be comparable in strength with [O I] \( \lambda 6300 \). If observations do not confirm this prediction then either the N is not as abundant as the [N II] lines seem to indicate, or else \( N_e > 10^8 \text{ cm}^{-3} \) in the region in which the neutral lines are formed, because at densities
of this order collisional de-excitation of \( N^0 \) is important. The electron
density in this region could be measured reasonably well from the
[\text{S II}] \( \lambda \lambda 6717, 6731 \) lines which, because of the lower ionization potential
of \( S^+ \), probably arise in the same region as \[ \text{O I} \] and [\text{N I}].

Finally we turn our attention to quasars, which have emission-line
spectra similar in many ways to those of Seyfert galaxies. The quasars,
however, have large red shifts, so that the ordinarily unobservable
ultra-violet spectrum is shifted into the region in which measurements
can be made. Thus we have the opportunity to observe lines of many
presumably abundant ions that are not observable in ordinary low-
velocity nebulae and galaxies. At the present time there is very little
published spectrophotometry of quasars’ spectra, though a beginning
in this direction has been made. Thus the simplified models of
Greenstein & Schmidt (1964), Osterbrock & Parker (1965) and Bahcall
& Kozlovsky (1969, 1970) have to be compared with crude tables of
lines that are observed to be present or not present, or that are observed
as strong, medium or weak lines. However, these models all seem to
agree in showing that quasars have an unusually low He content with
otherwise normal abundances of the heavier elements (to within a
factor of 3 or 5 considering the crudity of the observational data).
This is a most interesting and unexpected result and certainly should
be confirmed or disproved by more precise observations together with
physically more meaningful models.

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