Some Current Problems in the Theory of Late-Type Stars*

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Present uncertainties in the position of the main sequence for Population I and Population II stars, and in the ages and chemical composition of old galactic clusters and globular clusters are discussed in terms of galactic evolution and current views in cosmology.

I. INTRODUCTION

LATE-TYPE stars are the main stellar constituent of the galaxy and probably of the universe, both in number and in total mass involved. Practically all stars seem to pass through a “late” stage of evolution during their lifetime, at least during their pre-main sequence phase of gravitational contraction. Some of the most interesting problems of stellar astrophysics such as those relating to pre-main sequence evolution, the structure of hydrogen convection zones, their interaction with rotation and Ca II emission features, pulsation instability, thin-shell instabilities, the helium flash, are encountered in late-type stars. Although several of these questions will be mentioned in this paper, I shall be concentrating on some of the astronomical rather than the astrophysical aspects of the theory of late-type stars, although, obviously, the two cannot be separated. Of special interest is the question of the interpretation of the color-magnitude diagrams of star clusters using the physical theory of stellar evolution. In the case of the old star clusters, most of the members are “late” in spectral types, i.e., their surface effective temperatures are less than approximately 6500°K. It is interesting to discuss them in the context of the little we know about galactic evolution and the recent advances in cosmology concerning the early history of the universe.

II. MAIN SEQUENCE

Let us consider first the problem of the zero-age main sequence. This is an old problem, but an important one because it involves the calibration of galactic and intergalactic distances. Interest in this question has recently been stimulated by a discussion of the Hyades distance modulus by Hodge and Wallerstein (1966). Since the Hyades cluster is used as the standard of distances for Population I stars, these authors point out that any change in the Hyades distance modulus could mean a change in the distance modulus of distant stars in and outside the galaxy. This result would follow naturally from the assumption generally made that all Population I stars fall on the same main sequence. But is this a correct assumption to make? The effects of large variations in chemical composition on the position of the theoretical main sequence is well known, at least in qualitative terms, and is shown in Fig. 1 (see Demarque, 1967a). Perhaps such large variations in Z may seem excessive. We do, however, have observational evidence from spectrophotometric studies of field stars by van den Bergh and Sackmann (1966) and of red giants in the old galactic clusters M67 and NGC 188, by Spinrad (1967), pointing to the existence of stars with perhaps twice the heavy element content of the sun.

III. OLD GALACTIC CLUSTERS

In the case of NGC 188, there is some additional evidence for high Z related to the presence of a gap in the stellar distribution just above the main sequence in the color–magnitude diagram (Eggen and Sandage 1966). The discussion of the next paragraph suggests that the theoretical interpretation of this gap requires an estimate for Z in NGC 188 of the order of 0.06, at least double the previously accepted value. Let us first illustrate this point with the help of galactic cluster M67. Figure 2 shows the color–magnitude diagram of M67, obtained by photoelectric photometry by Eggen and Sandage (1964). The position of the gap above the main sequence is clear. The physical interpretation of this feature is the following: It corresponds to the latter
part of the hydrogen exhaustion phase for stars which on the main sequence possessed a convective core. This phase occurs on a Kelvin time scale, which is relatively very fast in contrast with the nuclear time scale of the phases of evolution immediately preceding or following it. Figure 3 describes two evolutionary tracks for a normal chemical composition \((X=0.67, Z=0.03)\) for \(1.0\) and \(1.4M_\odot\) due to Hallgren (1966). The star with \(1.0M_\odot\) does not have a convective core on the main sequence and follows a smooth evolutionary track; the star with \(1.4M_\odot\) exhibits a characteristic hook in its track. Figure 4 represents the intermediate case of \(1.2M_\odot\) for the same composition (Hallgren and Demarque 1966). Let us consider what happens in the interior of the star. Figure 5 describes, as a function of time, the mass fraction included in the convective core: There is an incipient convective core for \(1.0M_\odot\) and a well-developed convective core for \(1.2M_\odot\) (Hallgren 1966). This illustrates the delicate balance between the carbon–nitrogen cycle, whose efficiency depends sharply on the temperature and for this reason renders convection more likely, and the proton–proton chain with a much milder temperature dependence. Note that contrary to the case of massive stars (Morris, Demarque, and Percy 1965; Hartwick 1967) in which the convective core mass decreases with time, leaving behind a region of intermediate chemical composition, the convective core here grows in mass before it disappears abruptly. This growth of the convective core is a result of the increase in the relative importance of the carbon–nitrogen cycle over the proton–proton chain as the central region of the star gets hotter.

Although theory and observation are in satisfactory agreement in the case of M67, the fitting of the color–magnitude diagram of NGC 188 to theoretical models presents difficulties: in particular, it does not appear possible to reproduce the gap observed near \(V=15.5\) by Eggen and Sandage (1966), with photometric measurements. Figure 6 shows the results of the earlier study of NGC 188 by Sandage (1962), where the presence of the gap is suggested but the evidence is inconclusive. Evolutionary tracks obtained in 1964 (Demarque and Larson) for solar mass stars with the composition parameters \(X=0.67\) and \(Z=0.03\), shown in Fig. 7, fail to yield any evidence of a hook due to hydrogen exhaustion. The cluster age found is approximately \(9.5\times10^9\) yr. With a lower metal content \((Z=0.01)\), we found an age of \(12\times10^9\) yr and no evidence of a gap. More recently, Iben (1966) has reconsidered the cases of M67 and NGC 188 assuming an intermediate metal content \((Z=0.02)\). His age estimate for NGC 188 is \(11\times10^9\) yr; he finds no evidence of a gap either.

In an attempt to understand this discrepancy between theory and observation in a phase of stellar evolution considered well understood, additional models were obtained varying the initial composition rather drastically (Demarque and Schlesinger 1968), hoping at the same time to gain valuable information on the chemical composition of the cluster. In fitting the models to observation, two steps must be considered: (1) the position of the minimum luminosity at which a gap occurs is determined theoretically for each composition; (2) the distance modulus of the cluster, and therefore the absolute magnitude of the observed gap, is found a function of chemical composition since the cluster main sequence must be fitted to the zero-age main sequence with the proper composition. First, one notices, as illustrated in Fig. 8, that varying \(X\) at constant \(Z\) does not appreciably modify the shape of the color–magnitude diagram, but only affects the position of the gap with respect to the main sequence. This is due to the fact that the difference in bolometric magnitude between the main sequence and the flat part of the evolutionary track identified with the shell-burning stage is nearly independent of \(X\). On the other hand, it is not surprising that the lower the original \(X\), the more rapidly hydrogen will be exhausted at the

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**Fig. 1.** Main sequences for different compositions in the theoretical H-R diagram. All models were computed with \(l/H=1\) in the convection zone. The labels refer to the following compositions: (1) \(X=0.67, Z=0.10\); (2) \(X=0.67, Z=0.05\); (3) \(X=0.67, Z=0.03\); (4) \(X=0.67, Z=0.01\); (5) \(X=0.67, Z=0.001\); (6) \(X=0.67, Z=0\); (7) \(X=0.77, Z=0.03\); (8) \(X=0.57, Z=0.03\); (9) \(X=0.37, Z=0.03\). (After Demarque, P., Astrophys. J. 150, 945, 1967.)

**Fig. 2.** The color–magnitude diagram of M67 showing the gap at the turnoff point. (After Eggen, O. J., and Sandage, A. R., Astrophys. J. 140, 141, 1964.)
center of the star and the closer to the main sequence we find the gap.

The effect of varying the heavy element content $Z$ is shown in Fig. 1. On each main sequence, we have indicated the position at which the convective core reaches $4\%$ of the total mass of the star. Varying the...
metal content does not change this critical luminosity, but since it changes the position of the main sequence, it is possible to find a solution for a particular value of $Z$. We find $Z = 0.06$. On the other hand, it is interesting to note that variations in the helium content have a marked effect on the luminosity for the onset of core convection, but this effect is nearly exactly counteracted by the resulting shift in main sequence position. One of the results of this analysis is that the age of NGC 188 may be decreased to $6 \times 10^9$ yr, rather lower than the probable age of the galaxy ($10^{10}$ yr). It could also be that the age of M67 might have to be lowered in a similar fashion.

IV. GLOBULAR CLUSTERS

Turning now to Population II, let us consider the field subdwarfs. Are they, as their name suggests, subluminous for a given effective temperature, as compared to Population I stars? Or do they appear as subdwarfs only in a color–magnitude diagram or a spectral type–absolute visual magnitude diagram? Early semi-empirical studies (Eggen and Sandage 1962; Wildey, Burbidge, Sandage, and Burbidge 1962) suggested that, at least in the case of mild subdwarfs, they do not differ from Population I dwarfs in the color–magnitude diagram after corrections for differential line blanketing have been applied. In other words, they do not differ in the theoretical H–R diagram ($\log T_{\text{eff}}$ vs $M_{\text{bol}}$). Recent studies by Strom and Strom (1967) and by Cayrel (1968) based on model stellar atmospheres suggest that subdwarfs do indeed fall below the Population I main sequence in the theoretical H–R diagram.

In order to compare this information with stellar interior models, one needs information about the mass-luminosity law and the helium content of subdwarfs. At present, 85 Peg, a mild subdwarf, seems to be the only available candidate for a mass determination. The helium content cannot be measured directly since helium lines do not appear in the spectra of late-type stars, at least in the visual range. These problems, serious as they are, are rendered more serious by our inability to calculate theoretically accurate radii for late-type stars. Predictions based on the mixing length theory are very uncertain, and although there is hope that stellar hydrodynamists will eventually solve this difficult problem, there is a very real need for direct measurement of radii of late-type stars. To quote Eddington: “it is easy to perceive the inevitability of a conclusion when one is already persuaded by experiment of its truth.” The need to measure stellar radii is urgent and one should seriously consider the construction of a large-scale Michelson interferometer as recently proposed by Miller (1966). A better understanding of hydrogen convection zones could not only improve our age determinations, but in all likelihood
help bring about a solution of the lithium problem and that of the strength of Ca II emission lines.

At present, it seems reasonable to be guided by our knowledge of the solar radius and that of Population I stars about which we feel a little more confident, and to assume a ratio of mixing length to pressure scale height, \( l/H \), of the order of unity. This procedure should at least yield reasonable positions of the main sequence for different compositions. From a recent paper (Demarque 1967b), one can show the effect of variations in the ratio \( l/H \) on the position of the main sequence and on an evolutionary track. Some results are indicated in Figs. 9 and 10. Effects of uncertainties in opacities and in the treatment of the convection zone (Demarque, Hartwick, and Naylor 1968), are shown in Fig. 11.

Under the assumptions described above, the work of Strom and Strom (1967) and of Cayrel (1968) suggest a value of the helium content by mass \( Y \) between 0.25-0.30 in line with the primordial helium content found in theories of the primordial fireball (Peebles 1966; Wagoner, Fowler, and Hoyle, 1967).

The evolution of Population II stars along the giant branch, for the first time investigated in detail by Hoyle and Schwarzschild (1955) more than a decade ago, follows a now familiar course. An inert helium core forms, which increases in mass, decreases in radius and heats up as the star moves up the giant branch. The increased density leads to degeneracy of the electron gas in the core until at high luminosity the electrons contribute most of the pressure. Mestel (1952) has discussed in general the possibility of catastrophic events when nuclear burning starts in a degenerate region. In the core of red giants of low mass, helium begins to burn effectively around 80 million degrees. Since in a degenerate electron gas the pressure is practically independent of temperature, the increase in temperature due to the nuclear energy output does not lead to an expansion but only accelerates the rate of nuclear burning. A thermal runaway follows. Schwarzschild and Härm (1962, 1964) have shown in successive studies of this helium flash that the temperature ceases to increase only when the electron degeneracy is removed and a convective helium-burning core is formed. In the meanwhile, the temperature reaches 400 million degrees at the center of the star, the core luminosity is \( 10^{44}L_\odot \), the time scale of evolution of the order 0.1 sec, the surface of the star remaining practically unaffected. One of the first questions one asks in these circumstances is of course: Does the star mix completely at this stage of evolution? Although Schwarzschild and Härm's answer is no, the approximations that had to be introduced in their models renders this answer highly uncertain. More recent work by Thomas (1967) indicates that the flash could well occur in the outer part of the helium core instead of the center, as found by Schwarzschild and Härm because of their assumption of core isothermality, in particular if one introduces energy losses due to neutrino production. Eggleton's (1966) very interesting study on the effect of chemical composition on the development of the helium flash, which indicates a tendency toward an outer helium flash for increasing helium and metal abundance leads one to speculate that metal-rich stars may not follow the same unmixed course as Population II stars appear to do. Observationally, it is still unclear whether galactic clusters have horizontal branches in the sense of globular clusters.

![Fig. 7. Evolutionary tracks for stars with \( X = 0.67 \) and \( Z = 0.03 \), together with the locus for NGC 188 given by Sandage (1962). Ages in units of \( 10^8 \) yr are indicated along the tracks. (After Demarque, P. R., and Larson, R. B., Astrophys. J. 140, 547, 1964.)](image)

![Fig. 8. Evolutionary tracks illustrating the effect of the hydrogen abundance \( X \) on the shape of the evolutionary track. The lower the value of \( X \), the sooner the hydrogen exhaustion phase occurs. The track with \( X = 0.51 \) is due to B. M. Schlesinger.](image)
That complete mixing does not occur in globular cluster stars at the time of the helium flash [perhaps one should say the first helium flash, since there may be several successive flashes (Sugimoto and Yamamoto 1966)] can be verified easily in a much less arduous way than working through the helium flash. Let us assume that complete mixing occurs. The star then jumps discontinuously to the left of the H-R diagram, to the position appropriate for a helium-rich main sequence star. Larson (1965) has followed the evolution of such a star \( (1.0M_\odot, X=0.25, Z=0.001) \). The result is shown in Fig. 12. It is clear that both in shape and time scale this track does not describe the horizontal branch of globular clusters. One star in M13, Barnard 29, which appears to be helium rich and is located well above the horizontal branch of the cluster, may be on such a Larson track. It is marked on Fig. 12. Sandage (1953) has also reported a similar object in M3 and Arp (1958) found a number of possible candidates in several globular clusters. Partial mixing, however, cannot be ruled out, although recent models for horizontal branch stars constructed under the assumption of no mixing by Faulkner (1966) and by Faulkner and Iben (1967) seem to give a very satisfactory description of the observations.

Turning back to the helium problem, we note that by lumping all field subdwarfs in one group, we have tacitly assumed that they all have the same chemical composition. Since, in fact, we know that they differ in their metal contents by large factors, we may reasonably expect some variations in their helium contents. In the case of the globular clusters, we encounter some difficulties in determining the distance modulus and in fitting the main sequence. Two main methods have been extensively used in this context. In the fifties the universal value of \( M_v=0 \) for the RR Lyrae stars was usually adopted and whatever their chemical composition, globular clusters were assigned a distance modulus in accordance with this assumption. The respective main sequences of each cluster were then staggered as illustrated in Fig. 13. As it became clear that the absolute magnitude of the RR Lyrae stars is a function of their population characteristics, interest shifted to main sequence stars. As mentioned earlier, the work of Melbourne (1960), Eggen and Sandage (1962), and Wildey, Burbidge, Sandage, and Burbidge (1962) indicating that mild Population II stars superimpose on the Population I zero-age main sequence after due corrections for interstellar reddening and ultraviolet excess have been made, leads to a fitting of main sequences as described in Fig. 14. The recent research of Faulkner (1967), Strom and Strom (1967), and Cayrel (1968) show that this procedure is also likely to be incorrect. However, if one uses Sandage’s (1964) latest data on photoelectric observations of globular clusters, together with Sturch’s (1967) measurements of the reddening of RR Lyrae stars in the same clusters, it is interesting to note that one obtains ages of the order of \( 10^9 \) yr, in satisfactory agreement with the perhaps naïve estimate of the age of the universe given by the Hubble time.

It seems that better hope comes from a consideration of the horizontal branch members, which are relatively brighter and much easier to observe in detail. Furthermore, the presence of the RR Lyrae variables should yield more information on these stars. Christy’s
(1966) calculations indicate that the variables in M3 behave in the same way as envelopes calculated with approximately 30% helium. The same calculations also indicate that the range in bolometric magnitudes of the cluster variables is of the order of 0.3 mag, and nearly independent of helium content (only colors appear to be helium dependent). If one applies Christy's luminosity criterion to the clusters observed by Sandage, one then finds that the main sequences of the clusters are staggered due to variations in both the metal contents (approximately known) and their helium contents (which turn out less than 0.1 in Y). The suggestion arises that small variations, of a few percent, occur in the helium content of the globular clusters. Recently, Sandage and Wildey (1967) have interpreted the peculiarities of the color-magnitude diagram of NGC 7006 as due to helium deficiency with respect to M3. They find that using Faulkner's (1966) interior models for horizontal branch stars as a guide, it is possible to interpret NGC 7006 in terms of a small helium deficiency. This explanation should, however, still be considered as very tentative.

V. CONCLUSIONS

In summary, one may draw a few tentative conclusions:

(1) It appears that there is evidence, both observational and theoretical, that the old galactic clusters NGC 188 and M67 are richer in heavy element content than the sun. In particular, in order to explain the observed gap above the main sequence in NGC 188, one must set Z=0.06-0.07. The age of the cluster is then about 6×10^9 yr.

The presence of stars with such a high Z could raise questions on the calibration of the absolute magnitudes of Population I variable stars.

(2) There is one evidence from old stars in the galaxy that the helium content is nearly uniform in a first approximation, with Y=0.25-0.30 as a minimum, seemingly in agreement with primordial fireball predictions. It seems then possible to date the oldest globular star clusters within the presently accepted estimate of the Hubble time, with ages of the order 10 billion years. Horizontal branch stars would then have started on the main sequence with masses near 0.8\(M_\odot\).

(3) There is, however, an indication that, in a second approximation, the helium abundance may differ from cluster to cluster, the range in Y being less than 10%.
The outer clusters (such as NGC 7006) would then represent the primordial helium abundance, suggesting that some regions of the Galaxy have gone through a process of helium enrichment. This helium might have been produced in massive stars or perhaps in some kind of supermassive object or objects.

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