to spectra in the photographic infrared at the low dispersion used in this study. Thus we cannot determine if the very red stars are giants and/or supergiants, although Nassau, Blanco, and Morgan (1954) and Blanco and Nassau (1957) suggested that many of the stars with wedge-shaped, early M-type spectra (such as RM-WS 70, H-C 1 and H-C 6) are probably supergiants. This expectation has been confirmed by Sharpless (1966), who found that between 60 and 70 per cent of the stars with this type of spectrum are M supergiants.

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VIRIAL RELATIONS FOR UNIFORMLY ROTATING FLUID MASSES IN GENERAL RELATIVITY*

In a recent paper Boyer (1966) has reduced the equation governing the hydrostatic equilibrium of a uniformly rotating fluid mass in general relativity to a form which allows of immediate integration in the case the specific entropy is a constant throughout. Boyer’s form of the equation of hydrostatic equilibrium (in the more general case when the specific entropy is not necessarily a constant) leads directly to a class of moment relations (“virial theorems”) which can be used as checks, or as constraints, in the solution of the equations of hydrostatic equilibrium by a variational procedure as proposed by Hartle and Sharp (1965, 1967).

For axially symmetric systems, the metric can be reduced to the form

\[ ds^2 = g_{00}dx_0^2 + g_{11}dx_1^2 + g_{22}dx_2^2 + g_{33}(d\phi)^2 + 2g_{03}dx_0d\phi , \]

where the metric coefficients are functions of \( x_1 \) and \( x_2 \) only, and \( \phi (= x_3) \) plays the role of the “azimuthal angle.” Also, for the case of uniform rotation with an angular velocity \( \Omega \), the four-velocity \( u^i \) must be of the form

\[ u^i = (a, 0, 0, \beta) \quad \text{and} \quad \beta/a = \Omega/c = \text{constant} , \]

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where \( a \) and \( \beta \) can, separately, be functions of \( x_1 \) and \( x_2 \) (though \( \beta/a \) must be a constant). And we assume that the fluid is described, as usual, by the energy-momentum tensor

\[
T^{ij} = (\epsilon + p) u^i u^j - pg^{ij},
\]

where \( p \), the isotropic pressure, and \( \epsilon \), the energy-density (including that due to the rest mass), are both independent of \( x_0 \) and \( \phi \). Under these circumstances, Boyer has shown that the conditions

\[
T^{ij}_{\mu} = 0
\]

reduce to the pair of equations

\[
\frac{1}{2} (\epsilon + p) \frac{\partial}{\partial x_{\mu}} \log \left( g_{00} + \frac{2\Omega}{c} g_{03} + \frac{\Omega^2}{c^2} g_{33} \right) = -\frac{\partial p}{\partial x_{\mu}} \quad (\mu = 1, 2).
\]

Multiplying the equations for \( \mu = 1 \) and \( 2 \) by \( x_{1m} x_{2m}^{-1} \) and \( x_{1m-1} x_{2m} \), respectively, and integrating over the spatial volume of the fluid, we obtain

\[
\frac{1}{m_1} \int_v (\epsilon + p) x_{1m} x_{2m-1} \frac{\partial}{\partial x_{1}} \log \left( g_{00} + \frac{2\Omega}{c} g_{03} + \frac{\Omega^2}{c^2} g_{33} \right) dx_1 dx_2 = \frac{1}{m_2} \int_v (\epsilon + p) x_{1m-1} x_{2m} \frac{\partial}{\partial x_{2}} \log \left( g_{00} + \frac{2\Omega}{c} g_{03} + \frac{\Omega^2}{c^2} g_{33} \right) dx_1 dx_2
\]

\[
= 2 \int_v px_{1m-1} x_{2m}^{-1} dx_1 dx_2,
\]

where we have assumed that \( m_1 \) and \( m_2 \) are such that all the integrals converge and are arbitrary otherwise.

If the coordinates \( x_1, x_2, \) and \( \phi \) play the roles of the cylindrical polar coordinates, \( \varpi, z, \) and \( \phi, \) at infinity, then with the choices \( m_1 = 2 \) and \( m_2 = 1, \) equations (6), will, in the Newtonian limit, reduce to the relations provided by the second-order tensor virial theorem.

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October 30, 1966

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NON-AXISYMMETRIC DIFFERENTIAL MOTIONS IN GALAXIES HAVING AXISYMMETRIC MASS DISTRIBUTIONS

In the many investigations in stellar dynamics which deal with systems having axisymmetric, time-independent mass distributions, the only differential motion commonly considered is rotation. In this letter, we shall consider non-axisymmetric differential motions in such systems. The tendency to overlook or ignore these more general motions in the past has probably resulted from the prevalence of an intuitive feeling that departures from pure rotation could not occur except in conjunction with departures of the density from axisymmetry or time independence.

Consider a self-gravitating stellar system in which the stars are distributed in an inhomogeneous, axisymmetric disk of infinitesimal thickness and surface density \( \sigma(\varpi), \)