COMPUTER CORRECTION OF LINE PROFILES FOR INSTRUMENTAL ERROR

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A Fourier-transformation method for removing instrumental-profile errors is discussed, briefly. In this approximation method the observed spectrum is distorted (smearred) by the instrumental profile of the observing equipment. To study the distinctive features of the method, an artificial (idealized) spectral region is distorted by a given instrumental profile, and the inverse problem is then solved. Two approximations are needed to correct narrow solar lines to 0.5% accuracy for an instrumental profile 12 mÅ wide. As an illustration, artificial noise is superposed on the spectrum and the distortions are analyzed through successive approximations in the correction process.

The need to apply electronic digital computers to correct for the apparatus-function error—the instrumental profile—has arisen in connection with the problem of making adjustments in unfairly extensive portions of the solar spectrum having blended absorption lines. It is worth noting that the development of universal methods of correcting for this error over long spectral regions in which the intensity distribution cannot be represented analytically is of great importance also in fields other than solar research. The instrumental profile may have to be eliminated from a fairly large block of observational data in order to set up various physical measurements and to solve a variety of problems in optics, radio astronomy, and astrophysics.

1. The effect of the instrumental function \( h(y) \) in "smearing" the signal \( f(x) \) is described in the general case by the well known convolution-type equation

\[
g(x) = \int_{-\infty}^{\infty} f(x-y)h(y)dy,
\]

where \( g(x) \) is the observed signal, and the instrumental function should be normalized by the relation

\[
\int_{-\infty}^{\infty} h(y)dy = 1.
\]

Many authors have considered possible methods for finding the true value of \( f(x) \) from the observed quantity \( g(x) \). Fairly complete information on these investigations together with a bibliography has been given by Raftian [1], and a paper by Petrish [2] has also appeared recently.

The most general variant of a solution to our problem, in the case where the function \( g(x) \) cannot be represented by a convenient analytic formula, would be a solution by means of a Fourier transformation. In brief, the problem reduces to the following. If we denote the amplitudes of the Fourier transformations of the functions \( g(x) \), \( f(x) \), \( h(x) \) by \( G(\omega) \), \( F(\omega) \), \( H(\omega) \) respectively, then in view of the relation (1) we have (see, for example, [3]) \( G(\omega) = F(\omega)H(\omega) \). Hence

\[
f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{-i\omega x}d\omega.
\]

Substituting \( G(\omega) = \int_{-\infty}^{\infty} g(z)e^{i\omega z}dz \) into Eq. (2), we obtain

\[
f(x) = \int_{-\infty}^{\infty} g(z)h_0(z-x)dz,
\]

where

\[
h_0(z-x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{-i\omega (z-x)}d\omega.
\]
limits. In they limits should be the values \( z_1 \), \( z_2 \), corresponding to the boundaries of the region over which the observed \( g(x) \) is to be corrected. The frequency \( \omega \) is limited by the value \( \omega_{\text{max}} \), which should be established on the basis of definite physical premises.

The inverse convolution problem, as is well known, is not always solved correctly. In Eq. (2) the ratio \( G(\omega) / H(\omega) \) may assume infinitely great values in the frequency range where \( H(\omega) \) approaches zero more rapidly than \( G(\omega) \). Roughly speaking, an exact correction is not possible for spectral lines narrower than the instrumental profile. In practice, however, this situation is virtually excluded in modern work on the solar spectrum. We can demonstrate this fact by a very simple example. The narrowest lines observed in the solar spectrum have a full halfwidth of about 50 mA. Consider an even narrower line, with a Doppler halfwidth \( \Delta \lambda_D = 15 \text{ mA} \). In high-order spectra obtained with a good grating the instrumental profile has a full halfwidth of 10–20 mA. For simplicity, we assume that the instrumental profile is also Doppler, with a halfwidth \( \Delta \lambda_D = 7 \text{ mA} \). For a Doppler profile \( G(\omega) = I_0 \sqrt{\pi \Delta \lambda_D} \exp \left(-\frac{\omega^2 \Delta \lambda_D^2}{4}\right) \), where \( I_0 \) is the central intensity or depth of the line. Correspondingly, for the normalized instrumental profile \( H(\omega) = \exp \left(-\frac{\omega^2 \Delta \lambda_D^2}{4}\right) \).

Thus

\[
\frac{G(\omega)}{H(\omega)} = I_0 \sqrt{\pi \Delta \lambda_D} \exp \left(-\frac{\omega^2 \Delta \lambda_D^2}{4}\right).
\]

We see that the greater the difference \( b^2 = \Delta \lambda_D^2 - \Delta \lambda_D^2 \), the more rapidly \( G(\omega)/H(\omega) \) approaches zero with increasing \( \omega \), and the smaller the value of \( \omega_{\text{max}} \) required to correct the line with prescribed accuracy. By Eq. (5), for our example the relation of type (2) will take the form

\[
f(\Delta \lambda) = \frac{I_0 \sqrt{\pi \Delta \lambda_D}}{\pi} \int_0^\infty e^{-b^2/4 \cos \omega \Delta \lambda_D} d\omega.
\]

At the line center, where the error of the correction is greatest, Eq. (6) may be written after some elementary transformations in the form

\[
k = \frac{2}{\pi} \left( \int_0^\infty e^{-b^2/4 \omega_{\text{max}}} \omega_{\text{max}} \right)
\]

If we take the precision of the correction as 0.5%, we have

\[
\omega_{\text{max}} = 4/b.
\]

so that \( \omega_{\text{max}} = 4/b \). In our example, \( b = 13 \text{ mA} \) and \( \omega_{\text{max}} = 0.30 \text{ mA}^{-1} \). At \( \omega = \omega_{\text{max}} \) the ratio \( G(\omega)/H(\omega) \) amounts to less than 0.02 of its maximum value and declines rapidly with increasing \( \omega \).

Figure 1 presents a graph of \( G(\omega) \) for a very narrow line, unrealistic for the solar spectrum, whose halfwidth \( \Delta \lambda_D = 10.8 \text{ mA} \). This line (see following text) was obtained through the effect of the instrumental profile for the solar spectrograph of the Central Astronomical Observatory, USSR Academy of Sciences, on a very narrow Doppler line of halfwidth \( \Delta \lambda_D = 8 \text{ mA} \). Also plotted in Fig. 1 are the \( H(\omega) \) for this instrumental profile, as approximated by a Voigt function with parameters \( \beta_1 = 7.4 \text{ mA} \) and \( \beta_2 = 9.8 \text{ mA} \), and the ratio \( G(\omega)/H(\omega) \). Evidently this line also can be corrected with the required precision, although in this event higher values of \( \omega_{\text{max}} \) would be required.

2. In the general case \( g(x) \) and \( h(y) \) cannot be represented by elementary analytic functions. Since the instrumental profile will experience hardly any change over the long spectral intervals \([x_1, x_2]\) within which corrections are to be made, the problem of determining \( f(x) \) for a number of points in \([x_1, x_2]\) will reduce in this case to solving Eq. (3) for limits of integration from \( x_1 \) to \( x_2 \). The values of \( b(x) \) (or \( x \)) are given by the integral (4), whose upper limit is \( x_{\text{max}} \), while \( H(\omega) = \int_\sigma^x h(y) e^{i\omega y} dy \), where \([x-a, x+b] \) is the interval within which the instrumental function is "effective" on both sides of a given point \( x \).

Quite apart from the supplementary computations needed to determine \( H(\omega) \) and \( h(x) \), the problem of determining \( f(x) \) at one of the points of the block \([x_1, x_2]\) covering the spectrum from 2000 to 3000 A every 2–3 mA would require some \( 10^{8} \) operations on an electronic computer, and \( 10^{12} \) operations would be needed to correct for the whole interval. Evidently it is practically impossible to solve the problem in this variant unless one can manage in some way to unify the series of operations associated with the solution of Eq. (3), or can find some approximations of the expressions in the integrands of Eqs. (1)–(4) such that the computation process can...
be substantially shortened. The fairly well developed method of [4] for making the correction by means of tables of nomographs, with the observed line profile approximated by a Voigt function, can be applied only for isolated symmetrical lines and under conditions where the instrumental function may also be approximated by a Voigt function.

Some of the possible ways of solving the problem by transforming the expressions in the integrands have been considered in [1, 5, 6]. It appears to us that the most effective procedures for solving the problem are those where the desired quantity may be replaced by the observed quantity. For example, if we set $h(y) = \delta(y) + \varepsilon(y)$, where $\varepsilon(y)$ is a small correction to the $\delta$-function, then for the case of small $\varepsilon(y)$ the problem will reduce to a solution of an equation of the type

$$g(x) - f_1(x) = \int_{-\infty}^{\infty} g(x-y)\varepsilon(y)dy,$$  \hspace{1cm} (7)

in which $f_1(x)$ is an approximation to the desired function $f$. This treatment would seem to be efficient for a profile having a steep core and very weak short wings.

In arbitrary cases of approximation, however, certain errors are introduced and the asymmetry caused by the instrumental function is not taken into account. Errors of the latter type would in general be inadmissible in precise investigations of the displacements and asymmetry of spectral lines.

In view of these circumstances we have made an attempt to develop a method in which $h(y)$ would be utilized without any distortions or smoothings and in which the number of computational operations would be acceptable for evaluation on an electronic computer.

Assuming that a recording was being reduced of a spectrum obtained with a sufficiently narrow instrumental profile, that is, that $g(x) \approx f(x)$, we have preferred to solve the problem by approximations. Of all the procedures tried, the simplest method and a very efficient one was found to be the method of "double smearing" of the function $f(x)$. Performing a repeated convolution of the function $f(x)$, we have

$$g_1(x) = \int_{-\infty}^{\infty} g(x-y)h(y)dy.$$  \hspace{1cm} (8)

If the "action" of the operator $h(y)$ is insignificant, $g(x)$ will differ from $f(x)$ by a quantity approximately twice as great as the difference $g(x) - f(x)$ (Fig. 2), that is, $g_1(x) - f(x) = 2[g(x) - f(x)]$, so that the first approximation becomes

$$f_1(x) = 2g(x) - g_1(x).$$  \hspace{1cm} (9)

\footnote{While this paper was being prepared for publication we received a reprint of a paper by de Jager [7] in which a number of carbon lines in the solar spectrum are corrected accurately by this same method. De Jager points out that the fundamental principles of the method were worked out in 1932 by van Cittert [8].
The second and higher approximations may then be formulated as

$$g_n(x) = \int_{-a}^{b} f_{n-1}(x - y) h(y) dy,$$

$$f_n = f_{n-1} + g - g_n.$$  \hspace{1cm} (10)

The method is an extremely simple one and is particularly convenient for programming the problem on an electronic computer. The operations amount to a solution of the convolution integral, enabling \(g_1, \ldots, g_n\) to be determined with any required accuracy by ordinary summation:

$$g_n(x) = \Delta x \sum_{i=-m}^{m} f_{n-1}(x + i\Delta x) h(i\Delta x).$$  \hspace{1cm} (11)

Here \(\Delta x\) is the interval between adjacent values of the numerical quantities \(f_{n-1}(x)\). The sum (11) contains several hundred terms; that is, the number of operations is three orders of magnitude smaller than for solving an equation of the type (3).

3. The first trial of the method was carried out in the summer of 1964. Corrections were made for the short section of the solar spectrum \(\lambda 5377-5384\) \(\AA\), as recorded in the fifth order by B. Babii and V. Karpinskii with the double-diffraction monochromator of the spectrograph at the Central Astronomical Observatory, USSR Academy of Sciences [9, 10]. The core of the instrumental profile (Fig. 3) has a halfwidth of 11.8 mA, while the wings are quite extensive and stretch to 4380 mA from the center.

A computation of five approximations at an interval \(\Delta x = 2\) mA was performed on the M-20 digital computer of the Cybernetics Institute of the Ukrainian Academy of Sciences. R. Kostik and A. Gavyanina were responsible for programming the problem.

Figure 2 shows the corrected spectral region for one of the narrowest lines in the solar spectrum, \(\lambda = 5379.58\) \(\AA\) (Fe), which had previously been corrected manually by B. Babii by means of Voigt functions. The third approximation is not shown in the graph since it practically coincides with the second approximation. Only in the central part of the line do the second and third approximations differ by the amount \(= 0.5\%\). The dots in the figure represent the corrections derived manually by Babii from the Voigt functions.

The following point merits attention. Approximating the observed line and instrumental profile by a Voigt function provides an accuracy which in this case only negligibly exceeds the accuracy of the first approximation. The error comes from the fact that the approximate instrumental profile (Fig. 3) transfers a comparatively small amount of energy from the regions remote from the central point; that is, it has a more massive core and more gentle (weaker) wings.

In this case we could not test whether the instrumental profile introduced appreciable asymmetry, since the spectral line itself could have been asymmetric. In order to check on this effect, and also to estimate the accuracy of the method more carefully, we decided to use the instrumental profile to "smear" an artificially constructed portion of the spectrum having absorption lines, and then to deal with the correction problem. In Fig. 4 this portion of the spectrum is shown by the solid curve 3. Proceeding from left to right, lines with the following parameters have been constructed: I) \(\Delta \lambda_D = 30\) mA, \(r_D = 0.30\); II) \(\Delta \lambda_D = 150\) mA, \(r_C = 0.20\); III) \(\Delta \lambda_D = 40\) mA, \(r_C = 0.30\); IV) \(\Delta \lambda_D = 20\) mA, \(r_C = 0.60\); V) \(\Delta \lambda_D = 8\) mA, \(r_C = 0.60\). Lines II and III are blended.
It was pointless to consider broader lines than line II since they would suffer hardly any effects from the instrumental profile. Line IV is very narrow. Considering their width, lines I and III are very deep. Evidently such lines are not encountered in the solar spectrum. For testing purposes we have taken one other line known to be "unrealistic" for the solar spectrum; its halfwidth is approximately equal to the halfwidth of the core of the instrumental profile. In Fig. 1 we have already exhibited the Fourier spectrum of this line, as well as the instrumental profile.

The convolution of this portion of the spectrum with the instrumental function is shown in Fig. 4 by the solid curve 1. We observe that a considerable amount of the energy from the continuum is transferred into the central parts of the lines, especially in places where the gaps between adjacent lines are small. In many cases this effect may lead to large errors in the behavior of the continuum background. The central intensities in narrow, deep lines may be too high by 20%. These distortions of the true intensity distribution in the spectrum appear to be unexpectedly large in view of the fact that the instrumental profile considered is very narrow. In this case there is undoubtedly some influence from the weak, distinct wings, which frequently are not taken into account at all by observers.

The instrumental profile also introduces a perceptible displacement and asymmetry into the lines. On a magnified scale, Fig. 5a shows the axis of symmetry of the ideal line I (straight line), and the axis of symmetry of line I as distorted by the instrumental profile. The latter was found from the midpoints of the chords joining points of the line profile having the same intensity. Evidently the amount of displacement, which reaches 1 mA, changes, thereby rendering the line asymmetric.

The observed displacement effect toward the red arises from the fact that the total amount of energy transferred by the blue half of the instrumental profile is greater than the energy transferred by the red half of the instrumental profile into a given point of the spectral line. In the present case the "blue" half of the area of the normalized profile is $451 \cdot 10^{-3}$, and the "red" half is $446 \cdot 10^{-3}$. The difference amounts to only $5 \cdot 10^{-3}$ of the whole area of the profile, and yet this difference has an appreciable effect. Moreover, as Fig. 3 shows, the core of the instrumental profile actually has an asymmetry of opposite character: the "red" half of the core has a greater area than the "blue" half. This circumstance once more indicates the preponderant
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Figure 5. A) Asymmetry and displacement of the ideal line I due to the influence of the instrumental profile; B) noise of intensity 0.5% in the wing of line III, and its distortion after the fifth approximation.

In order to estimate how strongly noise is magnified in the correction process, we have introduced into the spectrum as distorted by the instrumental function several artificial types of interference in the form of triangular peaks and dips with amplitudes of 0.5% and 1% and half-periods of 4-12 mA. Here we have been guided by the actual maximum noise effects observed in tracings of the solar spectrum. Some of these effects were introduced into the wing of the III (Fig. 4, region B), others into regions a, b, c, d, which are far away from absorption lines.

For the correction process, five approximations were programmed. The first approximation is shown in Fig. 4 by a broken curve. The next four approximations and the noise are not plotted in Fig. 4 because of the small scale. For the cores of the four narrowest lines in the region corrected, these approximations are illustrated in Fig. 6 on a large scale. Line III is plotted together with a region where because of blending with line II there is a fairly sharp intensity peak. The true (undistorted) portions of the lines are drawn with broken curves. For line III the first through fourth approximations are shown for lines I and IV, all five approximations and for line V, the first, third, and fifth approximations.

We see that the correction of fairly broad lines of the type of line II to an accuracy of 0.5% requires only a single approximation. Lines such as I, III, and IV can be corrected to this accuracy only after three or four approximations. We emphasize that these lines are "limiting" examples of lines in the solar spectrum, in terms of the difficulty of correcting them. Hence, we may consider that a 0.5% accuracy in correcting a tracing of the solar spectrum recorded with equipment having an instrumental profile of the type considered in this paper would require two approximations. The same accuracy is reached in two approximations for the actual $\lambda = 5379.6$ A line we have corrected, with its 40-mA halfwidth (Fig. 2). We are assuming here, of course, that the errors in determining the instrumental profile are negligible.

The noise is shown on a magnified scale in Figs. 5B and 7. The dashed lines represent the spectrum undistorted by noise for regions a, b, c, d, B respectively. The dot-dash lines represent artificial noise with the parameters listed in Table 1. For regions a, b, c, d, the solid lines represent the noise as distorted after the first, third, and fifth approximations. For the noise in region B, in the wing of line III, it was convenient to take only the fifth approximation.

The following properties of the data exhibited in Figs. 7 and 5B should be noted.

The magnification ("dispersal") of noise during the approximation process is greater when the extent (period) of the noise is small.

At points where noise sets in the approximations yield auxiliary peaks directed opposite to the original noise. This effect, for the case where very narrow and faint lines are to be corrected—lines comparable in magnitude with large occurrences of noise—would enable actual lines to be distinguished from noise.

The noises B and b have the same parameters. Evidently the amount of "dispersal" of noise located in regions with a steep intensity gradient does not differ perceptibly from the same quantity for noise in the continuum.

For noise of given extent the range in the approximation process is proportional to the am-
Fig. 6. Correction of lines I, III, IV, and V by successive approximations. The dashed line shows the undistorted spectrum.

Fig. 7. Noise (dot-dash lines) and its distortion after the first, third, and fifth approximations (solid lines).

Amplitude of the noise. If two approximations are considered necessary to correct for a given instrumental profile, then for a 0.5% accuracy in the final result a precision of at least 0.2% would be needed in the observations. This degree of precision is fully attainable with modern photoelectric methods by averaging several recordings. One should also note that at the bottom of the lines the noise is magnified to a lesser degree in the correction process, since the "dispersal" of the noise is proportional to the amount of energy that has been extracted by the wings of the instrumental profile from the adjoining regions of the spectrum.

The severity of the requirements imposed on the observational precision, as well as the number of approximations needed to correct the spectrum with prescribed accuracy, will be higher or lower depending on whether there is an increase or de-

<table>
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<th>Designation of noise</th>
<th>Amplitude, %</th>
<th>Extent, mA</th>
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<tbody>
<tr>
<td>a</td>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>b</td>
<td>0.5</td>
<td>8</td>
</tr>
<tr>
<td>B</td>
<td>0.5</td>
<td>8</td>
</tr>
<tr>
<td>c</td>
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<tr>
<td>d</td>
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crease in the width of the instrumental profile and
the extent of its wings.

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