ON THE HEIGHT OF FORMATION OF H-ALPHA IN
THE SOLAR CHROMOSPHERE

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ABSTRACT

Several features of Hα limb spectra, including the line emission beyond the continuum limb, are analyzed to yield information about the height of formation of Hα in the solar chromosphere. It is concluded that de Jager’s suggested value for the height of unit optical depth at the line center (5000 km) is too high. For the chromospheric features in these spectra the height of formation lies in a range from 1500 km (above a sunspot) to 2700 km. This is consistent with the limb profile analysis which yields a range from 1600 to 3000 km for a hypothetical mean chromosphere.

I. INTRODUCTION

Since high-resolution Hα limb spectra give information about the height scales of chromospheric structures, they can also be used to estimate a mean height of formation for Hα.

Mattig (1962) has obtained spectra for the first four Balmer lines in which the slit is perpendicular to the limb and crosses a sunspot close to the limb. In his interpretation of the Hα spectra, Mattig assumes a knowledge of the structure of Hα in the undisturbed chromosphere and derives information about the chromosphere above a sunspot from his observations.

De Jager (1959) has given two values for h, the height at which radial optical depth at the center of Hα is unity; from eclipse observations, spectroheliograms, and model calculations h = 5000 km, and from Athay and Thomas’ (1958) theoretical approach h = 3500 km. Mattig accepts the former value without comment and, finding that above the sunspot h = 2160 km, deduces various properties about the chromosphere in this region. In his original paper, de Jager (1957) emphasizes the large difference in A-values computed from different models and the real physical difference in h for different solar structures.

We find the use of h = 5000 km questionable for the undisturbed chromosphere. Therefore, in this analysis of limb spectra, we prefer to use the evidence of disk features near the solar limb, together with the line emission beyond the continuum limb, to provide direct information on the height of formation of Hα in the various regions of the chromosphere.

At the Harvard-Smithsonian conference on the formation of spectral lines, Giovanelli (1965) attacked methods of analysis of chromospheric structure which ignore the presence of inhomogeneities and which obtain mean values in the chromosphere by treating its properties as depth-dependent only.

There is some merit in this complaint. In an analysis of the effect of horizontal structure on photospheric mean values, Wilson (1965b) found that, because of the non-linear relationship between source function and absorption coefficient, neglect of these horizontal fluctuations introduces errors of up to 10 per cent in photospheric mean values.

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Observations clearly show the chromosphere as an inhomogeneous layer where similar non-linear effects of the horizontal structure introduce uncertainty into any interpretation of mean chromospheric quantities such as smooth limb-darkening curves, a mean source function, and a mean Doppler width. However, our current knowledge of the structure of the chromosphere in terms of height in kilometers above the photosphere is so uncertain that there is considerable need for attempts to improve chromospheric mean values—if only to provide some framework on which to build a more detailed account of the structures.

Ideally the problem should be solved by computing line profiles using solutions of the transfer equation in inhomogeneous atmospheres. Currently attempts are being made to solve simple problems of this type, but the difficulties involved in these solutions are such that it seems unlikely that they can be applied to the real chromosphere for some considerable time. The present analysis makes some use of the concept of a mean chromosphere, but we recognize that the heights may be different in different chromospheric structures, and we attempt to analyze these structures separately.

II. OBSERVATIONS

a) Observing Technique

On August 2, 1964 (14:27 U.T.), a series of Hα spectrograms and filtergrams were taken at Sacramento Peak Observatory ("SPO") that show a sunspot region (SPO No. 77) and the chromospheric structure near the northwest limb of the Sun. The spectra were made with the slit placed across the P spot of the group and extending beyond the solar limb. The slit departs from the direction of the solar radius by only 3°, so the correction for a radial height scale is negligible. The half-width of the seeing function is 1000–1300 km as estimated from the shapes of the solar-limb profiles.

The observations were made with a 12-inch objective fed by a coelostat, with an image scale of 5384 km/mm. The spectra were taken in the second order of a 1200-line/mm Bausch and Lomb replica grating in a 13-m Littrow spectrograph. The spectral dispersion was 5.01 mm/A. Each spectrogram was accompanied by two on-band Hα filtergrams: (1) simultaneously with the spectrograph exposure (3.5 sec) the reflecting slit jaws of the spectrograph were photographed through an Hα Lyot (OPL 0.75 Â band pass) filter, and (2) 30 sec later the beam was automatically shifted to a separate camera system that photographed the same solar region through a Halle filter (0.5 Â band pass) with better spectral purity and a larger field than the slit-jaw camera. The exposure times were 4 sec for the slit-jaw camera and 1 sec for the Halle filter camera. During the spectrograph exposures, image motion perpendicular to the solar limb was reduced by using a servo-driven rocking plane-parallel prism to hold the limb steady. The error signal for the prism servo mechanism was generated by the imbalance between two solar cells placed in the image plane along the spectrograph slit (one cell crossed by the solar limb; the other cell located on the disk just inside the limb and partially masked).

b) Reduction of Observations

The combination of the slit-jaw filtergrams with the Halle camera filtergrams allowed us to find the position of the spectrograph slit in the chromospheric-mottling field, and thus establish the correspondence between features in the spectrograms and filtergrams. Figure 1 is a mosaic of a white-light sunspot and granulation photograph, an Hα spectrogram, and the corresponding Hα on-band filtergram of the sunspot region. The heights of the chromospheric and photospheric boundaries are indicated (CB and PB). In addition, the positions of the sunspot (S) and three chromospheric structures (i, ii, and iii) are marked. The intersections of the indicated slit line and the horizontal height positions in the filtergram mark the chromospheric mottles corresponding to the arched structures in the spectra.
The observation of chromospheric features arched toward the solar limb has been previously reported by McMath, Mohler, Pierce, and Goldberg (1956), Prokofyena (1957), and Simon and White (1966). Similar arch structures in sunspot spectra have been discussed by Severny and Bumba (1958) and Mattig (1959, 1962). The fundamental difference between the photospheric arched features (sunspots) and the chromospheric arches is that the sunspots appear as an absorption band crossing the Hα line, whereas the chromospheric arches appear as fine emission bands in the spectral line. In the original negatives the emission arches are very conspicuous out to $\pm 0.5$ Å from the line center, but their association with bright features in the continuum is not well established at this time. In prints of the spectra such as in Figure 1, diffuse absorption features are more visible on the limb side of the emission arches. It is not clear whether these absorption features are upward extensions of the bright solar features producing the emission arches or only due to the visual appearance of closely spaced emission bands superposed on a dark background.

Comparison of the spectra with the on-band Hα filtergrams shows that the bright chromospheric arches occur at points where the slit crosses the bright Hα flocculi outlining supergranulation cells (see Beckers [1964] for a description of characteristic chromospheric line structures). In this context the arches correspond to the bright emission features at the bases of spicule bushes (Cragg, Howard, and Zirin 1963) or bases of rosettes (Beckers 1964). We take the emission arches to be the spectroscopic picture of

![Figure 2](image-url)

**Fig. 2**—A computer plot of reduced microphotometer traces made along the slit at several wavelengths in the red side of Hα. The positions of the sunspot and the three spectroscopic arches (i, ii, iii) are indicated in the plot. The zero points of successive scans are offset by 0.05 of the continuum intensity at $u = 1$. 

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Fig. 1.—A mosaic of three observations showing the limb region as it appeared in white light (A), an Hα spectrogram with the slit across the sunspot and chromospheric structures (B), and an on-band (0.5-A bandpass) Hα filtergram showing the chromospheric mottling and the position of the spectrograph slit (C). The height positions CB and PB refer to the chromospheric boundary as defined by the emission beyond the limb in Hα and to the photospheric boundary defined by the continuum limb in the spectrogram. S marks the position of the sunspot in the continuum. In the spectrogram the three positions i, ii, and iii mark the continuum positions of three spectroscopic arches. The white-light sunspot photograph was taken from the SPO sunspot patrol courtesy of P. M. McIntosh.
upward-projecting structures. As such, the geometry of the arches gives information on the vertical height structure of the supergranulation boundaries in the low chromosphere.

Two spectrograms were reduced on the SPO microphotometer by making scans along the slit at many wavelengths in Hα (see Fig. 2). Positions of conspicuous emission arches were marked on the tracings as the plates were scanned. The positions of these emission features were then measured relative to the solar limb in the continuum. Assuming that the structures project radially out of the solar surface, we then computed the maximum vertical heights (measured at line center) given in Table 1 (see below). In addition, a photometric calibration was made from a standard wedge, and the intensity scale relative to the continuum intensity at the center of the solar disk was fixed by fitting to photoelectric measurements at μ = 0.2 and 0.3 (White 1962). Figure 2 shows reduced tracings at a few wavelengths on the red side of Hα from one plate. Positions of the sunspot and three chromospheric arches are indicated at each wavelength.

III. ANALYSES OF OBSERVATIONS

a) Limb Profiles

The most direct evidence about the structure of Hα in the chromosphere comes from the intensity profile beyond the continuum limb (see Fig. 2). Although detailed analysis in this region is difficult, because of the smearing of structures along the line of sight, such profiles are useful in giving approximate, averaged values of chromospheric quantities because the horizontal scale is simply the height above the photosphere.

Recently, Wilson (1966; hereinafter cited as “Paper I”) has calculated intensity profiles across the limb for a strong line (such as Hα) for which the absorption coefficient at line center is given by the simple exponential model

\[ k(z) = a e^{-kz}, \]

where \( z \) is the height above the photosphere and \( a \) and \( k \) are disposable parameters. For a large range of values of \( a \) and \( k \) it is shown that the “knee” in the profile of the logarithm of the intensity, defined to be the point at which the rate of change of the gradient is maximum, i.e., \( d^2 (\log I)/dT^2 = 0 \), corresponds to a tangential optical depth \( \tau_T \) of 1.3 ± 0.3 (i.e., a total tangential optical thickness of 2.6 ± 0.6). It is shown that this identification has considerable advantages over that used by de Jager (1959) and others, where the point of inflection of the profile is identified with unit tangential optical thickness (\( \tau_T = \frac{1}{2} \)).

By using a direct conversion factor, the height of \( \tau_T = 1.3 \) at the center of Hα is found to be 4860 ± 200 km above the corresponding point observed in the continuum radiation. For an exponential atmosphere defined by equation (1), tangential and radial optical depths are related by (see Paper I, § II)

\[ \tau_r/\tau_T = \sqrt{(2/kR_0\pi)}. \]

Thus at the top of the photosphere, where the scale height, \( 1/k \), is approximately 100 km and the solar radius, \( R_0 \), is \( 7 \times 10^8 \) km, \( \tau_T = 1.3 \) corresponds to \( \tau_T = 0.013 \). Taking the base of the chromosphere at \( \tau_T = 0.004 \) (Allen 1963), \( \tau_T = 0.013 \) is approximately 60 km below this level. Thus the height at which \( \tau_T = 1.3 \) in the center of Hα is 4800 ± 200 km above the base of the chromosphere.

In an exponential atmosphere defined by equation (1) it can be shown (e.g., Paper I, § II) that the heights at which radial optical depth and tangential optical depths are unity are related by

\[ h(\tau_T = 1) - h(\tau_r = 1) = \frac{\ln(kR_0\pi) - 0.693}{2k}. \]
Similarly
\[ h(\tau_T = 1.3) - h(\tau_r = 1) = \frac{[\ln(kR_\odot x) - 1.218]}{2k}. \]

Thus if the scale height in the chromosphere is known and equation (1) is assumed to hold over such a range, \( h(\tau_r = 1) \) can be found from equation (3). Of course, the scale height for Ha is not known with accuracy in this region, but a range between 500 and 1000 km is typical of current estimates. Inserting these values in equation (3) to give a rough estimate of the height of formation—\( h(\tau_r = 1) \)—yields a range from 1600 km to 3000 km. No claim is made for high accuracy even to these bounds for this rather large range. However, none of the assumptions or approximations made can explain the difference between these heights of formation and de Jager's value of 5000 km.

b) The Chromospheric Arches and the Sunspot

Since photographs of the disk in Ha indicate a highly non-uniform structure, it may be misleading to attempt to find a mean height of formation of Ha. It is probably more useful to give \( h(\tau_r = 1) \) at various points in the chromosphere where particular features make such an identification possible. Two such features are observed on this particular spectrogram; a set of three chromospheric arches and a sunspot. Although an arch is

| TABLE 1 |
| --- | --- | --- | --- |
| Heights of Formation of Ha in Chromospheric Features | Sunspot | Arch (i) | Arch (ii) | Arch (iii) |
| Heliocentric angle \( \theta \) | \( 67^\circ \pm 1 \) | \( 77^\circ \pm 1 \) | \( 80^\circ \pm 1 \) | \( 83^\circ \pm 1 \) |
| \( \tau(\text{Ha}) = \cos \theta \) | 0.39 | 0.23 | 0.17 | 0.12 |
| \( h^* \) (km) | 1900 ± 200 | 2600 ± 500 | 3000 ± 500 | 2900 ± 500 |
| \( d_e \) | 240 | 210 | 190 | 140 |
| \( h_e \) | 1560 ± 200 | 2290 ± 500 | 2710 ± 500 | 2660 ± 500 |

essentially a bright region while a sunspot is darker than its surroundings, they both extend through a considerable height in the solar atmosphere. Thus in the spectrogram they appear as bright or dark lines crossing the dark region of the spectral line from continuum to continuum. The archlike structure seen in the line is caused essentially by the difference in height of formation of the radiation in the line and in the continuum.

A full treatment of the structures would involve setting up chromospheric models for the absorption coefficient and source function, which include the inhomogeneity. The directional intensity must then be calculated at various frequencies across the line and at different positions across the inhomogeneity. There are considerable difficulties in such an analysis, however, not the least of which is the absence of any reliable model for the source function in the chromosphere remote from the inhomogeneity.

At present, therefore, it is sufficient to relate the displacement of the features at the line center from the corresponding feature in the continuum with a projected height difference \( \Delta h \). The actual chromospheric height difference between \( \tau(\text{Ha}) = \cos \theta \) and \( \tau_{5000} = \cos \theta \) in the inhomogeneity (\( \theta \) is the heliocentric angle of the inhomogeneity) is then given by \( h^* = \Delta h/\sin \theta \). In Table 1 the results for the sunspot and for the three arches are given. The probable errors quoted in Table 1 refer only to the uncertainties of measuring the positions of maxima or minima on the tracings. For the sunspot, the error arises at the line center where another dark feature associated with the plage lies close to the spot and makes it more difficult to determine the position of the intensity minimum on the tracing. The possible errors involved in measurements of the chromospheric arch heights are much greater, not only because the intensity changes are much less near
the line center but also because the photospheric feet of the arches are obscured by the noise in continuum tracings made with a short slit. Thus the appropriate level for the foot of the arch in the continuum must be inferred by extrapolating from points on tracings in the wings of the line.

To find $h_c$, the height above the base of the chromosphere at which $\tau(\text{Ha}) \approx \cos \theta$, $h^*$ must be corrected by subtracting the distance $d_c$ between $\tau_{5000} = \cos \theta$ and $\tau_{5000} = 0.004$. For the archlike structures, this correction is not important since $d_c$ (obtained from Allen's [1963] table of the mean photosphere) is in each case well within the range of probable error. However, it is shown in Table 1 for completeness.

In the sunspot, which may be accurately traced into the continuum, $d_c$ is just greater than the probable error and the correction is of some importance. Here, however, another difficulty arises, for currently there is no reliable model of the structure of a sunspot in the photosphere. Bray and Loughhead (1964) discuss such models as are available. They note that, while Mattig gives a model in which the umbra is slightly more opaque than the surrounding photosphere, Michard's model requires that the umbra be much more transparent. On Mattig's model, no further correction is necessary since $\tau_{5000} = \cos \theta = 0.39$ corresponds to approximately the same physical depth in the umbra and in the photosphere. Bray and Loughhead point out, however, that the evidence of the "Wilson effect" qualitatively supports Michard's more transparent model. In an analysis of Loughhead and Bray's (1958) photographs, Wilson (1965a) has put an approximate limit on the opacity of that spot at between one and two orders of magnitude less than the opacity of the photosphere. In a typical model for that spot the physical depth in the umbra at a point corresponding to $\tau_{5000} = 0.39$ is approximately 100 km below the corresponding point in the surrounding photosphere. For this reason $h_c$ for the sunspot is obtained from $h_c = h^* - d_c - 100$.

IV. DISCUSSION

Mattig's (1962) results for four different spots give values of $h^*$ ranging from 1680 to 2830 km. Considering the differences which must exist between individual sunspots, our own result for $h^*$ therefore is quite typical. Mattig's value (150 km) for the correction, $d_c$, differs slightly from ours (240 km), and he makes no correction for sunspot height differences in the photosphere. However, considering the spread of values for individual sunspots, these differences are marginal.

What does concern us is his unqualified acceptance of de Jager's value of 5000 km for the height of $\tau(\text{Ha}) = 1$ in the undisturbed chromosphere and his subsequent deductions from this. Because the height $h_c$ refers to unit optical path length along the line of sight, or $\tau(\text{Ha}) = \cos \theta$, the height of formation of Ha[$\tau(\text{Ha}) = 1$] in each of these features must occur at even smaller heights in the chromosphere. These observed heights of formation contradict de Jager's suggested value of 5000 km but are consistent with the deductions from the limb profiles (§ IIIa), which gave a range from 1600 km to 3000 km.

It is true that the chromosphere is not spherically symmetrical, and thus these results may be typical only of a particular region of the chromosphere. This point can be settled only when many more limb spectra of this type are analyzed. This analysis is presented here in the hope that it will lead to similar analyses in other regions not only in Ha but also in other chromospheric lines.

V. CONCLUSION

It is suggested that, if the concept of a mean chromosphere is to be preserved, the currently accepted value for $h[\tau(\text{Ha})] = 1$ is seriously in error and that something of the order of 2500 ± 700 km is more appropriate. However, we are of the opinion that, in view of the obvious inhomogeneities, the idea of a mean chromosphere is rather unsatisfactory and it is preferable to measure heights in Ha at particular points in the chromosphere, such as those discussed above.
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REFERENCES

Jager, C. de. 1959, Ency. of Physics (Berlin: Springer-Verlag), 52, 125.
Severny, A. B., and Bumba, V. 1958, Observatory, 78, 33.
———. 1965b, ibid., p. 1195.