THE MAGNETIC EFFECT AND SHOCK WAVE OF A METEOR

V. V. Ivanov and Yu. A. Medvedev

Translated from Astronomicheskii Zhurnal, Vol. 41, No. 6, pp. 1118-1127, November-December, 1964
Original article submitted January 30, 1964

It is shown that the shock wave of a meteor in the upper atmosphere reveals itself by an isothermal discontinuity. The width of the latter is computed. It is shown that ionization per unit length of the train is determined by the dimensions of the meteoric body. The electromotive force which gives rise to currents responsible for the magnetic effect of the meteor and also the magnitude of the effect are computed. The results are in good agreement with available experimental data.

Introduction

The study of electromagnetic effects accompanying the phenomenon of meteors can provide data on the phenomenon itself and, possibly, also the structure of the earth's atmosphere.

The magnetic effect of a meteor has been known for a long time [1] and has been described experimentally by A. G. Kalashnikov [2]. No detailed calculations of this effect have yet been made. The estimate given in [1] cannot be considered as a good starting point, since it was assumed there that the meteor becomes charged to a charge of \( q \approx \alpha L \sim 3C \), which is transported with the speed of the meteor (\( \alpha \) and \( L \) are the ionization per unit length and the length of the trail, respectively).

However, the charge \( Q \) cannot be retained by the meteoric body since in a conducting medium it will become screened in a period of time of the order of \( 1/\sigma \), where \( \sigma \) is the conductivity of the medium which, even if the ionization produced by the meteor itself is neglected, will be of the order of

\[
\sigma = \frac{n_0 e^2 \lambda}{2mv} \approx 10^8 \text{scm}^{-1};
\]

here \( n_0 \approx 10^5 \text{ cm}^{-3} \) is the electron density, \( e \), \( m \), \( \lambda \), \( v \) are the charge mass, mean free path, and average velocity of an electron at a height of 80-100 km.

In addition, it is not clear how a meteoric body could acquire such a charge. On becoming charged, the body acquires a potential which prevents the accumulation of charge. For example, the body could become charged as the result of electron emission, but the limiting charge which it would acquire in this case is given by

\[
\frac{eq}{R} = \mathcal{S},
\]

where \( R \) is the radius of the body and \( \mathcal{S} \) the energy of the electrons emitted (of the order of the temperature at which the meteoric material boils). Even if the dimensions of the body are of the order of a meter, the limiting charge is \( q \approx 10^{-10} \text{ C} \), which is very small.

Therefore, the nature of the emf giving rise to the current which in turn produces the magnetic effect of a meteor appears not yet to have been established.

A possible mechanism for the formation of an emf arising together with the meteor shock wave is considered in the present paper.

1. The Meteor Shock Wave (Isothermal Discontinuity)

It is known that sufficiently large meteoric bodies reach dense layers of the atmosphere. In this case the motion of the meteor is accompanied by a shock wave. The shock wave of a large body in the lowest section of its trajectory has been studied by V. A. Bronstein [4]. From the point of view of a theory of the magnetic effect, it is of interest to investigate the structure and

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shape of the head shock wave along a higher section of the trajectory (height \( \sim 70-90 \) km).

We introduce cylindrical coordinates \((R, z)\) whose origin is at the point where the body is found and whose \(z\) axis is directed against the motion. The pattern of flow around the body is axially symmetrical.

If it is considered that the meteor does not possess a characteristic aerodynamic shape, then the law of large Mach numbers \( M \) is valid (see [5]) when the shape of the shock wave and the drag coefficient of the body \( c_d \) are independent of the number \( M \). In the case under consideration \( M \sim 100 \).

Let \( d \) be the maximum dimension of the body in a direction normal to the flow (for a sphere \( d \) is the diameter); \( c_d^{1/4} \approx 1 \) to within 0.1 (see [6]).

As is known, the law of nondependence on the Mach number leads to the following equation for the shock-wave front:

\[
\frac{R}{d} = \kappa c_d^{1/4} \left( \frac{z}{d} \right)^{1/6} . \tag{1}
\]

The constant \( \kappa \) is calculated theoretically for a blunt semi-infinite cylinder. Analogous calculations have not been carried out for bodies of finite size, but we will make use of the experimental results reported in [5, 6]. The following expressions for \( \kappa \) have been taken from Fig. 5.8a on page 180 of [6]:

\[
\frac{z}{\sqrt{c_d} M d} \cdot 10^{-3} = 6, \\
\frac{R}{\sqrt{c_d} M d} \cdot 10^{-2} = 25.
\]

Substituting these values into (1) we find that

\[
\kappa = \frac{2.5}{\sqrt{6}}
\]

and formula (1) takes the form

\[
\frac{R}{d} = \frac{2.5}{\sqrt{6}} \left( \frac{z}{d} \right)^{1/6} . \tag{1a}
\]

The pressure behind the wave front, the shape of the shock wave being given, can be found from the stability condition. If \( \varphi \) is the angle between the normal to the wave front and the velocity of the meteor \( U_0 \), \( v \) being the velocity of the shock wave, then from the stability condition we have that

\[
\cos \varphi = \frac{v}{U_0} . \tag{2}
\]

The velocity of the shock wave and the thermodynamic properties of the medium in front of the wave determine the thermodynamic state of the gas behind the front with the help of the Hugoniot adiabatic.

For strong shock waves, the pressure \( p \), the temperature \( T \), and the density \( \rho \) behind the wave front are related to the pressure \( p_0 \), temperature \( T_0 \), density \( \rho_0 \) in front of the shock wave, and \( \gamma = c_p/c_v \) by (see [7], Sec. 85)

\[
\sqrt{\frac{p}{\rho_0} \frac{\gamma + 1}{2}} = U_0, \\
\rho = \frac{\gamma + 1}{\gamma - 1} \rho_0, \\
T = T_0 M^2 \frac{2(\gamma - 1)}{(\gamma + 1)^2}.
\]

Expressions (3) are asymptotic for \( M \gg 2/(\gamma - 1) \).

In the case of strong stationary shock waves in low-density gases (upper atmosphere), it appears that the regime of an isothermal discontinuity is realized. Indeed, in this case the front of the shock wave radiates strongly. The radiation overtakes the shock wave and heats the air in front of it to a distance of the order of one mean free path of the radiation. In the upper layers of the atmosphere the mean free path for radiation at \( M \sim 100 \) is \( \sim 10 \) m. Therefore, the change in temperature from \( T_0 \) to \( T \) occurs over a distance of the order of 10 m.

If the medium encountered by the shock wave is heated by radiation to the temperature behind the shock wave, then an isothermal discontinuity will occur.

Let us calculate at what values of \( M \) this will take place. As is known, the energy flux of equilibrium radiation emitted by the shock-wave front is given by

\[
\mathcal{F}_r = AT^4.
\]

For radiating bodies of sufficiently large dimensions, the constant \( A \) is simply the Stefan-Boltzmann constant:

\[
A = \sigma = 5.71 \cdot 10^{-5} \text{ ergs} \cdot \text{cm}^{-2} \cdot \text{sec}^{-1} \cdot \text{deg}^{-4}.
\]

If the radius of the radiating body \( R \) is less than the mean free path \( \Lambda \) of a quantum whose frequency corresponds to the maximum of the Planck distribution, then the constant \( A \) should be understood to mean \( \Lambda = R \sigma / \Lambda \). In the stationary case, the energy flux due to radiation is balanced by the energy flux \( \mathcal{F} \) associated with the heating up of the gas, \( \mathcal{F} = (2 \rho_0 / m_1) v (kT / \gamma - 1) \), where \( k \) is

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Boltzmann's constant and \( m \) the mass of the gas particles.

Equating these fluxes and taking into account the third of expressions (3), we find that the shock wave represents an isothermal discontinuity when

\[
M^2 \gg M_{cr}^2 = \frac{(y+1)\gamma}{4(y-1)\gamma^2} \sqrt{\frac{c_kT_0}{m_1 m_1 R \sigma T_0^\gamma}}. \tag{4}
\]

Substituting into (4) the values of the atmospheric parameters at a height of 70-90 km, we obtain \( M_{cr} \approx 20 \), which corresponds to \( U_0 \approx 6 \) km/sec. Therefore, we assume in the following that an isothermal discontinuity occurs in the case under consideration.

The temperature and pressure profiles in the shock wave are shown qualitatively in Fig. 1; \( \Gamma \) denotes the distance along the normal away from the shock-wave front, the coordinate origin being at the wave front. The pressure in front of the wave \( p' \) has been calculated in [7] and is given by

\[
p' = \frac{1}{2} [(y+1)\rho_0 + (y-1)\rho_0] \approx \frac{y-1}{2} p.
\]

2. The Width of the Isothermal Discontinuity

The isothermal discontinuity occurs over a distance of the order of a molecular mean free path (L). The latter quantity is not the same on the two sides of the discontinuity. It will be necessary to know to which one of them determines the width of the discontinuity. The law of conservation of momentum in an isothermal discontinuity with viscosity taken into account can be written as

\[
p + \rho u^2 - \eta \frac{da}{dr} = \rho_0 U_0 \delta. \tag{5}
\]

Here \( p \), \( \rho \), \( u \) are functions of \( \Gamma \). For gases the viscosity \( \eta \) depends weakly on the density and in an isothermal discontinuity may be taken to be constant.

In addition to the laws of conservation of momentum (5), the laws of conservation of mass and the condition of isothermality must hold on the discontinuity:

\[
\rho u = \rho_0 U_0, \tag{5a}
\]

\[
\frac{P}{\rho} = \frac{R}{\mu} T = \text{const}.
\]

Eliminating \( p \) and \( u \) from (5) and (5a), we obtain an equation for the density

\[
\frac{\rho_0 U_0}{\eta} \int_{\Gamma_0}^{\Gamma} \frac{\rho_0 U_0}{\rho} d\Gamma = \frac{P}{\rho} = \frac{R}{\mu} T = \text{const}.
\]

whose solution is

\[
r - r_0 = \frac{\eta \rho_0 U_0}{RT_0/\mu} \int \frac{d\rho}{\rho (\rho - \rho_1)(\rho - \rho_2)}.
\]

In the last formula, \( \rho_1 \) and \( \rho_2 \) are the air densities before and after the discontinuity.

Let us define the width of an isothermal discontinuity \( L_1 \) as the distance in which the density changes from \( 2\rho_1 \) to \( \rho_2/2 \).

From the preceding expression we find that

\[
L_1 = \frac{\eta \rho_0 U_0}{RT_0/\mu} \left[ \frac{1}{(\rho_2 - \rho_1)} \ln \frac{\rho_2}{\rho_1} - \frac{1}{(\rho_1 - \rho_2)} \ln \frac{\rho_1}{\rho_2} \right].
\]

For a strong shock wave we have

\[
\rho_2 = \frac{y+1}{y-1} \rho_1,
\]

\[
\rho_1 = \frac{y+1}{2} \rho_0,
\]

\[
\frac{R}{\mu} T_0 = \frac{2U_0^2(y-1)}{(y+1)^2},
\]

and to within \( (y-1)/(y+1) \) the width of the isothermal discontinuity can be represented by

\[
L_1 = \frac{\eta \rho_0 U_0}{\rho_0 U_0} \ln \frac{1}{2(y-1)(2-y)}.
\]

If the classical kinetic expression is used for \( \eta \) then we have

\[
L_1 \approx \frac{1.2 h}{\gamma y/1} \ln \frac{1}{2(y-1)(2-y)},
\]

i.e., the width of the discontinuity is governed by the mean free path of air molecules behind the shock front.
3. Electromotive Force Arising in a Shock Wave

One of the most important meteor parameters that can be measured experimentally (for example, by means of radar) is the ionization per unit length of trail. This quantity is also in many respects the factor that governs the magnetic effect. Therefore, let us make a preliminary calculation of $\alpha$ on the assumption that the ionization is produced by the shock wave.

On the shock-wave front the temperature is given by

$$T \approx T_0 \frac{2\gamma(y-1)}{(y+1)^2} M^2 \approx 10^6 \text{ deg}.$$  

The potential for single ionization of air is $I \approx 14 \text{ eV} [9]$ which corresponds to a temperature of the order of $1.4 \cdot 10^6 \text{ deg}$. As is known, at temperatures comparable with the ionization energy, the gas is practically completely ionized (singly, inasmuch as the potential of double ionization is $\sim 30 \text{ eV} [9]$). The critical temperature at which there is no appreciable ionization in air is $T_{\text{cr}} \approx 1.4 \cdot 10^4 \text{ deg}$.

It can be seen from (2) and (3) that each section of the shock wave has its own temperature. As we proceed away from the head part of the shock wave, the temperature decreases and at some distance $z_{\text{cr}}$ it becomes less than $T_{\text{cr}}$ (see Fig. 2). The Mach number corresponding to the critical temperature is

$$M_{\text{cr}} = \sqrt{\frac{2}{\gamma(y+1)}} \approx 10.$$  

The coordinates $z_{\text{cr}}$ and $R_{\text{cr}}$ are defined by the condition

$$\sin \varphi_{\text{cr}} = M_{\text{cr}}^{-1}/M,$$

from which, together with (1a), we find that

$$R_{\text{cr}} = \frac{d}{2 M_{\text{cr}}^2} \approx 7d.$$  

The ionization density per unit length of trail depends on the diameter $d$ of the meteoric body and the concentration of molecules of the medium. That fraction of the gas which strikes the shock wave with $R < R_{\text{cr}}$ becomes completely ionized and distributes itself along the trail. The gas which impinges on the meteor outside this region is not ionized. On the basis of these considerations, we obtain the following estimate of the quantity $\alpha$:

$$\alpha = \frac{\pi \rho}{4} \left( \frac{M}{M_{\text{cr}}} \right)^2 n = 4 \alpha_{\pi \rho d^2}.$$  

It appears that a measurement of $\alpha$ will allow us to determine the dimensions of the meteoric body sufficiently accurately. As is known, $\alpha = 10^{11-10^{12}} \text{ cm}^{-1}$ for bright meteors (see, for example, [10]). This corresponds to $d \approx \left( \frac{\alpha}{4 \pi n} \right)^{1/3} \approx 1/3 \text{ mm}$ ($n = 10^{13} \text{ cm}^{-3}$). For weak visual meteors $d \approx 0.1 \text{ mm}$; for bolides $\alpha \approx 10^{18}$ (see [1]) and the value of $d$ is found to be of the order of $1 \text{ m}$. These estimates agree with the dimensions found in [10] on the basis of other considerations.

It is clear from what has been said above that the medium which transmits the isothermal discontinuity is completely ionized (singly). There is no additional ionization taking place on the discontinuity itself, in contrast to the case considered in [4]. Since the electron mean free path is $\lambda > L_1$, the electron flux is conserved on the discontinuity. If $v_0(\alpha)$ and $n_0(\alpha)$ are the velocity of average motion and the electron density, then the above assertion means that

$$n_0(r)v_0(r) = \text{const},$$

where the value of the constant should be taken at a point directly in front of the discontinuity. As is known [11], the hydrodynamic equations for electrons have the form

$$\frac{\partial v_+}{\partial t} + v_+ \frac{\partial v_+}{\partial r} = \frac{kT}{nm} \frac{\partial n}{\partial r} - \frac{eE}{m} + \beta(v - v_+),$$  

(6)

here $E$ is the strength of the induced electric field, $e/m\beta$ is the electron mobility, and $\gamma$ is the velocity of the average motion of the medium. This system has to be augmented by the electrostatic equation

$$\frac{dE}{dr} = 4\pi e(n_i - n_e),$$  

(6a)

where $n_i$ is the ion density. Generally speaking, it is also necessary to write down the hydrodynamic equation for the ions with the influence of the
electric field on them specifically taken into account. However, dynamically the action of the field on the ions is less by a factor of \(m/m_1\) than its action on electrons (\(m_1\) is the mass of an ion). Therefore, we can assume that the ion density is governed by the usual hydrodynamic equations (5) and (5a). Then, Eqs. (6) and (6a) determine the electron distribution and the induced electric field.

The system of equations (6) and (6a) can be simplified in the case under consideration. In the region where ionization is significant, the temperature in the isothermal discontinuity is \(\sim 5 \times 10^4\) deg and the thermal velocity of an electron is \(v_T \approx (10^7 - 10^8)\) cm/sec \(> U_0 \approx 5 \times 10^6\) cm/sec. Let us evaluate the order of magnitude of the terms in (6). In the stationary case \(\partial v_e/\partial t = 0\). The other terms are \(v_e \partial v_e/\partial x \sim U_0 v_T \sim \frac{kT_e}{m_e} \frac{\partial n_e}{\partial x} \sim \frac{v_e^2}{L} \beta(v - v_e)\).

In the first approximation with respect to the parameter \(U_0/v_T \approx 10^{-4}\), we obtain from (6) and (6a)

\[
\frac{kT_e}{e} \frac{\partial n_e}{\partial x} = - eE, \tag{7}
\]

\[
\frac{dE}{dx} = 4\pi e [n_i(x) - n_e(x)].
\]

It is impossible to obtain an exact solution of equations (7). An approximate solution can be obtained by an expansion in a series in \((R_D/L_1)^2\), where \(R_D = (kT/4\pi e^2 n)^{1/2}\) is the Debye radius. In our case \((R_D/L_1)^2 \approx (10^{-4} - 10^{-5})\). In the zero-order approximation in the small parameter, system (7) has the solution

\[
n_i(r) = n_e(r),
\]

\[
E(r) = \frac{kT_1}{e} \frac{dn_i}{dr}.
\]

This approximation corresponds to an ambipolar mechanism of motion of the medium.

Thus, the emf arising on an isothermal discontinuity is

\[
\varepsilon_i = \frac{kT_e}{e} \ln \frac{\rho_2}{\rho_1} = \frac{kT_e}{e} \ln \frac{2}{\gamma - 1}.
\]

It should be noted that there exists a classical formula for the contact potential difference (for example, see [12]) due to the difference between the particle concentration in two touching metals.

In addition to the emf \(\varepsilon_i\), there is an additional emf \(\varepsilon_e\) in the region of heating — on the boundary of the ionization front:

\[
\varepsilon_e \approx \frac{kT_e}{e} \ln \frac{n}{n_0},
\]

where \(n\) is the concentration of air molecules at the height of the meteor trail. The total emf is given by

\[
\varepsilon = \varepsilon_i + \varepsilon_e \approx \frac{k}{e} \left( \frac{2}{\gamma - 1} + T_c \ln \frac{n}{n_0} \right) \approx 65. \tag{8}
\]

In the last expression we have taken \(n_0 \approx 10^5\) cm\(^{-3}\), \(n \approx 10^{13}\) cm\(^{-3}\).

4. The Magnetic Effects of a Meteor

A stationary distribution of charge in the discontinuity is possible only if there exists a flow of electrons along the trail away from the head of the meteor. Thus, a current \(I\) flows along the trail. The conductivity of gas inside the trail is much higher than the conductivity of the outer medium (ionosphere). Together with the current \(I\), other currents appear in the ionosphere in which electrons flow from the meteor head to the end of the trail.

Let \(r_1\) and \(r_2\) be the coordinates of the head and the end of the trail. Then, the distribution of current density \(j\) in the ionosphere with current \(I\) given is determined by the continuity equation

\[
\text{div} j = I(\delta(r - r_1) - \delta(r - r_2)),
\]

whose solution is

\[
\left. j(r) \right| = \frac{I}{4\pi} \left( \frac{r - r_1}{|r - r_1|^3} - \frac{r - r_2}{|r - r_2|^3} \right). \tag{9}
\]

If \(R\) is the electric resistance of the trail and \(R_1\) the resistance of the short-circuiting of the trail by the ionosphere, then the current \(I\) is given by Ohm's law:

\[
I = \frac{\varepsilon}{R + R_1}. \tag{10}
\]

The electric resistance of the trail is

\[
R = \frac{2mL}{\varepsilon^2} \int_0^L \frac{v dz}{\lambda n(z) S(z)} = \frac{2mL}{\varepsilon^2 \alpha} \int_0^L \frac{v dz}{\lambda},
\]

where \(n(z), v(z), S(z)\) are the electron concentration, the average thermal velocity, and the cross-sectional area of the trail; \(n(z)S(z) = \alpha\).
Neglecting the small high-temperature region near the meteor head, we find that

\[ R = \frac{4.1 \times 10^5 \sqrt{2kT_mL}}{e^{7a}} \Omega, \]  

(12)

where \( T_0 \approx 300^\circ\text{C} \).

The electric resistance of the short-circuiting of the trail by the ionosphere is governed by the minimum radius of curvature \( b \) of the boundary of the region with increased conductivity:

\[ R_i = 1 / 2\pi ab. \]  

(13)

The radius of the region with increased conductivity is determined by the mean free path of the radiation emitted by the isothermal discontinuity in the ionosphere.

As is known, \([13]\), the cross section for the absorption of ultraviolet radiation by air is \( \sigma_a \approx 2.5 \times 10^{-18} \text{cm}^2 \). If the density of particles in the ionosphere is \( N \approx 10^{14} \text{cm}^{-3} \), then \( b \) is found to be of the order of

\[ b \approx \frac{1}{N\sigma_a} \approx 4 \times 10^8 \text{cm} \]

and the resistance (13) is found to be \( R_i \approx 9 \Omega \).

If \( R > R_i \), the current in the trail is primarily governed by the resistance of the trail, on the other hand if \( R < R_i \), the current is independent of \( R \). Therefore, the magnetic effect exhibits saturation for \( R \approx R_i \) in the case of meteors with \( \alpha \gg \alpha_{cr} \), where

\[ \alpha_{cr} = \frac{4.1 \times 10^5 \sqrt{2kT_mL}}{9e^7} = 10^{14} \text{cm}^{-1}. \]

The magnetic field of a meteor is governed only by the current \( I \) flowing along the trail, since the magnetic field of the short-circuiting currents is equal to zero.

In fact, the short-circuit current density \( j \) consists of two terms, each of which is of the form \( r^2 \) [see (10)].

As is known, the vector potential of these currents is also of the form \( r^2 \) and the magnetic field, being the curl of the radial function, is zero. This is correct for an infinite ionosphere.

For the calculation of the magnetic effect \( \Delta H \) at an observation point situated outside the ionosphere, it is necessary to take into account the fact that the current is reflected from the lower boundary of the ionosphere. The distribution of the short-circuit current densities in this case will be

\[ j = \frac{I}{4\pi} \frac{r - r_1}{|r - r_1|^3} \frac{r - r_2}{|r - r_2|^3} \frac{r - r_3}{|r - r_3|^3} \frac{r - r_4}{|r - r_4|^3}, \]  

(14)

where \( r_3 \) and \( r_4 \) are the coordinates of the reflections of the points \( r_1 \) and \( r_2 \) in the lower boundary of the ionosphere (see Fig. 3). The distribution (14) is meaningful only in the conducting half-space. In the lower half-space \( j = 0 \).

Let us calculate the magnetic field of the current \( I \) and the short-circuit currents (14). From considerations of symmetry it can be easily seen that in the case of a "head-on" entry of the meteor into the atmosphere, the magnetic field at the point of observation \( R_0 \) is equal to zero, i.e., the short-circuit currents completely balance the contribution of the linear current \( I \). On this basis it can be shown that with an inclined entry the magnetic effect is different from zero and is produced by the current ABCD shown in Fig. 3,

\[ \Delta H = H_{AB} + H_{BC} + H_{CD}, \]  

(15)

where

\[ H_{AB} = \frac{I}{r_{AB}} (1 - \cos \alpha_{AB}), \]

\[ H_{BC} = \frac{I}{r_{BC}} (\cos \alpha_{BC} - \cos \alpha'_{BC}), \]

\[ H_{CD} = \frac{I}{r_{CD}} (1 - \cos \alpha_{CD}). \]

The symbols used are defined in Fig. 3. A numerical estimate from formulas (15), (9), (11), (12), and (13) with \( \lambda = 5 \text{cm}, R_0 \approx 100 \text{km}, \alpha = 10^{14} \text{cm}^{-1}, L \approx 30 \text{km yields} \)

\[ \Delta H \approx 0.8 \times 10^{-7} \text{Oe}. \]
which is in good agreement with the results obtained in [2].

LITERATURE CITED
