SURVEY OF THE PROBLEM

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Our purpose at this Conference is to discuss the current state of knowledge of the problem of the formation of spectral lines. In many ways this is a natural outgrowth of the 1962 Herstmonceux Conference on Stellar Atmospheres*; many problems raised there will be clarified here -- as we shall see. In these opening remarks my aim is to try to sketch a framework within which we can assess the relevance, to the whole problem, of the individual contributions.

Our problem ultimately reduces to this: We wish to deduce the properties of the atmosphere from our observations of the strength and shape of a spectral line or lines. This introduces two separable aspects: firstly, that of the basic atomic emission and absorption processes; secondly, the quantity of radiation which is generated, absorbed, and transferred through the atmosphere. The first aspect is essentially a question of radiation theory on an atomic scale; the second introduces the macroscopic aspects of radiative transfer from which predictions are made as to the observable spectral features which we must interpret. In the discussion of the transfer of radiation we can introduce, quite formally, certain bulk emission and absorption coefficients; to be meaningful these macroscopic quantities must be interpreted in terms of the microscopic properties of the gas and it is there that our central problem arises. Much of our discussions here will center on this question.

Before considering this further, however, a few more remarks on the general problem may be worthwhile. We might expect that the analysis of an observed line spectrum could be undertaken in two (interdependent) ways, one synthetic, one analytic. In the former procedure we would assume an atmospheric structure and compute the emergent radiation; if this did not agree with observation we could change the model and try again. Whether or not this would ultimately give a unique result is not considered here. The second general approach would proceed directly from the observed radiation and attempt to infer the excitation state of the gas, and hence its physical structure. This again will not obviously yield unique results.

Since examples of each of these procedures are to be discussed by individual speakers it is worthwhile to review the pertinent features of each.

1. The Synthetic Approach

Let us restrict attention initially to the formation of spectral lines in the absence of a background continuum radiation. The equation of transfer for a homogeneous plane parallel geometry can then be written in the well-known form

\[ \mu \frac{dI}{d\tau} = I - S, \tag{1} \]

where \( I(\mu, \tau) \) is the monochromatic specific intensity of the radiation traveling in a direction making an angle \( \cos^{-1}\mu \) to the normal to the atmosphere; \( \tau \) is the line optical depth, and the source function \( S(\tau) \) is the ratio, at depth \( z \), of the volume emission coefficient \( \varepsilon \) to the absorption coefficient (per unit length) \( k \). Since equation (1) may be integrated immediately once \( S \) is known, the fundamental problem in the analysis of spectral lines lies in the determination of the source function.
By definition, $S_\nu$ is the ratio $\epsilon_\nu / \kappa_\nu$. These quantities depend linearly on the populations of the upper and lower states of the transition and so, to specify $S_\nu$, we face the threefold problem of determining (i) the profile (i.e., frequency dependence) of $\kappa_\nu$, (ii) the profile of $\epsilon_\nu$, and (iii) the ratio $n_u / n_f$ of the populations of the upper and lower states. In general, however, $n_u / n_f$ depends on the strength of the radiation field in the given line* — and so on $S_\nu$ — and the transfer equation can only be made determinate if we either assume a form for the population ratio or if we compute it from other considerations. An example of the first procedure is the assumption of LTE (when the population ratio is given by the Boltzmann relation). We shall prefer, however, not to prejudge the result but shall simply suppose that at each point in the gas the population of each level remains constant in time; this obviously includes LTE as a special case.

The price we pay for this generality is that we must now determine $S_\nu$ for each line produced by the atom. This is hardly feasible, however, because of the interlocking of a given line with all others, and in practice progress has been made by restricting consideration to atoms with a limited number of levels. It has never been quite clear how this restriction limits the practical application of the procedure; however, recent advances, to be reported at this Conference, now make feasible an early resolution of this question.

Most attention has been given to the case of complete redistribution — in which $\epsilon_\nu$ and $\kappa_\nu$ have the same frequency dependence — so that the line source function is frequency independent. In this simple (and widely applicable) case it can be readily shown (cf. Jefferies, 1963) that we can write the source function in a given line in the form

$$S(\tau) = \lambda b + \bar{\omega} \int J_\nu \phi_\nu dv,$$  \hspace{1cm} (2)

* And on those of all other lines and continua.
where \( J_v \) is the mean intensity, \( \phi_v \) is the profile of the absorption coefficient, and \( \lambda, b, \) and \( \overline{\omega_0} (=1-\lambda) \) are functions of the ambient gas conditions and of the radiation intensities in all other lines and continua formed by the model atom. Thus \( \lambda, b, \overline{\omega_0} \) -- and in general \( \phi_v \) -- are functions of depth; their specific forms, i.e., their dependence on transition rates, follow in a straightforward fashion from the statistical equilibrium equations.

The simple two-level case, formulated by Milne (1930), and more satisfactorily by Thomas (1957), was solved in an Eddington approximation by Jefferies and Thomas (1958) and has recently been extensively discussed by Avrett and Hummer (1965). While this work has thrown much light on the physics of the problem it was inadequate in its restriction to a two-level atom. An extension to discuss the resonance lines of a three-level atom (Jefferies, 1960) brought to light the important fact that the source functions for multiplet lines should show a common depth variation over a substantial part of the atmosphere while differing markedly (in general) from their LTE values. This conclusion was confirmed by Waddell (1962) from his solar D line observations.

Nevertheless it was clear, as emphasized at the Herstmonceux Conference, that further progress was blocked by the mathematical problem involved in the solution of the basic transfer equation (1). This has now been overcome by a variety of workers using either the differential equations (1) and (2) (Feautrier, 1964; Rybicki, 1965); or the corresponding integral equation (Avrett, 1965; Kalkofen, 1965)

\[
S(\tau) = \lambda b + \overline{\omega_0} \int S(t) K(t, \tau) \, dt, \tag{3}
\]

where the kernel \( K(t, \tau) \) has a simple form (cf. Kalkofen, 1965, for details). There now seems no difficulty in principle to the determination of the excitation state for multilevel atoms and we may therefore look forward to rapid progress in our understanding of the excitation...
of quite complicated atoms in stellar atmospheres. Some results are to be presented later in this Conference.

This is, however, only one aspect of the problem. The excitation of a gas is strongly dependent on the strengths of the collision transitions; these are implicit in the parameters $\lambda$ and $b$ of equation (2). Thanks to the work of Seaton, Burgess, van Regemorter and their colleagues (see, for example, Seaton, 1962) we can now make much more confident estimates of collision cross sections than was possible a few years ago. Further work is still needed, however, especially for heavy particle collisions and in the determination of cross sections for transitions between close lying levels.

Although complete redistribution is probably acceptable for line formation in a stellar atmosphere, there will be cases -- like La transfer in a nebula -- where a scattered photon in fact preserves a good deal of frequency coherence. Hummer (1962) has studied the physical basis of this more general formulation and has recently managed to solve the associated transfer problem. His results, to be discussed at this Conference, indicate that in practice we may still hope to obtain reasonable results from the much simpler case of complete redistribution.

Finally, we have so far discussed only the question of line formation in the absence of a background continuum. However, depending on the ratio of line to continuum absorption coefficient, the introduction of continuous opacity can produce a substantial change in the radiation transfer (as pointed out by Jefferies and Thomas, 1958). The physical reason for this, of course, lies in the possibility of mixing of line and continuum photons. If, as we usually assume, the background continuum is formed in LTE (i.e., continuum photons are not immediately re-emitted following absorption), then the diffusion length of a photon is automatically reduced if continuous absorption is significant. This complication has received further attention in
the past few years; we shall hear from Hummer (1965) of some further results later in the Conference.

It is, perhaps, worthwhile to add a few comments on future directions which these studies might take. Firstly, there is no longer any serious difficulty in principle to the computation of the radiation emerging from a gas, but there are at least two shortcomings in the development which may limit its application in practice. I refer specifically to limitations inherent in the restricted atomic model used to represent the actual energy level structure of the atom; and the limitations inherent in the representation of an atmosphere by a homogeneous layer. There is the additional limitation of uncertainty in atomic data (cross sections and f-values), but this problem is of an essentially different character.

The limitation of the atomic model can be studied in a variety of ways, e.g., by varying the number of levels retained, but it should be emphasized that it is much better to include upper levels of an atom even in an approximate manner than to ignore them completely (as is often done). A preliminary analysis can show, for example, that under the given atmospheric conditions all levels down to a certain principal quantum number are essentially in LTE with respect to the continuum. It is then a simple matter to include these as "effective continuum" states. This type of procedure was anticipated by Giovanelli (1948) and is used by Bates et al. (1962) in their study of the decay of a hydrogen plasma.

Visual observations of the solar surface plainly indicate its inhomogeneous nature. Undoubtedly future studies must take this into account both in reformulation of the transfer equation for such media and in observation of parameters other than the mean center-limb profile, as averaged over inhomogeneities. In attacking this problem it is essential that theory and observation go hand in hand so that the most meaningful higher moments of the intensity distribution which enter the theoretical formulation should be feasible to obtain
observationally while, as far as possible, the features actually observed should be uppermost in mind in seeking the best theoretical formulation.

2. The Analytic Approach

The inferential problem has to date received comparatively little attention outside the LTE framework. In the past few years, however, it has become clear that we are now in a position to perform such an analysis -- at least for certain multiplet pairs -- with the aid of two widely applicable results: a) that the line source function is essentially frequency independent across the line, and b) that the line source functions of two close lying lines of a common multiplet have the same value at a given physical depth. These principles allow an a priori analysis (within the assumption of homogeneity) of the center-limb variation of the line profiles of two multiplet lines -- applications to the solar D lines and to the Balmer lines are to be given later in the Conference.

For a fully satisfactory analysis we should be able to account for the profiles of each line emitted by the atom. To do this we must understand the relative depth dependences of the source function of each line; future studies of the "synthetic approach" should therefore concentrate on the elucidation of this question. For example, while analysis of the solar H and K lines of CaII could be carried out using observations of those lines alone, a far more satisfying analysis -- and a more stringent test -- would be provided by a simultaneous analysis of H, K, and the infrared triplet of CaII. This, however, would only be possible if we knew the relative depth dependences of the corresponding line source functions.
References


