LETTERS TO THE EDITOR

1285

tion is similar to that of many radio galaxies where often the optical galaxy is isolated between the two radio components. This is the first time that this peculiarity has been observed in a quasi-stellar source. Therefore, for MSH 14-121 we cannot associate in an obvious way the peculiar continuous optical spectrum, characteristic of the quasi-stellar sources, with the radio spectrum, as in 3C 48 and 3C 196 (Matthews and Sandage 1963); i.e., the optical and radio emission come from different regions.

P. Veron
February 23, 1965
Mount Wilson and Palomar Observatories
Carnegie Institution of Washington
California Institute of Technology

REFERENCES


RADIO-FREQUENCY OPTICAL DEPTHS OF PLANETARY NEBULAE

The recent measurements of radio-frequency radiation from planetary nebulae at 750 Mc/s and 1410 Mc/s by Menon and Terzian (1965) are over an order of magnitude better in signal-to-noise ratio than previous attempted measurements of these objects. As Menon and Terzian state, their observations show that most of the bright planetaries they measured are thermal sources with finite self-absorption. In this letter their data are used to make a somewhat more quantitative estimate of the radio-frequency optical depths in the observed planetaries, and the resulting values are compared with expectations based on data derived from optical observations.

We adopt the simplest possible model, namely, that a planetary may be represented as a homogeneous, spherical nebula, and since real planetaries are much more complicated than this, the derived optical depths, although having some quantitative significance, are far from exact. The thermal radiation from such a model planetary can be specified by two parameters, the temperature $T$, and the total optical depth at frequency $\nu$ measured along any diameter of the nebula, $\tau(\nu)$. The emergent intensity $I(\nu, \tau)$ at any point on the surface of the nebula at an angle $\theta (<\pi/2)$ to the outward normal is then

$$I(\nu, \tau) = B(\nu, T) \left[ 1 - e^{-\tau(\nu) \cos \theta} \right], \quad (1)$$

and the outward flux $\pi F(\nu, \tau)$ is

$$\pi F(\nu, \tau) = \frac{2\pi B(\nu, T)}{[\tau(\nu)]^2} \left\{ \frac{1}{2} \left[ \tau(\nu) \right]^2 + \left[ 1 + \tau(\nu) \right] e^{-\tau(\nu)} - 1 \right\}. \quad (2)$$

In the limit of no self-absorption [$\tau(\nu) \rightarrow 0$], this expression becomes

$$\pi F(\nu, \tau \rightarrow 0) = \frac{2\pi B(\nu, T) \tau(\nu)}{3}, \quad (3)$$

so that including the effects of absorption

$$\frac{\pi F(\nu, \tau \rightarrow 0)}{\pi F(\nu, \tau \rightarrow 0)} = \frac{3}{[\tau(\nu)]^3} \left\{ \frac{1}{2} \left[ \tau(\nu) \right]^2 + \left[ 1 + \tau(\nu) \right] e^{-\tau(\nu)} - 1 \right\} = f[\tau(\nu)], \quad (4)$$

where $f(0) = 1, f(\infty) = 0$, and in general $f[\tau(\nu)]$ gives the fraction of the thermal radiation emitted by the nebula which escapes to space, that is to say, the nebula’s emission

© American Astronomical Society • Provided by the NASA Astrophysics Data System
efficiency. Numerical values of \( f(\tau(\nu)) \) are plotted in Figure 1, together with corresponding values for another possible model, an infinite plane-parallel slab with optical diameter \( \tau(\nu) \), observed in the normal direction, for which

\[
\frac{\pi F(\nu, \tau)}{\pi F(\nu, \tau \to 0)} = \frac{1}{\tau(\nu)} \left\{ \frac{1}{2} - E_2[\tau(\nu)] \right\} = f_1(\tau(\nu)).
\] (5)

To apply these results to the thermal radio-frequency radiation from planetary nebulae, we use the free-free opacity (per unit volume),

\[
\kappa(\nu) = \frac{4N_eN_iZ^2\alpha^6}{3 (2\pi)^{1/2}(m_kT)^{3/2}c^2}\ln\left(\frac{(2kT)^{3/2}}{\pi \gamma^{5/2}m_h^{1/2}e^3\gamma Z}\right)
\] (6)

in cgs units (Oster 1961), taking the average charge on the ions \( Z = 1 \). Except for the weak logarithmic dependence, the ratio of opacities at two frequencies is essentially inversely proportional to the squares of the frequencies and independent of density and temperature. A short list of relative optical depths (which are the same as relative opacities) at various frequencies is given in Table 1, calculated for the assumed temperature \( T = 10000^\circ K \). Then the ratio of fluxes at two frequencies for the spherical homogeneous model planetary may be written

\[
\frac{\pi F(\nu_1)}{\pi F(\nu_2)} = \frac{B(\nu_1, T)\tau(\nu_1)f[\tau(\nu_1)]}{B(\nu_2, T)\tau(\nu_2)f[\tau(\nu_2)]} = \frac{\nu_1^2\tau(\nu_1)f[\tau(\nu_1)]}{\nu_2^2\tau(\nu_2)f[\tau(\nu_2)]},
\] (7)

Table 1

<table>
<thead>
<tr>
<th>Frequency ( \nu ) (Mc/s)</th>
<th>Relative Optical Depth ( \tau(\nu)/\tau(1410) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>750</td>
<td>3.75</td>
</tr>
<tr>
<td>1410</td>
<td>1.00</td>
</tr>
<tr>
<td>3000</td>
<td>0.205</td>
</tr>
<tr>
<td>8000</td>
<td>0.0259</td>
</tr>
</tbody>
</table>

Fig. 1.—Emission efficiency \( f(\tau) \) for a model spherical, homogeneous nebula, and \( f_1(\tau) \) for a model infinite, plane-parallel nebula observed normally, as functions of \( \tau \), the total optical depth of the nebula.
and for any pair of frequencies the right-hand side can be calculated as a function of say \( \tau(\nu_1) \), since \( \tau(\nu_1) / \tau(\nu_2) = \text{constant} \). Using this model, the observed ratio of fluxes at 750 and 1410 Mc/s measured by Menon and Terzian (1965) can be substituted on the left-hand side, and the right-hand side can be solved to find \( \tau(1410 \text{ Mc/s}) \), with the results listed in Table 2. All the planetaries measured by them are included, except NGC 3242, which has a definitely non-thermal spectrum with \( F(1410)/F(750) < 1 \), and NGC 3587, which also appears to have a slightly non-thermal spectrum, although it is considerably fainter and probably less accurately measured than the other objects.

The measurements of NGC 7027 at 1410 Mc/s and 750 Mc/s show that its optical depth is quite large (Menon and Terzian 1965), and in fact the observed ratio is slightly larger than that predicted for the limit \( \tau(\nu) \to \infty \). However, Menon and Terzian also measured this planetary at 3000 Mc/s, where its optical depth is smaller and the method correspondingly more accurate, and from the ratio of fluxes at this frequency and at 1410 Mc/s the optical depth can be found, with the result listed in Table 2. From Menon and Terzian’s measurements there is no doubt that \( \tau(1410 \text{ Mc/s}) \) is quite large for NGC 7027, but the exact numerical value could possibly be better determined by observations at still higher frequencies. For instance, with \( \tau(1410 \text{ Mc/s}) = 23 \) as given in Table 2, the predicted flux at 8000 Mc/s is \( 22 \times 10^{-26} \text{ W m}^2 \text{ (c/s)} \).

### Table 2

**Derived Radio-Frequency Optical Depths of Planetary Nebulae**

<table>
<thead>
<tr>
<th>Nebula</th>
<th>( \tau(1410) )</th>
<th>Nebula</th>
<th>( \tau(1410) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 6720</td>
<td>0.2</td>
<td>NGC 6543</td>
<td>1.0</td>
</tr>
<tr>
<td>NGC 6853</td>
<td>0.3</td>
<td>IC 418</td>
<td>&gt; 1.3</td>
</tr>
<tr>
<td>NGC 7662</td>
<td>0.3</td>
<td>NGC 7027</td>
<td>23</td>
</tr>
<tr>
<td>NGC 7009</td>
<td>0.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The derived radio-frequency optical depths are mostly in fairly good accord with expectations derived from optical data. The list of Table 2, ordered by optical diameter, is also approximately a list in order of mean H\( \beta \) surface brightness, as should be expected since both quantities depend on density and nebular diameter in the same way. Furthermore the absolute value of the optical depth for a typical bright high-density planetary with \( N_e \approx N_i \approx 10^4 \text{ cm}^3 \), and a diameter \( \approx 0.07 \text{ pc} \) (O’Dell 1963) is \( \tau(1410 \text{ Mc/s}) \approx 1 \), again in agreement with the observational data. However, NGC 7027 is seen to be quite different from the other planetaries in Table 2, and must be either much denser or much larger than a “typical” planetary. It is also known to be atypical in spectrum (Aller, Bowen, and Minkowski 1955) and in form, and furthermore its central star (if it exists) is unusually subluminous, since its apparent magnitude is fainter than 18 (Liller 1955), though an average planetary with the size and surface brightness of NGC 7027 should be expected to have a central star with \( M_{pg} \approx 0 \) (O’Dell 1963), corresponding at its photometrically derived distance of 1770 pc (O’Dell 1962) to \( m_{pg} \approx 13 \) (including an allowance for interstellar extinction). Therefore NGC 7027 is seen to be an object that is deviant in properties (probably including the amount of nebular mass) from the large majority of planetary nebulae, for which a statistical theory based on the idea that they are similar objects in different evolutionary stages seems to give consistent results (Shklovsky 1956; O’Dell 1962, 1963; Harman and Seaton 1964). Other nebulae which may be more nearly similar to NGC 7027 than to typical planetaries are NGC 2440 and NGC 6302, both of which have complicated knotted, filamentary structure reminiscent of NGC 7027, and neither of which has any apparent central star.