THE $K_{332}$ LINE PROFILE AND THE STRUCTURE OF THE SOLAR ATMOSPHERE

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The Thomas-Jeffries theory for the source function is applied to a computation of the profile of the Ca II line $K_{332}$. Comparison with the observations shows that the $K_2$ component originates in a quasi-isothermal region, and $K_3$ in a region with a sharp temperature rise. The distribution of temperature, electron density, and neutral-hydrogen density with height in the chromosphere is derived for active and undisturbed regions. For a given height, the temperature is somewhat lower in the active regions, with the high-temperature rise setting in at a lower height in the undisturbed regions.

It has often been remarked that the strong lines of ionized calcium offer a convenient opportunity for determining the physical characteristics of the solar atmosphere. But, despite the large number of papers dealing with the H and K lines, no theoretical profiles have so far been computed for the lines, and this hinders the interpretation of the various models of the solar chromosphere. The few attempts that have been made [1, 2] represent only a rough approximation.

Calculation of the line profile. In this paper we shall construct a profile for the $K_{332}$ line by using the theory developed by Thomas and Jeffries [3, 4, 5]. Morton and Widing [6] have already applied the theory to a calculation of the solar Lyman-$\alpha$ line. They demonstrated that the $L_\alpha$ line is formed in a region having a sharp temperature rise. However, in deriving the expressions for the line profile, Morton and Widing make the additional assumption that the Doppler halfwidth of the line is constant over the chromosphere. Since this quantity is fully determined by the temperature in the case of the $L_\alpha$ line, the assumption is not valid, and indeed it contradicts the main conclusion of the paper. It is in fact impossible to determine the temperature, as Morton and Widing have done, in the region of line formation by comparing the observed separation of the line peaks in Angströms with the values computed theoretically for the separation in Doppler halfwidths, under the assumption of constant $T$. This procedure yields too high a temperature in the region of line formation. Morton and Widing obtain a temperature $T_{ep}$ (associated with the peaks) of $\sim 70,000-90,000^\circ$K, and for the central portions of the line as much as $200,000^\circ$K. The authors adopt the following form for the source function in the continuum:

$$S_c = S_1 \left(1 + Ae^{-x^2}\right).$$

Here $S_1$ is the Planck function for the temperature $T_{min}$ corresponding to the base of the chromosphere; $\tau_0$ is the optical depth at the center of the line; and the parameter $A$, which expresses the amount of radiation at the boundary of the region of line emission, is taken as $10^8$ in [6]. From this formula, we can compute the temperature $T_{max}$ at the upper boundary of emission of the $L_\alpha$ line. If $T_{min} = 4000^\circ$K, then for $A = 10^8$ we have $T_{max} \approx 10,000^\circ$K, while for $T_{min} = 5000^\circ$K we have $T_{max} \approx 23,000^\circ$K, so that in any case $T_{max}$ is much less than the temperatures obtained in [6]. The assumption that $\Delta \lambda_D$ is constant is more appropriate for the $K_{332}$ line of Ca$^+$, in which the Doppler width depends primarily on the value of the turbulent velocity $\xi$. It has been shown [7] that the turbulent velocities in the chromosphere reach a value of 5-6 km/sec at a height of only 1000 km, but above this level they change little. Khokhlova [6] finds that in order to explain the K-line emission, it is best to adopt a constant turbulent velocity of 7.5 km/sec.

In the papers of Thomas and Jeffries, an expression is obtained for the source function $S_L$ for the strong resonance lines, under the assumption that the electron density is constant in the region of line formation. This assumption appears to be an important defect in the theory. Nevertheless, we shall use the Thomas-Jeffries expression for constructing the profile. We defer until

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later a discussion of the extent to which variability of \( n_e \) would affect our results.

In the case where collisions predominate over photoionizations, we have, for an atom with two discrete levels,

\[
S_L = \frac{\int \varphi \Phi d\nu + eB_{\nu_0}(T_e)}{1 + \varepsilon} \quad (2)
\]

Here \( \varphi \nu \) is the line absorption coefficient \( \bar{\varphi} = \frac{1}{4\pi} \int \varphi \, d\omega \),
and \( \varepsilon \) is the ratio of the number of collisional excitations to the number of photoexcitations; this quantity may be written as

\[
\varepsilon = 0.74 \cdot 10^{11} \frac{\bar{Q}_0 \sigma_x T_e^{\frac{3}{2}}}{1 + X_0} \int \Phi \nu_0^{-2} f_0^{-1},
\]

where \( \bar{Q}_0 \) is the mean inelastic cross section in atomic units, namely 10 [9], \( X_0 \) is the energy of excitation in units of \( kT_e \), and \( f_0 \) is the oscillator strength of the transition.

In Eq. (2), \( \bar{\varphi} \nu \) is found by solving the equation of transfer in the Eddington approximation, and replacing the integral by a Gaussian sum over three points that are determined by means of zeros of Hermite polynomials.

For the source function in the continuum, we adopt the form

\[
S_c = S_1 (1 + \beta \tau_0 + A e^{-\tau_0}),
\]

where \( \tau_0 \) is the optical depth at the center of the line, \( \beta = 1.5 \tau_0 \), and \( \tau_0 \) is the ratio of the opacity in the continuous spectrum to the opacity at the center of the line. This quantity is also assumed not to vary with height in the chromosphere. The term \( \beta \tau_0 \) may be neglected for the K line [10]. We introduce a second exponential term into the expression for \( S_c \), in order to represent the break in the rise of temperature in the chromosphere at a definite height:

\[
S_c = S_1 (1 + A_1 e^{-\tau_0} + A_2 e^{-\tau_0}).
\]

Moreover, we shall use the conclusion mentioned as to the weak height dependence of \( \Delta \lambda_D \), and also the assumption of constant \( n_e \). We then have

\[
I_{\nu} = \frac{1}{S_1} \left[ 1 + A_1 \sigma_1 \left( \frac{\Phi_{\nu}}{q_{0\nu} + \Phi_{\nu}} + \frac{3}{k_{0\nu} + \Phi_{\nu}} \right) + A_2 \sigma_2 \left( \frac{\Phi_{\nu}}{q_{0\nu} + \Phi_{\nu}} + \frac{3}{k_{0\nu} + \Phi_{\nu}} \right) \right],
\]

where

\[
\Phi_{\nu} = e^{-\tau_0}; \quad y = \frac{\Delta \lambda}{\Delta \lambda_D}; \quad \sigma_1 = \frac{\varepsilon}{1 + \varepsilon - \sum \frac{\alpha_1}{1 - \frac{q_{0\nu}}{\Phi_{\nu}}}},
\]

\[
\sigma_2 = \frac{\varepsilon}{1 + \varepsilon - \sum \frac{\alpha_2}{1 - \frac{q_{0\nu}}{\Phi_{\nu}}}}.
\]

The coefficients \( k_j \) and \( L_j \) are found from the conditions

\[
1 + \varepsilon = \sum \frac{d_i}{1 + k_j x_i^2},
\]

and

\[
1 + \frac{A_1 \sigma_1}{1 - \frac{q_{0\nu}}{\Phi_{\nu}}} + \frac{A_2 \sigma_2}{1 - \frac{q_{0\nu}}{\Phi_{\nu}}} = \sum \frac{L_j}{1 - \frac{k_j x_i^2}{}},
\]

Here \( x_i \) and \( \alpha_i \) represent the division points and weights of the Gaussian quadrature. For \( \bar{Q}_{12} = 10 \), \( \varepsilon = 10^{-5} n_e \). With a mean value \( n_e \approx 10^{10} \), we obtain \( \varepsilon = 10^{-5} \). The values of \( k_i \) for \( \varepsilon = 10^{-5} \) have been taken from [10], and are

\[
k_1 = 0.665, \quad k_2 = 0.046, \quad k_3 = 3 \cdot 10^{-4}.
\]

For given \( \varepsilon \), the expression (5) may be transformed into a more convenient form for computation by substituting into it the solution of the system of equations (9), namely

\[
I_{\nu}/S_1 = 1 + \frac{A_1 \sigma_1}{1 + \frac{A_1 \sigma_1}{1 - \frac{q_{0\nu}}{\Phi_{\nu}}}} + \frac{A_2 \sigma_2}{1 + \frac{A_2 \sigma_2}{1 - \frac{q_{0\nu}}{\Phi_{\nu}}}} - \sum_i \Pi_i R_i,
\]

where

\[
\Pi_i = 1 + \frac{A_1 \sigma_1}{1 + \frac{A_1 \sigma_1}{1 - \frac{q_{0\nu}}{\Phi_{\nu}}}} + \frac{A_2 \sigma_2}{1 + \frac{A_2 \sigma_2}{1 - \frac{q_{0\nu}}{\Phi_{\nu}}}}; \quad R_i = \prod_j \frac{k_j x_i}{1 - \frac{k_j x_i}{}}.
\]

For \( \varepsilon = 10^{-4} \), the matrix is

\[
\beta_{ij} = \begin{pmatrix} 0.389 & 0.226 & 0.035 \\ -0.345 & 0.534 & 0.084 \\ -0.035 & -0.010 & 0.704 \end{pmatrix},
\]

while for \( \varepsilon = 10^{-7} \)

\[
\beta_{ij} = \begin{pmatrix} 0.393 & 0.225 & 0.040 \\ -0.350 & 0.534 & 0.095 \\ -0.043 & -0.682 & 0.855 \end{pmatrix}.
\]
Thus, even as \( \epsilon \) changes by three orders of magnitude, \( S_{1j} \) hardly changes at all. The central part of the profile is determined mainly by the quantities \( k_1, k_2, S_{1j} \). All these quantities depend weakly on \( \epsilon \), that is, on \( n_e \). At the same time the wings of the line depend essentially on \( k_2 \sim 0.1 \epsilon^{1/2} \). The choice of the quantities \( c_1 \) and \( A_1 \), which determine the wings of the line, therefore depends critically on the assumption of constant \( n_e \), and on the selected value of \( n_e \). The quantities \( c_2 \) and \( A_2 \), which determine the central part of the line, and primarily the ratio \( I_p/I_0 \) of the intensity at the peaks to the central intensity and the separation \( \Delta \lambda_p \) of the peaks, depend very little on \( \epsilon \). This last result might have been anticipated by keeping in mind the differing role of the two exponential terms in the expression (5). The quantities \( A_1 \) and \( c_1 \) govern the region with a weak temperature rise which enters fully into the region of \( K_{232} \) emission. The assumption that \( n_e \) is constant does not hold sufficiently accurately over so extensive a region. The quantities \( A_2 \) and \( c_2 \) determine the sharp rise in temperature at definite \( \tau_\phi \). This region enters into the region of line emission only over a very small interval of heights. We may regard \( n_e \) as constant over so small an interval without substantial error.

It is very laborious to fit the profiles with the four parameters \( A_1, A_2, c_1, c_2 \). The task is facilitated somewhat if we note the different roles of these parameters, as mentioned above. Furthermore, to facilitate the fitting procedure we may transform formula (10) into

\[
I_p/S_1 = a_0(y) + A_1a_1(y, c_1) + A_2a_2(y, c_2);
\]  

(13)

here

\[
a_0 = 1 - \frac{3}{i=1}R_i;
\]

(14)

\[
a_1 = \sigma_1 \left( \frac{1}{1 + c_1/\Phi_y} - \frac{3}{i=1} \frac{R_i}{1 - c_1/x_4} \right);
\]

(15)

\[
a_2 = \sigma_2 \left( \frac{1}{1 + c_2/\Phi_y} - \frac{3}{i=1} \frac{R_i}{1 - c_2/x_4} \right).
\]

(16)

Figure 1 shows two theoretical profiles in relative units, for \( \mu = 1 \). As \( c_2 \) decreases, \( I_p/I_0 \) and \( \Delta \lambda_p \) increase. Figure 2 presents profiles of the \( K_{232} \) line obtained by N. S. Shilova from a flocculus associated with the large chromospheric flare of July 18, 1961. The profiles show that \( I_p/I_0 \) varies over quite a wide range.

The following relation has been found to hold [11] with a high correlation coefficient (0.97):

\[
I_0 = 0.91 I_p - 0.02.
\]

(17)

Let us take \( I_0 \) equal to 0.07 for an undisturbed region [12], and 0.20 for a typical facula. Then \( I_p/I_0 \) will be 1.52 in the undisturbed region, and 1.21 in the active region. Thus, the high-temperature gradients in the active and undisturbed regions must also differ. In the active region the ratio \( I_p/I_0 \) will therefore decrease, the peaks will coalesce, and the widths of the profile and the peaks will increase, in agreement with observation [11]. The values of the temperature parameters are found to be as follows: for the active region, \( A_1 = 15 \), \( A_2 \approx 5 \cdot 10^4 \) to \( 10^5 \), \( c_1 = 5 \cdot 10^{-4} \), \( c_2 = 0.4 \) to 0.5; for undisturbed regions, \( A_1 = 15 \), \( A_2 = 10^4 \), \( c_1 = 5 \cdot 10^{-4} \), \( c_2 = 0.075 \) to 0.10.

Observed values are given in [11] for the widths of \( K_{232} \) line profiles. By comparing the measured width with that computed theoretically, we can find \( \Delta \lambda_p \) and the value of the turbulent velocity \( \xi \); the latter is 9...
km/sec and is close to the value obtained by Khokhlova [8].

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<tr>
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<tbody>
<tr>
<td>( \frac{W_{e-1}}{W_e a_1} )</td>
<td>1.5</td>
<td>1.2</td>
<td></td>
<td>Profile width</td>
</tr>
<tr>
<td>( \frac{\Delta \lambda p a_1}{\Delta \lambda p_1} )</td>
<td>1.8</td>
<td>1.8</td>
<td></td>
<td>Peak separation</td>
</tr>
<tr>
<td>( \frac{I_{\theta 1}}{I a_1} )</td>
<td>3.5</td>
<td>1.4</td>
<td></td>
<td>( K_3 ) intensity</td>
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We can examine the variation of the profile from center to limb by means of formula (6). Since \( \mu \) enters the expression (6) as a coefficient of the quantities \( c_1 \), \( c_2 \), and \( k_j \), a decrease in \( \mu \) will have the same effect as a decrease in those quantities: In accordance with the observations, the profile expands, the wings rise, the peak separation and the ratio \( I_p / I_0 \) increase. Since the absolute value of \( I_p \) depends mainly on the quantities \( A \), the intensity in the peaks does not vary from center to limb. Table I gives the ratios of the profile shape parameters for \( \mu = 1 \) and \( \mu = 0.1 \), using both experimental [11] and theoretical data. The qualitative agreement is satisfactory, except for \( I_0 \) which is not determined reliably enough by the quantity \( A_1 \).

Distribution of temperature with respect to height. Equation (5) allows us to obtain the temperature as a function of \( \tau_0 \). Taking \( S_1 \) to be the Planck function for \( T = 4000^\circ \text{K} \), we find

\[
T = \frac{15.95}{\log \left( \frac{10^4}{S_e S_1} + 1 \right)}
\]  

(17)

Using the above values of \( A \) and \( c_0 \), we can obtain the quantity \( S_e / S_1 \), and thereby also \( T(\tau_0) \).

It is easily seen that the value \( c_1 = 5 \cdot 10^{-4} \) is somewhat too high. The absorption coefficient in the K line for \( \xi = 7.5 \) km/sec is \( \approx 0.8 \cdot 10^{-12} \). At a height of 1000-1500 km in a homogeneous chromosphere [15], we obtain \( \tau_0 \approx 5 \cdot 10^{-6} \). Then \( T = 4000-4500^\circ \text{K} \). At this temperature the electrons in the chromosphere would be furnished mostly by the metals, but this conflicts with the observations [13]. Furthermore, eclipse observations give a value for the temperature at the base of the chromosphere of not lower than 4500^\circ \text{K}, and even higher at a height of 1000 km.

The overestimate of the quantity \( c_1 \) is due to our basic assumption of a constant \( n_e \). Since \( n_e \) actually decreases with height, in order to obtain the observed profile we have to adopt a weaker drop in the temperature toward the base of the chromosphere than in the case of constant \( n_e \). In other words, the region having a temperature \( T \approx 5500^\circ \text{K} \) should set in at a smaller geometric height.

To tie the temperature in with the height, we shall use the values of the function \( \Delta \psi (h) = n_e^2 \exp [2h] \) derived by Ivanov-Kholodnyi and Nikol'skii [14] from the observations of Athay and Menzel [15]. In the region \( \tau_0 > 1 \), we use the Saha equation. For \( h \approx 1000 \) km, \( n_e \approx n_0 \). Then the Saha equation for hydrogen may be written as follows:

\[
\log n_H = -15.38 + \log \Delta \psi \frac{6.85 \cdot 10^4}{T}.
\]  

(18)

The ionization potentials of hydrogen and singly ionized calcium are close together. We can therefore obtain directly from (18) the value of \( \log n_{CaII} \), which is almost entirely in the ground state. Thus formula (18) allows us to determine \( \tau_0 (h) = f_1 (T(h)) \). On the other hand, from formula (5) we obtain \( T = f_2 (\tau_0) \). Simultaneous solution of these two equations gives \( T(h) \) and \( \tau_0 (h) \). In solving the equations we obtain \( n_H (h) \) as a by-product. Figures 3, 4, and 5 show the relations \( T(h) \), \( \tau_0 (h) \), and \( n_{CaII} (h) \) for active and undisturbed regions. The value of \( c_1 \) has been taken equal to \( 10^{-5} \) (here the value of \( \tau_0 \) is found to be \( \approx 10^5 \) at a height of 1000 km). The broken curves in Fig. 3 indicate the behavior of the

\[ T_e \]

\[ U \]

\[ 5300 \]

\[ 5500 \]

\[ 1000 \]

\[ 5000 \]

\[ 7000 \]

\[ h \]

Fig. 3. \( T_e \) as a function of height for an active (A) and an undisturbed (U) region.
temperature for $c_1 = 5 \cdot 10^{-4}$, and the dotted curves show the temperature rise above 7000 km for an active region and 4000 km for an undisturbed region. At these heights, $\tau_0$ very rapidly becomes much smaller than unity.

We see from Fig. 3 that the temperature in the active region is somewhat lower than in the undisturbed region at the same height. The break in the temperature variation starts at a lower height in the undisturbed than in the active region. This latter fact should be especially well established, since it depends little on the adopted value of $n_e$, as we have remarked above.

Eclipse observations of the K line allow us to establish directly the boundary of the quasi-isothermal region. One can show that as the temperature increases, the relative Ca II content falls sharply. Figure 6 presents ionization curves for Ca I, Ca II, and Ca III, obtained by the method given in [16]. The figure shows that for a temperature substantially greater than 7000°K, the relative Ca II content falls off by several orders of magnitude. One might have expected that at high temperatures, the increasing relative Ca III content would cause recombinations to play a more important part in exciting the $4f^2_{5/2}$ level of the Ca II atom than would electron collisions.

We shall demonstrate that this is not so. The number of elementary recombinations at a temperature $T_e = 7500°K$ may be written [17] in the form

$$Z_{\text{rec}} = 1.68 \cdot 10^{-13} n_e n_{\text{Ca III}}.$$  \hfill (19)

As the temperature rises, the numerical coefficient in this expression progressively decreases:

$$Z_{\text{rec}} < 10^{-13} n_e n_{\text{Ca III}}.$$  

The number of excitations by electron collision for $Q_0 = 10^{-15}$ and $T_e = 40,000°K$ is

$$Z_{\text{col}} = 2 \cdot 10^{-7} n_e n_{\text{Ca II}}.$$  \hfill (20)

Hence

$$Z_{\text{rec}} / Z_{\text{col}} < 10^{-6} n_{\text{Ca III}} / n_{\text{Ca II}}.$$  \hfill (21)

From Fig. 6 we find $n_{\text{Ca III}} / n_{\text{Ca II}} = 10^8$. Thus, recombinations are at least three orders of magnitude less effective than electron collisions.

The intensity of the radiation in the K line of Ca II is proportional to the probability $W$ of excitation by electron collision. Figure 7 shows the temperature dependence of the quantity $n^4 W / \Sigma n^4$ in relative units. As the temperature increases to several tens of thousands of degrees, the curve $n^4 W / \Sigma n^4$ falls by two or three orders of magnitude.

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Fig. 7. Relation between \(n(1)W/\Sigma n(1)\) and \(T_e\).

Fig. 8. The K-line radiation according to Vyazanitsyn's data for the 1945 and 1952 eclipses.

We consider now how the intensity of the K-line radiation should vary with height in the chromosphere. Along the branch with a low temperature gradient, the K-line radiation should be due mainly to the fall in electron density. Here it is self-absorption that plays the main part in depressing the gradient. As the temperature rises, a sharp increase in the gradient of the line radiation begins at a definite height, and self-absorption vanishes completely. Figure 8 displays the relation between the K-line radiation and height according to Vyazanitsyn's data [18] for the 1945 and 1952 eclipses, for the undisturbed chromosphere. The increase in the gradient at a height \(h \approx 4000\) km stands out quite clearly. The value of the height at which the increase in the gradient sets in is close to the value we have obtained. Figure 9 presents a similar relation for the 1941 eclipse. Here the increase in the gradient is less well defined. It is entirely possible that the height at which the temperature rise sets in may fluctuate from eclipse to eclipse.

Unfortunately, we do not have analogous observations at our disposal for the K line in active regions of the chromosphere. However, it is known that in active regions the K line can be followed to considerable heights, of the order of 10,000 to 12,000 km. The sharp temperature rise to several tens of thousands of degrees should rapidly cause the K line to cease its emission in the chromosphere. Figures 8 and 9 show that the boun-
dary of the emitting region lies 2000-3000 km above the point where the gradient changes. If this interval holds for an active region also, we might conclude that the point where the temperature variation changes sharply lies at a height of 7000-9000 km in active regions of the chromosphere. This matter can be settled conclusively by obtaining observations of the K line in an active region of the chromosphere.

The relation \( n_e(h) \) has been obtained from \( \Delta \psi(h) \) and \( T(h) \), and is shown in Fig. 10. The temperature and electron-density distributions obtained here agree, on the whole, with the representation developed by Ivanov-Kholodnyi and Nikol’skii [14] for the structure of the solar chromosphere in active and undisturbed regions. A comparison of Figs. 3 and 10 shows that for heights having equal temperatures the electron density in an active region is three times as great as in an undisturbed region, as would follow also from the observations.

Conclusions. Despite the rough character of our discussion, associated mainly with the assumption of constant \( n_e \), we believe it is possible to advance the following conclusions.

1. The Thomas-Jeffries theory provides an entirely satisfactory description of the profile of the K line.

2. The central part of the K line originates in a region having an extremely sharp rise in temperature.

3. The relation between temperature and height, as obtained through the function \( \Delta \psi(h) = n_e^2 T^{-4/3} \) shows that for equal heights the temperature in active regions is somewhat lower than in undisturbed regions.

4. Analysis of eclipse observations yields the height at which the sharp temperature rise begins in undisturbed regions. This value of the height \( h \approx 4000 \) km agrees with that obtained by us from an analysis of the K-line profile.

To improve the line profile further, one should revise the fundamental initial formulas by incorporating the height dependence of \( n_e \) and \( \Delta \lambda \). Introducing a varying \( n_e \) should change the model at heights \( h \leq 3000 \) km. It would also be desirable to obtain reliable profiles in active and undisturbed regions.

I consider it a pleasant obligation to express my thanks to G. M. Nikol’skii for useful discussions.

LITERATURE CITED


All abbreviations of periodicals in the above bibliography are letter-by-letter transliterations of the abbreviations as given in the original Russian journal. Some or all of this periodical literature may well be available in English translation. A complete list of the cover-to-cover English translations appears at the back of this issue.