PHOTOELECTRIC PHOTOMETRY OF DIFFUSE NEBULAE

DONALD E. OSTERBROCK AND RALPH E. STOCKHAUSEN
Washburn Observatory, University of Wisconsin
Received September 28, 1959

ABSTRACT

Measurements are presented of the fluxes in H/3 from two diffuse nebulae, made with interference filters in a photoelectric photometer attached to a 5-inch F/5 refractor. For one of the nebulae a published radio-frequency observation is also available, and comparison of these two measurements appears to confirm the theory of hydrogen emission, though the accuracy of the measurements is low and the agreement is only just barely within the limits of precision. The Zanstra method is used to calculate the fluxes of ionizing Lyman continuum radiation of the hot stars involved in the nebulae from the H/3 measurements and from published radio-frequency measurements, and these fluxes are compared with the predictions of published model atmospheres. In all cases the deduced far-ultraviolet flux is either equal to or less than the flux expected according to the models; this result can be understood if the ionizing radiation is not completely absorbed in all the nebulae studied. Methods for improving the accuracy of further photoelectric measurements of emission nebulae are given.

I. INTRODUCTION

Diffuse nebulae have been intensively studied in recent years, by both direct photography and spectra, but only a little quantitative photometry has been done, although the photometric results—that is, the amounts of energy radiated in the various emission lines—would be most important for understanding the nebulae. An extensive survey in which nebulae were sampled photographically with a slit spectrograph was made by Johnson (1953), while a detailed photographic photometry of four bright nebulae was carried out by Boggess (1954; see Aller 1956). Also, photographic measurements have been made of a larger number of Magellanic Cloud nebulae by Doherty, Henize, and Aller (1956). Photoelectric photometry has the advantage that extended objects can be measured directly, without point-by-point integration as in photographic work and some results have been obtained with an H/3 interference filter by Strömgren and Hiltner (Strömgren 1951). Planetary nebulae have been intensively measured photoelectrically, both with interference filters and with a scanning spectrograph, by Liller and Aller (1954) and by Liller (1955). For observing diffuse nebulae, which have large angular diameters, a small telescope with a correspondingly small scale is advantageous because a large area can be measured at a single setting (see, e.g., Whitford 1936; Kron 1958; Johnson 1960), and we have therefore used a 5-inch aperture, F/5 telescope, together with interference filters, for the nebular program outlined below.

In the next section the measurements which lead to the H/3 fluxes received at the earth from two different diffuse nebulae are described. In Section III the measured flux for one nebulae is compared with the published radio-frequency continuum flux from the same nebulae; the ratio of these fluxes is predicted by the theory of the hydrogen recombination spectrum and the comparison therefore serves as a test of the theory. Then the H/3 measurements, as well as published radio measurements, are used to compute by Zanstra's method the Lyman continuum radiation from the hot stars involved in the nebulae, and a final summary completes the paper.

II. PHOTOELECTRIC MEASUREMENTS

The measurements of diffuse nebulae were made with a small refractor, especially constructed for this program by Mr. Lloyd McElwain in the Washburn Observatory shop. The objective is a coated, cemented achromat (sold by A. Jaegers) of 128-mm aperture (stopped down to 4 1/2 inches in the telescope) and 628-mm focal length, and at
the focal plane there is a brass wheel, carrying several round diaphragms of different sizes, any one of which can be rotated to the optical axis. The cold box in which the photoelectric cell is mounted is hinged and can be swung away from the optical axis so that an eyepiece on another hinged mount can be used to examine the field visually. A filter slide that holds as many as four 2-inch square filters is near the focus, between the lens and the diaphragm wheel, and the whole telescope is mounted equatorially at the Pine Bluff station, about 15 miles west of Madison. With this 5-inch refractor the scale is 5.6/\text{mm}, and the largest diaphragm used, 9.0 \text{ mm} in diameter, isolates a circular field 50\arcmin in diameter.

### Table 1

<table>
<thead>
<tr>
<th>Wave Length A</th>
<th>1P21 Response</th>
<th>Nebular-Line Filter</th>
<th>Comparison Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Transmission</td>
<td>Over-all Response</td>
<td>Transmission</td>
</tr>
<tr>
<td>4284</td>
<td>1320</td>
<td>0.000</td>
<td>0.003</td>
</tr>
<tr>
<td>4423</td>
<td>1308</td>
<td>0.000</td>
<td>0.088</td>
</tr>
<tr>
<td>4536</td>
<td>1290</td>
<td>0.004</td>
<td>0.284</td>
</tr>
<tr>
<td>4590</td>
<td>1281</td>
<td>0.004</td>
<td>0.354</td>
</tr>
<tr>
<td>4655</td>
<td>1270</td>
<td>0.051</td>
<td>0.178</td>
</tr>
<tr>
<td>4739</td>
<td>1244</td>
<td>0.118</td>
<td>0.054</td>
</tr>
<tr>
<td>4784</td>
<td>1239</td>
<td>0.172</td>
<td></td>
</tr>
<tr>
<td>4828</td>
<td>1221</td>
<td>0.254</td>
<td></td>
</tr>
<tr>
<td>4861</td>
<td>1210</td>
<td>0.317</td>
<td>0.017</td>
</tr>
<tr>
<td>4873</td>
<td>1200</td>
<td>0.380</td>
<td></td>
</tr>
<tr>
<td>4920</td>
<td>1180</td>
<td>0.491</td>
<td></td>
</tr>
<tr>
<td>4959</td>
<td>1160</td>
<td></td>
<td>0.014</td>
</tr>
<tr>
<td>4970</td>
<td>1155</td>
<td>0.513</td>
<td>0.004</td>
</tr>
<tr>
<td>5022</td>
<td>1132</td>
<td>0.390</td>
<td></td>
</tr>
<tr>
<td>5131</td>
<td>1072</td>
<td>0.159</td>
<td></td>
</tr>
<tr>
<td>5251</td>
<td>0994</td>
<td>0.076</td>
<td></td>
</tr>
<tr>
<td>5494</td>
<td>0768</td>
<td>0.018</td>
<td></td>
</tr>
<tr>
<td>5946</td>
<td>0316</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td>6744</td>
<td>0008</td>
<td>0.000</td>
<td>0.002</td>
</tr>
</tbody>
</table>

The filters used for the measurements were commercially available Bausch and Lomb interference filters bound with colored-glass blocking filters; the nebular-line filter has a nominal peak wave length of 4970 A, a full width at half-maximum of 250 A, and is bound with a Corning 3387 filter, while the comparison filter has a peak wave length of 4600 A, width of 160 A, and is bound with a Corning 3389 filter. Thus the nebular-line filter transmits the three strong emission lines H\beta \lambda 4861, [O iii] N2 \lambda 4959, and N1 \lambda 5007, while the comparison filter transmits a band between H\beta and H\gamma, in which there are no strong emission lines in diffuse nebulae. The transmission-curves of these filter combinations were measured in the laboratory of the Washburn Observatory, using a Perkin-Elmer double-pass monochromator with 10-A resolution, and the results are listed in Table 1.

At the telescope the measurements were made with a 1P21 photomultiplier, operated at 750 volts anode potential, and the signal was fed into a chopper-stabilized d.c. amplifier followed by a Brown recorder. The gain and attenuation settings on the amplifier were measured with a Student’s potentiometer operated as a voltage source and are known to have better than 1 per cent accuracy. The relative sensitivity of the photomultiplier as a function of wave length was not directly measured for these observa-
tions; instead, an average curve was used that is the result of measurements made in 1956 by Dr. J. D. Bahng of the sensitivities of three different 1P21 photomultipliers. This adopted photomultiplier sensitivity is listed in Table 1, and the resulting calculated over-all sensitivities of the filter-photomultiplier combinations are listed and are also displayed graphically in Figure 1. The transmission of the objective varies only slightly through the whole range of wave length used, so the curves of Figure 1 show the over-all sensitivity of the photometric system.

![Diagram of sensitivity of interference filter-photocell combinations as functions of wave length](image.jpg)

Fig. 1.—Sensitivity of interference filter-photocell combinations as functions of wave length

The two diffuse nebulae NGC 281 and NGC 2175 were selected for measurement because each is bright, has a convenient size (between 20' and 40' in diameter), has a well-identified exciting star, and is inclosed for a good part of its circumference by an ionization front, indicating that a considerable fraction of the ionizing radiation is absorbed within the nebula (Spitzer 1954; Pottasch 1956, 1958; Osterbrock 1957). The nebular measurements were made with large diaphragms (2.2-mm diameter = 12' for NGC 281, 9.0-mm = 50' for NGC 2175), and sky measurements were made with the same diaphragms outside, but in the immediate vicinity of, the nebulae. In the case of NGC 281, the nebular measurements were made with the diaphragm centered at six different points selected so that they cover, with some overlapping, the entire nebula as it appears on the National Geographic Society–Palomar Observatory Sky Survey print, and the sky measurements were made at five different points surrounding the nebula. In the case of NGC 2175, the diaphragm inclosed the entire nebula, so only a single position was measured; two sky positions were measured on either side of the nebula, and, in addition, measurements were made with a smaller diaphragm (1.1 mm = 6') of the central star in the nebula, HD 42088, to check that its contribution to the measured deflection was negligibly small.

However, the light of the large number of faint stars scattered over the face of the
nebula is by no means negligible, and so the sky measurements, together with the comparison-filter measurements, were used to correct the nebular-line measurements. The sky measurements were used to determine the average ratio of the nebular-line to comparison-filter deflections; the measured flux in this case results from stars, from the night airglow, and from whatever weak skylight there is that is caused by scattering of city lights. The scheme was to use the intensity measured with the comparison filter at the nebular position, together with the average ratio of nebular-line to comparison-filter intensities from the sky measurements, to calculate the flux in the nebular-line band due to the sky. This sky component was subtracted from the measured nebular-line-filter intensity, and the difference was attributed to the effect of the nebular emission lines alone. A small correction (amounting to 6 per cent of the final nebular flux) was made for the non-zero transmission of the comparison filter at the nebular-line wave lengths. The final measured nebular fluxes, expressed in units of deflections on the recorder, are each judged, from their internal consistency, to have an accuracy of about ±25 per cent, the errors resulting almost entirely from uncertainties in the sky correction and (with the small diaphragm used on NGC 281) from miscentering of the telescope on the selected positions in the nebula and uncertainties in the corrections for overlapping of the measured areas. The 50′ diaphragm used for NGC 2175 is larger than the nebula, so that the only source of error in this case is the sky correction, which, however, is rather considerable because a large area of blank sky is included within the diaphragm.

The nebular measurements were converted to energy units by means of comparison measurements made both on stars and on planetary nebulae with known fluxes. The measured deflection, \( D \), for any source can be written

\[
D = C \left( \sum s_\lambda F_\lambda + \int s_\lambda F_\lambda d\lambda \right),
\]

where \( s_\lambda \) is the sensitivity of the telescope-filter-photomultiplier system (listed in Table 1), \( F_\lambda \) is the flux either in an emission line (in the sum) or per unit wave-length interval in the continuum (in the integral), and \( C \) is the constant to be evaluated (Liller 1955). The stars that were measured for calibration are 10 Lacertae, spectral type O9 V; e Orionis, B0 Ia; and HR 483, G2 V (Johnson and Morgan 1953). For each of these stars, comparison of \( V \), the measured visual magnitude (Johnson and Morgan 1953; Johnson and Harris 1954), with \( V = -26.73 \), the visual magnitude of the sun (Stebbins and Kron 1957), gives the flux relative to the sun's flux at the representative visual wave length \( \lambda 5560 \) Å. Since the flux from the sun is known as a function of wave length (Minnaert 1953), the flux from the star at \( \lambda 5560 \) Å can be calculated, and then the flux at any other wave length can be found by using available relative spectrophotometric results. For the two early-type stars 10 Lacertae and e Orionis, the relative fluxes have been taken from unpublished measurements by Whitford and Code, made with a photoelectric scanner on the 60-inch telescope at Mount Wilson, while for the G2 V star HR 483 the solar measurements (Minnaert 1953) have been used. These stellar photoelectric measurements were used not only for calibration purposes but also to determine the extinction coefficient for reducing all the measurements, nebular and stellar, to a common zenith distance.

The planetary nebulae used for calibration were NGC 7662 and IC 418, both of which have been measured photoelectrically by Liller (1955). Liller's results are expressed in his Table 6 in terms of \( S \), the surface brightness at the outer boundary of the nebula (assumed spherical for reduction purposes); this can be converted into the flux \( F \) by the relation

\[
F = \left( \frac{A}{2.06 \times 10^5} \right)^2 S,
\]

where \( A \) is the radius, in seconds of arc, used in the reduction. The various calibrations for each of the two nebulae (which are in different parts of the sky and could not both

© American Astronomical Society • Provided by the NASA Astrophysics Data System
be observed conveniently the same night) are compared in Table 2. For each nebula the
average calibration constant, which is a straight mean weighting the stellar and the
planetary methods equally, is probably correct to within a mean error of about 25 per
cent.

Finally, the nebular deflections, which result from the combined effect of N1, N2,
and Hβ, must be split up to give the physically significant fluxes in the individual emis-
sion lines. The ratio of fluxes \( F_{\text{N1}}/F_{\text{N2}} = 3.0 \) is a well-determined value both theoretically
(Garstang 1951) and observationally (Liller and Aller 1954) and is independent of the
excitation conditions. The ratio of hydrogen to forbidden-line strengths on the other

### Table 2

<table>
<thead>
<tr>
<th>Object</th>
<th>( C ) (ergs/cm(^2)sec/ unit deflection)</th>
<th>Object</th>
<th>( C ) (ergs/cm(^2)sec/ unit deflection)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 Lac.</td>
<td>1 88×10(^{-11})</td>
<td>e Ori</td>
<td>1 66×10(^{-11})</td>
</tr>
<tr>
<td>HR 483</td>
<td>1 50×10(^{-11})</td>
<td>IC 418</td>
<td>1 35×10(^{-11})</td>
</tr>
<tr>
<td>Average of two stars</td>
<td>1 69×10(^{-11})</td>
<td>Average</td>
<td>1 51×10(^{-11})</td>
</tr>
<tr>
<td>NGC 7662</td>
<td>1 16×10(^{-11})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average</td>
<td>1 42×10(^{-11})</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 3

<table>
<thead>
<tr>
<th>Nebula</th>
<th>Flux (erg/cm(^2) sec)</th>
<th>Nebula</th>
<th>Flux (erg/cm(^2) sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 281</td>
<td>1 0×10(^{-9})</td>
<td>NGC 2175</td>
<td>1 9×10(^{-9})</td>
</tr>
</tbody>
</table>

band varies with physical conditions; we have adopted, as an average value for diffuse
nebulae, \( F_{\text{N1}}/F_{\beta} = 1.2 \) (Aller 1942; Johnson 1953). The fluxes in H\(\beta\) from the two
nebulae, expressed in energy units per unit area per unit time at the top of the earth’s
atmosphere, can then be calculated and are listed in Table 3.

These final results are uncertain by a factor of approximately 1.7, resulting from the
errors in the deflections themselves (25 per cent), the errors in the calibration (25 per
cent), and the error in the assumed ratio \( F_{\text{N1}}/F_{\beta} \) (20 per cent). Future measurements of
this type can be made much more accurately by using narrow-band interference filters
to isolate a single emission line and to reduce the sky correction, by using two comparison
filters on either side of the emission line, so that the sky correction can be accurately
interpolated, and by spending more time on the calibration, so that several stars and
planetaries can be intercompared.

### III. Theory and Comparison

The hydrogen emission lines in a typical diffuse nebula are caused by recombination
of hydrogen ions and electrons, followed by downward radiative transitions leading ulti-
mately to the ground state, and the theory of this process has been worked out at many
levels of approximation (Zanstra 1927; Baker and Menzel 1938; Searle 1958), the most
nearly accurate treatment being that of Burgess (1958). The theory predicts, among other things, the relative intensities of the various emission lines; comparisons of observed hydrogen-line ratios with the theory have been made for planetary nebulae by several authors (Aller 1951; Aller and Minkowski 1956; Burgess 1958) and for diffuse nebulae by Johnson (1953). In recent years, observations of the thermal radio-frequency emission of some diffuse nebulae have become available (Haddock, Mayer, and Sloanaker 1954; Haddock 1957; Westerhout 1958), and, since the theory also predicts the ratio of optical-line emission to radio continuous emission, a comparison over a very wide range in wave length is thus possible. Such a comparison has already been carried out by Boggess (1954), using results of photographic photometry for the bright southern nebulae M8, M16, M17, and M20. There are no published radio observations of NGC 281, but a measurement at 22 cm of NGC 2175 is available (Westerhout 1958), and the present section contains a comparison of the predicted and observed ratios of radio-frequency to optical fluxes from this nebula.

The emission coefficient per unit volume for $H\beta$ may be written

$$j_\beta = 16 b_4 A_4 \frac{h}{2\pi m_k T} \left( \frac{k}{2\pi m_k T} \right)^{3/2} \exp \left( \frac{T}{k_e} \frac{N_p N_e}{N_p N_e} \right) = f(T) N_p N_e,$$

where $N_p$ and $N_e$ are the number densities of protons and electrons, respectively, per unit volume, and all the other symbols are defined, for instance, by Seaton (1954). The values of $f(T)$ are given numerically by Burgess (1958) for temperatures of 10000° and 20000°, and we shall use his computed values for case B, corresponding to a nebula optically thick in all the Lyman lines. This is undoubtedly the most nearly correct of the cases that have been worked out (Aller 1951), but, since the optical radius of the nebula in the Lyman continuum is unity (Strömgren 1939), the actual physical case must be one approximating the assumptions of case B rather closely for the lower energy levels, but going over to a case intermediate between A (optical depth zero) and B (very large optical depth) for energy levels near the ionization limit. The calculations made according to case B cannot be badly in error, however, because the tables of Burgess (1958) show that captures to all the levels with principal quantum number $n > 12$ make a contribution of less than 10 per cent to the $H\beta$ emission coefficient, and so it is clear that the exact solution is not too sensitive to the assumptions made concerning the very high levels.

The free-free thermal emission coefficient per unit frequency interval in the radio region may be written (Smerd and Westfold 1949) thus:

$$j_\nu = 4 \pi \kappa_\nu B_\nu = \frac{8 \pi \kappa_\nu k T \nu^2}{c^2},$$

where

$$\kappa_\nu = \frac{4Z^2 e^6 N_e N_i}{3 \left[ 2 \pi (m k T)^{1/2} \right]^{1/2} \nu^2} \ln (1 + \frac{\nu_0}{\nu}).$$

The last factor in equation (5) is a logarithmic function of temperature and density, which, for the representative values 10000° K and 50 electrons/cm$^3$, has the value 24.8; if we adopt this constant in place of the slowly varying factor, the emission coefficient becomes

$$j_{1390} = 4.59 \times 10^{-37} \frac{N_e N_i}{\nu_{1/2}} \text{ergs/cm}^3 \text{sec cps}.$$

The ratio of the radio-frequency emission coefficient to the $H\beta$ emission coefficient thus is

$$\frac{j_{1390}}{j_\beta} = \frac{4.59 \times 10^{-37}}{N_p} N_i \left( \frac{T \nu_{1/2} f(T)}{N_p} \right).$$
which is independent of electron density but depends on the ratio of positive-ion density (since all ions contribute to the free-free processes) to hydrogen-ion density (which determines the hydrogen-line emission coefficients). However, nearly all the ions are hydrogen, so we can safely use for the helium abundance the result of Mathis (1957) for the Orion Nebula, \( N_{\text{He}}/N_{\text{H}} = 0.13 \), and neglect altogether the other ions; hence \( N_{\alpha}/N_{e} = 1.13 \) to a high degree of accuracy. The resulting ratios of calculated emission coefficients at 10000°K and 20000°K are given in Table 4; since the nebulae are presumably optically thin at both wave lengths, this ratio is also the ratio of emergent fluxes. It may be noted that the computed ratios depend only weakly on the temperature, the relative strengths among the different Balmer lines exhibit this same behavior to an even more marked degree (Baker and Menzel 1938). The physical reason in both cases is the same, namely, that the capture and collisional cross-sections of hydrogen ions all depend on electron velocity in approximately the same way.

Before the observations can be compared with this calculated ratio, the observed flux in H\( \beta \) from NGC 2175 must be corrected for interstellar extinction. The central star in this nebula is HD 42088, spectral type O6 (Morgan, Code, and Whitford 1955), color index \( B-V = +0.07 \) and hence visual absorption derived from the color excess \( A_V = \)

<table>
<thead>
<tr>
<th>( T ) (°K)</th>
<th>( j_{\text{vis}}/j_{\beta} ) (per cps)</th>
<th>( T ) (°K)</th>
<th>( j_{\text{vis}}/j_{\beta} ) (per cps)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000</td>
<td>4 24×10^-14</td>
<td>20000</td>
<td>5 63×10^-14</td>
</tr>
</tbody>
</table>

1.17 mag. (Hiltner 1956). According to the interstellar absorption-curve of Whitford (1958), the absorption at H\( \beta \) is then \( A_\beta = 1.38 \) mag, or the observed flux must be multiplied by a factor of 3.57 to find the flux in H\( \beta \) that would be observed if there were no interstellar absorption. The uncertainty in the absorption correction for the star is only about ±0.3; however, there is an additional source of error that is difficult to evaluate, namely, the possibly uneven absorption across the face of the nebula. The final result is that the flux in H\( \beta \) from NGC 2175, received at the earth and corrected for extinction, is \( 6.8 \times 10^{-9} \) erg/cm\(^2\) sec, with an uncertainty by a factor of about 1.8.

The flux at 1390 Mc from this same nebula is \( 7.0 \times 10^{-25} \) watts/m\(^2\) cps or \( 7.0 \times 10^{-22} \) ergs/cm\(^2\) sec cps, according to the measurements of Westerhout (1958). The uncertainty in the radio-frequency measurement is about a factor of 1.5, made up of the probable errors in the measured intensity (20 per cent), in the correction for the beam width (20 per cent), and in the calibration to energy units (20 per cent).

Thus, finally, the observed ratio of fluxes from NGC 2175 is \( F_{1390}/F_{\beta} = 10 \times 10^{-14} \), with an uncertainty by a factor of the order of, or slightly larger than, 2. The observed ratio is therefore over twice as large as the theoretically computed ratio for a temperature of 10000°, the generally adopted value for a diffuse nebula (Spitzer and Savedoff 1950; Liller and Aller 1954; Mathis 1957), and is less than twice the computed ratio for an assumed temperature of 20000°, but the observational uncertainties are large enough that the measurements and theoretical calculations can just be reconciled. We may therefore conclude that there is approximate agreement between the available theory of the hydrogen recombination spectrum and the ratio of observed radio-frequency and H\( \beta \) fluxes in this one nebula but that there is a suggestion that the observed ratio may be slightly larger than the computed. More accurate and more numerous observations, particularly optical measurements, will be needed to settle this point finally.
IV. LYMAN ULTRAVIOLET STELLAR RADIATION

Measurements of nebular hydrogen emission-line or emission-continuum radiation lead directly, through the theory of the hydrogen recombination spectrum, to deductions concerning the far ultraviolet radiation emitted by the hot stars involved in the nebulae. For in a nebula in a steady state, the number of photoionizations of hydrogen atoms into protons and electrons just balances the number of recaptures of electrons by protons to form atoms; and, since there is a definite relation (depending only on cross-sections and temperature) between the number of captures to all states and the number of captures to any particular state, it follows that there is a relation between the number of photoionizations and the total emission in any hydrogen line or continuum. If it is further assumed that the ionizing radiation from the star is completely absorbed in the nebula, that is, that the nebula is optically thick, then it is clear that the amount of nebular hydrogen radiation is directly related to the amount of stellar far-ultraviolet radiation. This method, due to Zanstra (1927), determines the temperature of the involved star if it is assumed to emit black-body radiation; recent work on model atmospheres of hot stars shows this assumption to be untenable, but the method nevertheless always measures the total number of ionizing quanta (\( \lambda < 912 \) Å) emitted by the star and absorbed in the nebula.

It has been particularly emphasized by Wurm (1951, 1956) that in many planetary nebulae the absorption of ionizing radiation is not complete and that care is therefore necessary in interpreting the measurements. The diffuse nebulae we selected for measurement were chosen because each is bounded on a large part of its perimeter by an ionization front, so that in these directions there is complete absorption; however, some escape of ionizing radiation probably does occur, because the ionization fronts do not completely surround the nebulae. We return to this point below, after first discussing the observations as if the absorption were complete.

The number of H/3 photons emitted per unit volume \( q_\beta \) is just the emission coefficient divided by the energy per photon, so, from equation (3), we obtain

\[
q_\beta = j_\beta \frac{1}{h\nu_\beta} = f(T) \frac{N_p N_e}{h\nu_\beta},
\]

where \( \nu_\beta = 6.17 \times 10^{14} \) sec\(^{-1} \) is the H/3 frequency. On the other hand, the total number of captures, not counting those that simply lead to re-emission of another ionizing photon, is

\[
q_{LC} = 2 A \left( \frac{2 kT}{\pi m} \right)^{1/2} \beta \phi_\beta(\beta) N_p N_e,
\]

where all the symbols are defined by Spitzer (1948). The ratio \( q_{LC}/q_\beta \) can thus be computed as a function of temperature alone and has the value 9.0 at 10000° K and 9.1 at 20000° K. Likewise, by using the ratio of emission coefficients given in Table 4, the ratio \( q_{LC}/q_{1390} \) of ionizing photons absorbed by hydrogen to continuum photons emitted per cycle per second at 1390 Mc can be computed; hence the radio observations can also be used to analyze the stellar far-ultraviolet radiation.

The data are collected in Table 5, in which \( Q_\beta \) stands for the flux of H/3 photons received at the earth from the nebula and is found by the conversion of the observed results given in Table 3 into numbers of quanta. This observed flux has then been corrected for interstellar extinction, using in each case the color excess of the central star in the way explained in detail for NGC 2175 in Section III, and the corrected flux has then been used to compute \( Q_{LC} \), the flux of Lyman continuum photons from the star, that would be incident on the earth in the absence of extinction.

The number of objects available for study has been increased by including in Table 5
four nebulae observed at 1390 Mc by Westerhout (1958), but not measured optically; for these sources the column \( Q_{1390} \) gives the flux of photons per cycle per second with frequencies near 1390 Mc, and this is then used to compute \( Q_{LC} \) at an assumed temperature of 10000° K. The discrepancy between the optical and the radio-frequency determinations of the flux of ionizing photons from the central star of NGC 2175 has already been described in the previous section and is just within the combined observational uncertainties.

We have next compared the derived ultraviolet fluxes with the directly measured fluxes of visual radiation from the involved hot stars, in order to find a quantity independent of distance that is also predicted by published model stellar atmospheres. The calculations are summarized in Table 6, in which for each nebula the exciting stars, mostly taken from Sharpless (1954) but also from Minkowski (1949) and Osterbrock (1957) in some cases, are listed. For each star the spectral type, from Morgan, Code, and Whitford (1955) is listed, as well as \( m_0 \), the visual apparent magnitude, corrected for interstellar extinction, from Hiltner (1956). The exceptions are HD 5005 br, for which the photometry is from Hiltner and Johnson (1956), and BD+66°1674, for which an estimated spectral type of O8 and the photometric results of Sandage (Osterbrock 1957) have been used. The visual magnitudes have been converted into fluxes of photons in the 1000-A range between 5000 and 6000 A by using the solar values, namely, \( m_0 = V = -26.73 \) (Stebbins and Kron 1957) corresponds to \( 5.20 \times 10^{16} \) photons/cm² sec 1000 A, from integration of the fluxes tabulated by Minnaert (1953). The fluxes of visual photons from all the O stars in each nebula have been summed and are listed in Table 6, along with the resulting values of \( Q_{LC}/Q_V \), the final observationally determined ratios of far ultraviolet to visual photon fluxes. It may be noted that for the nebulae in which Hß was measured, the ratio is nearly independent of the calculated interstellar extinction, but not quite, because of the small difference between Hß (X 4861) and the mean visual wave length (about X 5540), while, for the nebulae in which the radio-frequency radiation was measured, the extinction correction enters with full force. (The straight mean of the two determinations of \( Q_{LC} \) was used for NGC 2175.)

Now, finally, these “observed” ratios, which are in a sense color indices between the Lyman and the visual regions, can be compared with the predicted values of the same ratios according to model stellar atmospheres. Computed ratios for three different models of hot stars are listed in Table 7, in which the effective temperatures and spectral types both come directly from the references given, while the ratios \( Q_{LC}/Q_V \) were calculated from the tabulated fluxes.

Comparison of Tables 6 and 7 shows that the observed ratios are generally somewhat smaller than the predicted values for O stars, that is, that the models have fluxes in the
Lyman region that are too large. However, for HD 42088 in NGC 2175 the observed ratio is about the same as the interpolated computed ratio for O6, and hence it might be argued that the models are, in fact, correct and that in the other nebulae listed in Table 6 the absorption of ionizing photons is not complete, so that some of these photons escape and are therefore not counted in the observed $Q_{lc}$. The difficulty is that direct photographs of, for instance, NGC 281 and NGC 2175 are not strikingly different; both are bounded by obvious ionization fronts on about half their projected perimeters, so that it is not clear why these two objects should differ by a factor of the order of 2 in absorption of Lyman continuum radiation. Of all the nebulae studied, the close pair IC 1848

<table>
<thead>
<tr>
<th>Nebula</th>
<th>Star</th>
<th>Spectrum</th>
<th>$m_0$</th>
<th>$Q_v$ (photons/cm² sec 1000 Å)</th>
<th>$Q_{lc}/Q_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 281</td>
<td>[HD 5005 br]</td>
<td>O6</td>
<td>6 53</td>
<td>$2.58 \times 10^4$</td>
<td>3 2</td>
</tr>
<tr>
<td></td>
<td>[HD 5005 ft]</td>
<td>O9 f</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NGC 2175</td>
<td>[BD+66°1675]</td>
<td>O6</td>
<td>6 38</td>
<td>$2.96 \times 10^4$</td>
<td>8 8</td>
</tr>
<tr>
<td>S 274-8</td>
<td>[BD+66°1674]</td>
<td>O7</td>
<td>4 82</td>
<td>$1.98 \times 10^4$</td>
<td>4 2</td>
</tr>
<tr>
<td></td>
<td>[BD+66°1674]</td>
<td>(O8)</td>
<td>5 39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IC 1805</td>
<td>[HD 15570]</td>
<td>O5 f</td>
<td></td>
<td>$2.99 \times 10^4$</td>
<td>1 6</td>
</tr>
<tr>
<td></td>
<td>[HD 15629]</td>
<td>O5</td>
<td>6 17</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[HD 15558]</td>
<td>O6</td>
<td>5 29</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[BD+60°512]</td>
<td>O6</td>
<td>6 95</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[BD+60°512]</td>
<td>O6 5</td>
<td>7 26</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[BD+60°497]</td>
<td>O7</td>
<td>6 13</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[BD+60°498]</td>
<td>O9 V</td>
<td>7 38</td>
<td></td>
<td></td>
</tr>
<tr>
<td>IC 1848+Anon No. 1</td>
<td>[HD 17520]</td>
<td>O7</td>
<td>4 90</td>
<td>$2.34 \times 10^4$</td>
<td>2 6</td>
</tr>
<tr>
<td></td>
<td>[BD+60°586]</td>
<td>O7</td>
<td>6 62</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[HD 18326]</td>
<td>O8 V</td>
<td>6 38</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[BD+59°562]</td>
<td>O8 V</td>
<td>7 39</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NGC 2237-46.</td>
<td>[HD 46223]</td>
<td>O5</td>
<td>5 66</td>
<td>$2.40 \times 10^4$</td>
<td>5 4</td>
</tr>
<tr>
<td></td>
<td>[HD 46150]</td>
<td>O6</td>
<td>5 41</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[HD 46485]</td>
<td>O8</td>
<td>6 35</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[HD 46149]</td>
<td>O8</td>
<td>6 11</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[HD 46056]</td>
<td>O8</td>
<td>6 65</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[HD 46202]</td>
<td>O9 V</td>
<td>(6 88)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reference</th>
<th>Spectral Type</th>
<th>$T_e$</th>
<th>$Q_{lc}/Q_v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Underhill (1951)</td>
<td>O5 V</td>
<td>44 600°</td>
<td>12 9</td>
</tr>
<tr>
<td>Traving (1957)</td>
<td>O9 V</td>
<td>37 450°</td>
<td>4 0</td>
</tr>
<tr>
<td>Underhill (1950)</td>
<td>O9 5 V</td>
<td>36 800°</td>
<td>3 1</td>
</tr>
</tbody>
</table>

© American Astronomical Society • Provided by the NASA Astrophysics Data System
and Anon No. 1 (Sharpless 1954) appear from their photographs to be most nearly completely bounded by ionization fronts, but the observed ratio $Q_{LC}/Q_V$ nevertheless deviates strongly from the computed value. Perhaps more definite statements could be made from a very detailed study of photographs of the nebulae, but a more promising line of approach appears to be the measurement of the hydrogen radiation of a larger number of nebulae to somewhat higher precision. It would then be possible to interpret the observed upper limit to the observed $Q_{LC}/Q_V$ ratio as a lower limit that must be satisfied by an adequate model atmosphere. At the present moment we can only say that there is rough agreement of observations with the models, with the suggestion that incomplete absorption occurs in some nebulae.

V. SUMMARY

The observations listed in Section II show that a small telescope equipped with interference filters and a photoelectric photometer can be used to measure quantitatively the emission-line radiation from diffuse nebulae, and the calculations summarized in Sections III and IV show that these measurements, together with radio-frequency measurements, provide useful information on the theory of the recombination spectrum and on the Lyman continuum radiation emitted by the involved hot stars. Previous calculations of emission coefficients are confirmed, and the far ultraviolet flux predicted by a model stellar atmosphere is confirmed at least in one case (NGC 2175). Though in the other cases the stellar flux of Lyman radiation deduced from the observations is less than that predicted by the models, the discrepancy can be explained in terms of incomplete absorption in these nebulae.

More precise conclusions could be drawn if more nearly complete and more accurate observations were available. The main limit to the accuracy of the present work arose from the sky corrections, and it is clear that narrower band filters, accurately centered on Hβ and [O III] N1, together with comparison continuum filters both longward and shortward of the emission-line wave lengths, will make possible much greater precision in future measurements. The most satisfactory calibration seems to be with emission-line objects (such as planetaries) that were themselves compared with stars with a photoelectric scanning spectograph; direct comparison with stars requires that the filter transmission-curve be measured with high accuracy.

This program has been supported by funds made available to the Graduate School of the University of Wisconsin by the Wisconsin Alumni Research Foundation. We are particularly indebted to A. D. Code, R. C. Bless, T. E. Houck, and L. McElwain for their help in various stages of the work.

REFERENCES

———. 1951, ibid., 113, 125.

© American Astronomical Society • Provided by the NASA Astrophysics Data System
———. 1960, in press.
———. 1956, *Centr. Osservatorio Astrofisico dell'Università di Padova in Asiago*, No. 73, p. 77.