A STUDY OF TWO COMET TAILS

DONALD E. OSTERBROCK
Mount Wilson and Palomar Observatories
Carnegie Institution of Washington, California Institute of Technology
Received February 12, 1958

ABSTRACT
Photographic observations of the directions of the tails of Comet Baade (1954h) and Comet Haro-Chavira (1954k) are given. These comets both have large perihelion distances, and during the period of observation their heliocentric distances ranged between 3.9 and 5.0 a.u. The position angle of the tail as projected on the plane of the sky is not correlated with the position angle of the projection on the sky of the line from the sun to the comet, that is, the tail does not lie in the radial direction. The position angle of the tail is generally not too different from the position angle of the projection onto the sky of the orbit behind the comet. The observations indicate that the comet tail lies in the orbital plane roughly midway between the radial and tangential directions. Therefore, the material of the tail must be subjected to a resisting force roughly equal to the radial repulsive force of the sun. This resisting force can be understood as being caused by the interplanetary gas, provided that the comet tail contains a large fraction of hydride molecules, but the possibility that it consists of solid particles small in comparison with the wavelength of light cannot be ruled out.

I. INTRODUCTION
Comets are relatively faint objects except when they are close to the sun, and for this reason most of the observations of their physical features refer to relatively small heliocentric distances (Beyer 1956). With the 48-inch Schmidt telescope of the Palomar Observatory it is possible to obtain direct plates of faint objects in short exposure times (8 minutes with a 103a-O plate to reach the limit set by the sky background) and at an adequate scale (67″/mm). Therefore, plates can be taken incidentally if any other program is going on at the telescope. For these reasons it was decided to take a series of plates, as opportunities became available, of comets relatively distant from the sun, in order to make a physical study of their tails. Comet Baade (1954h), with a perihelion distance of 3.87 a.u., and Comet Haro-Chavira (1954k), with a perihelion distance of 4.07 a.u., proved to be the first suitable subjects for this program, and the observations of the directions of their tails are discussed below.

II. OBSERVATIONS
All the plates were taken with the 48-inch Schmidt telescope, mostly on 103a-O or 103a-J plates, but occasionally on panchromatic plates with a yellow or red Plexiglass filter. The images of the comets on these plates all show a nucleus, a small coma, and a tail ranging in length from 3″ to 8″, nearly straight but usually slightly curved and with little, if any, internal structure. The 1955 and 1956 plates of Comet Baade show that one sector, subtending roughly 30°–60° at the nucleus, is brighter than the rest of the head. This fan or sector has previously been described by Roemer (1955), Jeffers (1956), and Van Biesbroeck (1957). It is approximately, though apparently not exactly, opposite the tail in projection on the sky; the radius of the fan is only about 30″, and the 48-inch Schmidt does not have sufficient scale for measurements of any accuracy to be made on the orientation of this feature. There do not appear to be any marked differences between the blue and yellow or red images of the comet on the few occasions for which there are two plates taken within a few days.

On each plate the position of the nucleus and the position angle of the tail were measured. To determine the position, the co-ordinates of the nucleus and two nearby
reference stars were estimated to the nearest 0.1 mm, using a superimposed transparent millimeter grid. As the plates were all bound to a thick cover glass before being handled, these estimated co-ordinates may be in error because of the parallax caused by the finite distance between the images on the plate and the grid. The rectangular co-ordinates were then reduced by the two-reference-star method of König (1951), using star positions for 1950.0 from either the AGK2 catalogues or the Yale zone catalogues, to determine the right ascension and declination of the nucleus. Experience in the determination of a large number of positions of nebulae or nebulous features from Schmidt plates by this method has shown that the maximum error to be expected is about 15" in each co-ordinate, which, though it could be considerably reduced by a more nearly accurate measuring technique, is quite sufficient for the purpose of the present paper.

To determine the position angle of the tail, an ink line was drawn on the cover glass parallel to the axis of the tail, and the angle between this line and another line drawn through the two reference stars was measured. The position angle of the tail was then computed, using the known right ascensions and declinations of the reference stars. In nearly all cases it proved possible to choose the reference stars so that no dimension in the configuration of the comet and the two stars was over 30' and so that the angle measured between the two lines was between 45° and 135°. The major error in the measured position angle of the tail then results from uncertainties in judging just where the axis of the tail lies; it is not perfectly symmetric, and different people bisect it in different ways. All position-angle measurements were made independently by Mrs. Carol Tifft and by the writer; each of her final measured values was the mean of either two or three measures, made independently at intervals longer than one week, while my values are either one measure or the mean of two. The average difference without regard to sign between the two sets of measurements is 2°, and the maximum, 6°; both figures are comparable with the corresponding differences for Mrs. Tifft's different measures of the same plates. The adopted measured values are the means of the measured values of Mrs. Tifft and of the author, weighted equally. The observational data are listed in Table 1 (Comet Baade) and Table 2 (Comet Haro-Chavira), in which the first column gives the date of observation (U.T.); the next two columns, \( \alpha \) and \( \delta \), the position for the epoch 1950.0; and the fourth column, \( \theta \), the position angle of the tail, on the customary system 0° = north, 90° = east, etc. The other parts of these tables deal with the reduction and interpretation of the observations and are discussed in the following sections.

### Table 1

<table>
<thead>
<tr>
<th>Date</th>
<th>( \alpha )</th>
<th>( \delta )</th>
<th>( \theta )</th>
<th>( \phi )</th>
<th>( \psi )</th>
<th>( h )</th>
<th>( r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1954</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>July 31 1844</td>
<td>14h47m18s</td>
<td>+66°38'5</td>
<td>132°</td>
<td>99°</td>
<td>138°</td>
<td>+0</td>
<td>5</td>
</tr>
<tr>
<td>Aug. 1 2253</td>
<td>14 46 8</td>
<td>+66 41 3</td>
<td>131</td>
<td>98</td>
<td>138</td>
<td>+0 9</td>
<td>5 03</td>
</tr>
<tr>
<td>1955</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sept. 18 4986</td>
<td>7 28 46</td>
<td>+48 50 4</td>
<td>18</td>
<td>290</td>
<td>29</td>
<td>+0 8</td>
<td>3 88</td>
</tr>
<tr>
<td>Sept. 26 4570</td>
<td>7 28 42</td>
<td>+48 20 8</td>
<td>21</td>
<td>286</td>
<td>29</td>
<td>+0</td>
<td>3 89</td>
</tr>
<tr>
<td>Nov. 19 5285</td>
<td>6 41 28</td>
<td>+44 43 2</td>
<td>10</td>
<td>252</td>
<td>19</td>
<td>+1</td>
<td>3 96</td>
</tr>
<tr>
<td>Dec. 8 3340</td>
<td>6 6 28</td>
<td>+41 54 4</td>
<td>9</td>
<td>223</td>
<td>13</td>
<td>+1</td>
<td>4 00</td>
</tr>
<tr>
<td>1956</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feb. 8 2430</td>
<td>4 43 26</td>
<td>+26 59 6</td>
<td>6</td>
<td>85</td>
<td>2</td>
<td>+0</td>
<td>4 17</td>
</tr>
<tr>
<td>Mar. 14 2108</td>
<td>4 39 49</td>
<td>+21 1 2</td>
<td>6</td>
<td>82</td>
<td>2</td>
<td>+0</td>
<td>4 29</td>
</tr>
<tr>
<td>Apr. 9 1552</td>
<td>4 48 56</td>
<td>+18 2 1</td>
<td>6</td>
<td>86</td>
<td>0</td>
<td>+0</td>
<td>4 39</td>
</tr>
</tbody>
</table>
COMET TAILS

III. GEOMETRICAL INTERPRETATION

The measured position angles of the comets' tails were compared with the position angle of the projection on the sky of the radius vector from the sun prolonged through the comet and also with the position angle of the projection on the sky of the orbit behind the comet in its motion. There are many ways in which these position angles can be computed, but the scheme described below was found to be especially convenient to use with a hand computing machine.

The vector from the earth to the comet may be written

\[ \mathbf{r} = p \cos \alpha \cos \delta \mathbf{i} + p \sin \alpha \sin \delta \mathbf{j} + p \sin \delta \mathbf{k} = \mathbf{r} + \mathbf{R}, \tag{1} \]

where the notation used is that of Herget (1948). Therefore, for any variation in the vector from the earth to the comet, the corresponding variations in the right ascension and declination may be found by solving the equation

\[ d \rho = d \rho_1 \mathbf{i} + d \rho_2 \mathbf{j} + d \rho_3 \mathbf{k} = d (\mathbf{r} + \mathbf{R}) \tag{2} \]

**TABLE 2**

Observations and Computations for Comet Haro-Chavira (1954k)

<table>
<thead>
<tr>
<th>Date</th>
<th>a</th>
<th>(\delta)</th>
<th>(\theta)</th>
<th>(\phi)</th>
<th>(\psi)</th>
<th>(h)</th>
<th>(r)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1955</td>
<td>6h44m32s</td>
<td>+59°45' 7''</td>
<td>223°</td>
<td>279°</td>
<td>210°</td>
<td>+1 1</td>
<td>4 21</td>
</tr>
<tr>
<td>Sept 25</td>
<td>4403</td>
<td>+60° 2 1</td>
<td>226</td>
<td>278</td>
<td>211</td>
<td>+1 4</td>
<td>4 20</td>
</tr>
<tr>
<td>Sept 26</td>
<td>4236</td>
<td>+60° 2 6</td>
<td>221</td>
<td>278</td>
<td>211</td>
<td>+0 9</td>
<td>4 20</td>
</tr>
<tr>
<td>Sept 26</td>
<td>4399</td>
<td>+77° 53 6</td>
<td>216</td>
<td>232</td>
<td>213</td>
<td>+1 0</td>
<td>4 11</td>
</tr>
<tr>
<td>Nov. 20</td>
<td>4236</td>
<td>+83 3 2</td>
<td>195</td>
<td>191</td>
<td>195</td>
<td>+0 2</td>
<td>4 10</td>
</tr>
<tr>
<td>Oct. 8</td>
<td>3174</td>
<td>+83 6</td>
<td>195</td>
<td>191</td>
<td>195</td>
<td>+0 2</td>
<td>4 10</td>
</tr>
<tr>
<td>1956</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Feb 9</td>
<td>1455</td>
<td>+78 23 7</td>
<td>104</td>
<td>37</td>
<td>113</td>
<td>+0 7</td>
<td>4 08</td>
</tr>
<tr>
<td>Mar. 13</td>
<td>1663</td>
<td>+77 30 9</td>
<td>114</td>
<td>15</td>
<td>120</td>
<td>+0 4</td>
<td>4 09</td>
</tr>
<tr>
<td>Apr. 9</td>
<td>1663</td>
<td>+79 52 4</td>
<td>126</td>
<td>1</td>
<td>129</td>
<td>+0 3</td>
<td>4 12</td>
</tr>
<tr>
<td>May 11</td>
<td>4226</td>
<td>+84 47 2</td>
<td>154</td>
<td>358</td>
<td>156</td>
<td>+0 5</td>
<td>4 17</td>
</tr>
<tr>
<td>June 2</td>
<td>2660</td>
<td>+86 49 6</td>
<td>229</td>
<td>53</td>
<td>232</td>
<td>+3 8</td>
<td>4 22</td>
</tr>
<tr>
<td>June 29</td>
<td>2219</td>
<td>+81 18 5</td>
<td>300</td>
<td>86</td>
<td>294</td>
<td>+1 1</td>
<td>4 28</td>
</tr>
<tr>
<td>July 13</td>
<td>2663</td>
<td>+77 21 9</td>
<td>311</td>
<td>87</td>
<td>304</td>
<td>+0 7</td>
<td>4 32</td>
</tr>
<tr>
<td>Aug. 5</td>
<td>2518</td>
<td>+70 30 3</td>
<td>326</td>
<td>78</td>
<td>316</td>
<td>+0 8</td>
<td>4 38</td>
</tr>
<tr>
<td>Sept 2</td>
<td>1705</td>
<td>+62 10 2</td>
<td>336</td>
<td>63</td>
<td>326</td>
<td>+0 7</td>
<td>4 48</td>
</tr>
<tr>
<td>Sept 28</td>
<td>1615</td>
<td>+55 3 1</td>
<td>343</td>
<td>47</td>
<td>335</td>
<td>+0 7</td>
<td>4 57</td>
</tr>
<tr>
<td>Oct. 7</td>
<td>1531</td>
<td>+52 49 8</td>
<td>346</td>
<td>41</td>
<td>338</td>
<td>+0 8</td>
<td>4 60</td>
</tr>
</tbody>
</table>

in the form

\[ \rho \cos \delta \, d \alpha = - \sin \alpha \, d \rho_1 + \cos \alpha \, d \rho_2, \tag{3} \]

\[ \rho d \delta = - \cos \alpha \, \sin \delta \, d \rho_1 - \sin \alpha \, \sin \delta \, d \rho_2 + \cos \delta \, d \rho_3. \]

For a comet in a parabolic orbit, the radius vector may be written

\[ \mathbf{r} = qP (1 - \tan^2 \frac{1}{2} v) + 2qQ \tan \frac{1}{2} v = qP (1 - \beta^2) + 2qQ\beta, \tag{4} \]

where

\[ \beta + \frac{1}{2} \beta^3 = \frac{k (t - T)}{2^{1/2} q^{3/2}} \tag{5} \]

is Kepler's equation. Therefore, to find the position angle of the projection of the
radius vector on the sky, we may take the variation in $\theta$ to be in the direction of the radius vector, i.e.,

$$d\theta = C_1 r = r,$$

where the constant is chosen to have the value unity for convenience in computing. The procedure is thus first to solve equation (5) for each date to find $\beta$, then to compute $r$ according to equation (4), and then to use the components of $r$, which are the same as the components of $d\theta$ by equation (6), to compute $\rho \cos \delta = c$ and $\rho \sin \delta = d$ from equation (3). Finally, $\phi$, the position angle of the projection of the prolonged radius vector on the plane of the sky, is computed as

$$\phi = \tan^{-1} \frac{\rho \cos \delta \, da}{\rho \, d\delta} = \tan^{-1} \frac{c}{d}.$$

The elements used in the computation for Comet Baade were those of Cunningham (1955), while for Comet Haro-Chavira those of Merton (1956) were used.

To find the position angle of the projection of the orbit behind the comet onto the plane of the sky, we may take $d\theta$ to be in the direction opposite to the velocity,

$$d\theta = -C_2 r = qP \beta - qQ,$$

where this time the constant was chosen as $2\hat{\beta}$. The components of $d\theta$ from equation (8) lead, by the scheme described above, to values of $\rho \cos \delta \, da = a$ and $\rho \, d\delta = b$, and these values in turn are used to compute $\psi$, the position angle of the projection on the sky of the orbit behind the comet, by the equation

$$\psi = \tan^{-1} \frac{\rho \cos \delta \, da}{\rho \, d\delta} = \tan^{-1} \frac{a}{b}.$$

The computed angles $\phi$ and $\psi$ are listed in Tables 1 and 2 for the two comets and may be compared directly with the observed position angles of the comets' tails. It is clear in both cases that there is little correlation between the observed position angle and $\phi$, the position angle of the projected radius vector, and that therefore the tails of these two comets do not point directly away from the sun. The observed position angle is, on the other hand, in most cases fairly close to $\psi$, the computed position angle of the projection onto the sky of the orbit behind the comet. This same result, namely, that for comets at large heliocentric distances the projection of the tail onto the sky lies nearly in the opposite direction to the projection of the comet's velocity, has been found by Beyer (1955) for a large number of comets at somewhat smaller heliocentric distances (less than 3.2 a.u.). It must be noted, however, that the result is partly due to a purely geometric effect; for a comet at a heliocentric distance of say 4 a.u. is also at a geocentric distance of between 3 and 5 a.u., and the radius vector from the sun to the comet is therefore roughly perpendicular to the plane of the sky, while the orbital velocity vector, which is roughly perpendicular to the radius vector, lies more or less in the plane of the sky. Therefore, any component of the tail in the direction of the radius vector has a relatively small projection on the sky, while any component in the direction of the orbit is relatively large in the same projection.

The correlation between the observed position angle of the tail and the computed position angle of the projection of the orbit at least indicates that, to a good approximation, the tail lies in the orbital plane of the comet. We shall therefore make the assumption that the tail is exactly in this plane and on this basis can solve for the angle it makes with the direction of the orbit behind the comet. For suppose we write $e_\Omega$ as a unit vector in the direction of the prolonged radius vector, and $e_\alpha$ as a unit vector in the direction of the orbit behind the comet. These unit vectors are not orthogonal...
COMET TAILS

except at perihelion; in general, the angle between them is \( \gamma = \cos^{-1} \left[ -\beta / \sqrt{(1 + \beta^2)} \right] \).

A unit vector in the direction of the tail may be written

\[ \mathbf{n} = C_2 (\mathbf{e}_y + h \mathbf{e}_z) = C_4 \left[ -C_2 \hat{r} + AC_1 \hat{r} \right]. \]  
(10)

It is easy to show that, with \( C_1 \) and \( C_2 \) defined as indicated above,

\[ h = A \sqrt{(1 + \beta^2)} \]  
(11)

gives the component of the tail in the direction of the prolonged radius vector, relative to the component unity in the direction of the orbit behind the comet. Now, since equation (3) shows that the numerator and denominator of the tangent of the position angle are both linear in the components of a vector along the tail, the observed position angle of the tail must be given by

\[ \theta = \tan^{-1} \left( \frac{a + A_c}{b + A_d} \right), \]  
(12)

where \( a, b, c, \) and \( d \) are defined in the discussions preceding equations (9) and (7). Once equation (12) has been solved for \( A \), equation (11) may be used to find \( h \), the component of the tail in the direction of the prolonged radius vector relative to the component unity in the direction of the orbit behind the comet. The values of \( h \) derived from the observations in this way are listed in the seventh columns of Tables 1 and 2, and these columns give most directly the observational result which can be compared with a physical theory. It may be seen that \( h \) is in nearly all cases positive and between 0.5 and 1.5 in absolute value; hence the tail lies roughly halfway between the direction radially away from the sun and the direction of the orbit behind the comet.

Any errors in the derived values of \( h \) are caused almost entirely by errors in the measured position angles, which are much less precise than the other data used in the calculations. These errors were estimated by making duplicate calculations with various assumed values of the position angle for several of the dates for which there are observations. For Comet Haro-Chavira, for most dates the absolute value of \( \delta h / \delta \theta \) is approximately 0.1 per degree, and, since the individual measured position angles appear to have a random error of about 2° or 3°, the individual values of the ratio of components in the two directions should have random errors of 0.2 and 0.3; hence fluctuations smaller than this must be treated with reserve. For certain dates, however, the geometrical situation is peculiar, and the uncertainty in \( h \) is much larger. The two most extreme cases are the dates December 8.3174, 1956, for which \( \delta h / \delta \theta = -2.1 \) per degree, and June 2.2660, 1956, for which \( \delta h / \delta \theta = -0.8 \) per degree. The derived values of \( h \) for these dates are therefore almost entirely meaningless (they are the most discrepant from the run of the other values in Table 1). For Comet Baade, \( \delta h / \delta \theta \) is, in absolute value, about 0.1 per degree, and the derived values of \( h \) are therefore certain to within 0.2 or 0.3.

The last column in Table 1 and Table 2 lists the computed values of \( r \), the heliocentric distance of the comet.

IV. PHYSICAL INTERPRETATION

The observations and reduction procedure described above show that, in each of the two comets studied, the tail lies in a direction approximately halfway between the radial direction and the direction of the orbit just behind the comet. This result must be interpreted as meaning that the material of the tail is subjected not only to a repulsive force, directed radially outward from the sun, but also to a roughly equal resisting force, directed opposite to the instantaneous velocity of the comet. For it is clear that if only repulsive forces are involved, together with centrifugal and Coriolis accelerations, and if material is ejected from the nucleus in all directions, then close
to the nucleus the tail will lie in the radial direction. This conclusion may be checked in, for instance, the review by Bobrovnikoff (1951) of the Bessel-Bredikhin theory of cometary forms. It might be imagined that material is ejected from the nucleus in only one direction and that this defines the direction of the tail. This explanation cannot, however, be supported, because the bright fan observed in Comet Baade (see Sec. II) indicates that in this comet an appreciable amount of material is ejected in a direction away from the tail and is then turned around by an external force. The fact that both comets have heads completely surrounding their nuclei also indicates that the direction of the tail is not fixed by the initial ejection process alone.

The radial force on the material in the comet tail is presumably due to light-pressure. Recent work by Biermann (1951, 1953) has shown that the repulsive forces on long, straight tails of comets close to the sun are probably due chiefly to the interaction of the charged corpuscular radiation from the sun with the ions in the tail. These tails generally have considerable filamentary structure, show ionic bands (CO$^+$ and N$_2^+$) in their spectra, and are directed nearly radially from the sun; all these facts can be explained on Biermann's theory. On the other hand, the more amorphous, curved, neutral tails can still be understood on the basis of the older light-pressure theory (see, e.g., Wurm 1943), though it is by no means certain that corpuscular radiation does not also make some contribution to the force in these tails. The long, straight tails usually disappear rapidly with increasing heliocentric distance, and it seems fairly clear that the material in the tail of Comet Baade and of Comet Haro-Chavira is mostly neutral and that the repulsive force involved is probably light-pressure.

The tangential force, opposite to the direction of motion, apparently must be understood as resulting from interaction of the material in the tail with a resisting medium, which in turn must be identified with interplanetary matter. There does not seem to be any other mechanism that will give rise to a tangential resisting force on the neutral material of a comet tail. In the remainder of the present section it will be demonstrated that the rather scanty knowledge we have of interplanetary material and of the composition of comets does indeed suggest that a resisting force of about the correct magnitude can be expected. Observations of the polarized component of the zodiacal light show that the interplanetary density of free electrons is about 600/cm$^3$ in the plane of the ecliptic, near but inside the earth's orbit (Behr and Siedentopf 1953). The variation of the density with heliocentric distance and with distance from the plane of the ecliptic is poorly determined, however (van de Hulst 1956), so we can only make a rough guess at the density in the region in interplanetary space where Comets Baade and Haro-Chavira were observed. In the calculations that follow, a value of 20 electrons/cm$^3$ has been assumed as the density at a representative point distant 4 a.u. from the sun and at a celestial latitude of 45°. This assumed density is between the observed interplanetary density near the earth's orbit (600 electrons/cm$^3$) and the estimated interstellar density in a spiral arm (2 electrons/cm$^3$), so it is probably not wrong by more than one power of ten. The sun's ultraviolet radiation ionizes hydrogen out to this distance, so that the electron density is the same as the proton density and there are essentially no neutral atoms.

Biermann (1953, 1957) has pointed out that near the sun all the interplanetary gas may in fact be corpuscular radiation with a radial velocity of about 1000 km/sec. If the velocity of the interplanetary material striking the tails of Comets Baade and Haro-Chavira were this high, it would of course exert an almost purely radial force, not a tangential resisting force. The velocity estimate is derived from the time of flight of magnetic storms, the observed velocities of hydrogen atoms in great aurorae, and the accelerations observed in long, filamentary comet's tails. The first two of these certainly refer to particles emitted during large flares (see, e.g., Smyth 1956), and the filamentary comet-tail accelerations are also strongly correlated with enhanced solar activity (Biermann 1952). Therefore, even though the velocities of corpuscular rays emitted
at such times are of the order of 1000–3000 km/sec, the initial velocities of particles emitted during normal geomagnetically quiet periods may well be considerably smaller. Furthermore, if the initial velocity of the particles emitted from the sun is rather smaller than 1000 km/sec, the velocity of the particles at 4 a.u. distance from the sun may be quite a bit lower, both because of the gravitational attraction of the sun and because of the slowing-down effect of collisions with other charged particles. We shall therefore tentatively assume that at this distance the protons and electrons are essentially at rest and that the relative velocity of the comet with respect to the interplanetary gas is the same as its orbital velocity with respect to the sun. Our problem is thus to compute the relative values of the radial force due to light-pressure and the tangential force due to the motion of the comet through a resisting medium.

First we consider the forces on a solid particle, assumed spherical, with radius \( s \). The radial force is

\[
F_r = \frac{F}{c} \pi s^2 Q(s),
\]

where \( F \) is the flux of radiation, \( c \) is the velocity of light, and \( Q(s) \) is the mean efficiency factor of the particle for radiation pressure. The tangential force, on the other hand, is

\[
F_t = N_H m_H V^2 \pi s^2,
\]

where \( N_H \) is the interplanetary proton density, \( m_H \) is the mass of the proton, and \( V \) is the velocity of the comet with respect to the interplanetary gas. The ratio is, therefore,

\[
\frac{F_r}{F_t} = \frac{F Q(s)}{N_H m_H V^2 c}.
\]

All calculations will be made for a representative distance of 4 a.u.; at this distance the numerical values are \( F = 8.5 \times 10^4 \) ergs/cm\(^2\) sec and \( V = 21 \) km/sec for a comet in a parabolic orbit. With an assumed density \( N_H = 20 \) protons/cm\(^3\) the result is

\[
\frac{F_r}{F_t} = 2 \times 10^4 Q(s).
\]

If the particles are comparable in size with the wave length, \( Q(s) \) is of the order of magnitude 1 (van de Hulst 1957), and the radial force is much larger than the tangential force. The observed situation in which these two forces are approximately equal can be obtained only if the efficiency factor has a value of approximately \( 10^{-4} \). If the efficiency factor is this small, the particles must be small in comparison with the wave length, and the efficiency factor for scattering must be close to inversely proportional to the fourth power of the wave length (van de Hulst 1957). Thus, if the comet tail were composed of particles alone, it would appear rather blue; for instance, on the \( B - V \) photometric system it would be approximately 0.25 mag. bluer than the sun and would therefore have a color index \( B - V \) about 0.4 (Johnson and Morgan 1953). There do not seem to be any published measurements of the colors of the tails of distant comets; such measurements would possibly answer the question of whether these tails could be composed of particles small with respect to the wave length of solar radiation.

Next we consider the forces on a molecule in a comet tail. The radial force due to light-pressure on a single molecule is

\[
F_r = \frac{\pi e^2 f R^2}{m c^4} \rho \rho_r = \frac{2\pi^2 \hbar^2 \pi^2 \lambda^2}{m c^4} \frac{1}{e^{\hbar c/\hbar k T} - 1}
\]

© American Astronomical Society • Provided by the NASA Astrophysics Data System
(Wurm 1943), where $R$ is now the radius of the sun, $r$ is the heliocentric distance of the comet, $f$ and $\lambda$ are the $f$-value and mean wave length, respectively, for the transition involved, $T$ is the Planck temperature of the sun at the wave length involved, and the other symbols stand for the usual physical constants. In Table 3 details are given of calculations of the radial force for a number of diatomic molecules which may be expected in comets' tails, using $f$-values recommended by Herzberg (1950) and temperatures estimated from the curves published by Pettit (1951) and Tousey (1953). The Planck temperature at 1100 Å is derived from estimates of ionization caused by solar ultraviolet radiation in the earth's atmosphere (van de Hulst 1953). There is very little solar radiation at this wave length, and the radiation force on $H_2$ is correspondingly quite small, as Table 3 indicates.

The tangential resisting force on the molecules is given by

$$F_t = N_H m_H V^2 \sigma,$$

where $\sigma$ is the cross-section for scattering of protons with velocity $V$ by a molecule at rest; for our representative distance of 4 a.u. this velocity is 21 km/sec, corresponding to a proton energy of 2.2 electron volts. There is very little experimental evidence on the cross-sections at these low energies, but some simple theoretical estimates can be made. We first consider the case of $H_2$, in which the main interaction at large distances is between the charge of the proton and the induced dipole moment of the molecule. A classical calculation of the scattering cross-section for this process has been made by Zwicky (1923). The cross-section for scattering through an angle smaller than $\theta$ is given approximately by

$$\sigma (\theta, V) = \frac{3^{1/2} \pi^{3/2} a^{1/2} e}{2M^{1/2} V^2} \cot^{1/2} \theta,$$

where $a$ is the polarizability of the molecule, $M$ is the reduced mass of the system, and $e$ is the charge of the proton. This is an approximation valid for small angles $\theta$; the exact expression for large angles involves elliptic integrals and is tabulated numerically in the paper by Zwicky (1923). The cross-section diverges as $\theta \to 0$, but the momentum transfer cross-section (Massey and Burhop 1952), which is the cross-section to be used in equation (18), does in fact converge. We can take, as a sufficient approximation for our purposes, the value of the cross-section given by equation (19) with $\cot \theta = 1$, i.e.,

$$\sigma = \frac{3^{1/2} \pi^{3/2} a^{1/2} e}{2M^{1/2}} = 1 \times 10^{-15} \text{ cm}^2.$$
Now there is a laboratory measurement of the scattering cross-section of H₂ for slow protons (Simons, Fontana, Muschlitz, and Jackson 1943). Their result for the observed cross-section for scattering through an angle greater than a certain angle is about twice as large as the classically calculated result given above. It is uncertain how much of this discrepancy is due to quantum effects not taken into account in the calculation used and how much is due to observational inaccuracies, which are considerable at these low energies (Massey and Burhop 1952). We shall use an intermediate value, $\sigma = 1.5 \times 10^{-16}$ cm², but we must realize that the computed results are uncertain by at least a factor of 2. For other diatomic molecules which may be expected in comet tails, there are no experimental measurements of scattering cross-sections available. Rough calculations, summarized in Table 4, have been made, using band polarizability estimates (Hirschfelder, Curtiss, and Bird 1954) and modifying equation (20) to give the adopted value for hydrogen:

$$\frac{\sigma_2}{\sigma_1} = \left( \frac{a_2}{a_1} \frac{m_1}{m_2} \right)^{1/2}, \quad (21)$$

where the subscript 1 refers to the collision of a proton with the H₂ molecule, and the subscript 2, to the collision with any other molecule. It must be emphasized that the assumed polarizabilities are in many cases only rough interpolations in a table that is itself not the result of direct measurements, so that the accuracy of the tabulated cross-sections is quite low; there is no doubt, however, that all are of the order of $10^{-15}$ cm².

Several of the molecules listed in Table 4 have permanent electric dipole moments, and the interaction between these moments and the charge of the proton may be expected to contribute to the scattering. If the dipole rotates so that during the collision its axis always points to the proton, it is simple to show that the cross-section for scattering through an angle greater than $\theta$ in this case is

$$\sigma (\theta, V) = \frac{\pi^2 \mu e}{MV^2} \cot \theta \quad (22)$$

in the limit of small angle scattering, where $\mu$ is the value of the dipole moment. If we again take as a rough mean the cross-section for $\cot \theta = 1$ and use dipole moments estimated by interpolation in the lists published by Le Fevre (1953), we obtain the

\begin{table}[h]
\centering
\caption{Estimated Cross-Sections for Scattering of 21 km/sec Protons by Various Molecules}
\begin{tabular}{|c|c|c|c|c|}
\hline
Molecule & Polarizability & Dipole Moment & Cross-Section & Cross-Section \\
& ($a$ cm$^3$) & ($\sigma$ cm$^2$) & ($\mu$ esu) & ($\sigma$ esu) \\
\hline
$\text{H}_2$ & 7.9$\times 10^{-26}$ & 1.5$\times 10^{-16}$ & 0 & 4$\times 10^{-16}$ \\
$\text{CH}$ & 6.5 & 1 & 0 & 6 \\
$\text{NH}$ & 6.7 & 1 & 1 & 0 \\
$\text{OH}$ & 6.7 & 1 & 1.5 & 10 \\
$\text{C}_2$ & 23 & 2 & 0 & 2 \\
$\text{CN}$ & 18 & 2 & 0 & 2 \\
\hline
\end{tabular}
\end{table}
results given on the right side of Table 4. Again the accuracy is low, but there is no doubt that the permanent dipole moment does not increase the cross-sections in order of magnitude over the values calculated from the polarizability alone. The actual cross-sections for the dipole moment are even smaller than the values listed in Table 4, because the alignment of the molecule with the direction to the proton is not perfect throughout all collisions.

These approximate calculations show that, for all the diatomic molecules listed in Table 4, the cross-section for collisions with protons moving with a velocity of 21 km/sec may be taken to be $2 \times 10^{-18}$ cm$^2$, with an uncertainty of a factor of roughly 3. The tangential resisting force per molecule is then, according to equation (18), $F_t = 3 \times 10^{-25}$ dyne/molecule. Comparison with Table 3 shows that this force is considerably larger than the radial force of radiation on H$_2$, comparable with or somewhat larger than the radial force on CH, NH, and OH and somewhat smaller than the radial force on CN and C$_2$. Therefore, if the comet tail were composed entirely of H$_2$, we should expect it to lie very nearly in the direction of the orbit behind the comet; if it were composed entirely of CH, NH, and OH, we should expect it to lie in an intermediate direction between the radial and tangential directions; and if it were composed entirely of CN and C$_2$, we should expect it to lie more nearly, but not exactly, in the radial direction. Since, according to physical models of comets (Whipple 1950, 1951; Donn and Urey 1956), a considerable fraction of the matter in the tail probably consists of diatomic hydride molecules, the observed fact that the tail lies in an intermediate direction can be understood as resulting from this process. According to these models, there may also be stable polyatomic molecules such as CH$_4$, NH$_3$, and H$_2$O in the tail, as well as polyatomic fragments such as CH$_3$, CH$_2$, and NH$_2$. It would be interesting to make similar calculations of the forces to be expected on these molecules, but the physical data, particularly the $f$-values for transitions in the visible spectrum, are not available in the published literature. According to the models quoted above, solid particles of sizes larger than the wave length of the maximum of the solar energy-curve are to be expected in comets, and, if this is correct, the discussion at the beginning of this section shows that the bulk of the material in the tail cannot be dust but can, according to the latter part of the section, be molecular.

I am indebted to many people who have helped me in the course of this investigation: to Drs. L. C. Cunningham, S. Herrick, and E. Roemer, who prepared and sent me ephemerides in advance of publication; to Dr. G. O. Abell, Mr. C. E. Kearns, and Dr. L. Plaut, who took many of the plates for me; to Dr. W. Baade, who permitted me to use his three plates of Comet 1954h; to Mrs. Silvia Marcus, Mrs. Carol Tift, Mr. J. L. C. Ford, and Mr. W. B. Lindley, who performed the bulk of the numerical work involved; and to Drs. J. L. Greenstein, R. Minkowski, S. B. Nicholson, and M. Schmidt, who discussed with me many of the problems that arose.

REFERENCES

Beyer, M. 1955, Mitteilungen der Hamburger Sternwarte in Bergedorf, 22, 305
Cunningham, L. E., 1955, I.A.U. Circ., No. 1489


