TEMPERATURES AND ELECTRON DENSITIES IN FLARES AS DERIVED FROM SPECTROSCOPIC DATA

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(Communicated by R. G. Giovanelli)

(Received 1957 March 12)

Summary

It is shown that model flares with electron concentrations and temperatures in the ranges $5 \times 10^{11} - 10^{13}$ cm$^{-3}$ and $10^4 - 1.5 \times 10^4$ deg. K respectively have emissions consistent with the observed hydrogen and helium line intensities in solar flare spectra. Some difficulties are shown to exist in interpreting the observed great line width of Hα and a possible solution is given.

1. Introduction.—Over the past twenty years a good deal of observational data on solar flares has been obtained from studies both of their radiation at visible and radio frequencies and of their associated terrestrial effects such as SID’s. Surveys of various aspects of present knowledge are given, e.g. by Ellison (1, 2, 3) and Kiepenheuer (4).

Data on spectra of flares are, however, rather meagre and show considerable variation from one observer to another and so, presumably, from flare to flare. Comparison of the spectra given by Allen (5), Ellison (6) and Richardson and Minkowski (7) shows that this variation is very marked in the helium lines D$_3$(3D–2P) and λ 6688(3P–2P); the former appears, when at all, sometimes in emission and sometimes in absorption while the latter is seen only in emission from a few intense flares (3). Spectra covering the Balmer series limit, which will be of interest in the subsequent discussion, have been obtained by Richardson and Minowski (7) and by Suemoto (8).

The interpretation of flare observations in terms of a single model consistent with more than one aspect of the whole data does not seem to have been undertaken. The earliest attempt was by Giovanelli (9), who showed that, unless self-absorption of flare Lyman radiation were taken into account, temperatures of the order of $10^5$ deg. K or more were required to account for the Hα intensity. In a more recent investigation Svestka (10) has interpreted the well known relationship between Hα central intensity and line width in terms of a model with temperature $\sim 1.4 \times 10^4$ deg. K and hydrogen abundance $N_H$ of the order of $10^{16}$ cm$^{-3}$; Svestka’s analysis, however, assumed thermodynamic equilibrium in a flare and his deductions are consequently open to some doubt. Suemoto (8), confining his attention to an analysis of the Balmer line H10, has concluded that if $T=10^4$ deg. K, the electron concentration is $1.1 \times 10^{15}$ cm$^{-3}$.

There is as yet no agreement as to the mechanism responsible for the wide Hα wings. Ellison and Hoyle (11) and Mustel and Severny (12) have suggested a Stark broadening mechanism. Goldberg, Dodson and Müller (13) employed a damping type of absorption coefficient in their analysis although they rightly pointed out that the main problem in connection with the Hα width is to account
for a sufficiently large number of π-state H atoms in the line of sight. Their estimate for this number, \( \sim 10^{15} \text{cm}^{-2} \), has been derived on the assumption of natural broadening but they have made no attempt to link this figure with flare thickness, temperature and hydrogen concentration.

Existing flare models thus seem to account for some observational features to the neglect of others. In attempting to gain fuller consistency with the main observed spectral features of flares, we are faced with considerable lack of knowledge of the structure of a flare in depth. Probably the most important factors in deciding the emission characteristics are the maximum electron concentration and the temperature in this region, together with a thickness; this may be the actual thickness for a flare which is more or less uniform in height or a scale height if conditions vary through the flare.

We adopt as a flare model a layer, of uniform electron concentration \( N_e \) and in local statistical equilibrium at an electron temperature \( T \), parallel to the colder solar photosphere which it overlies. The geometric form chosen is unimportant in the present context, for the computed emergent intensity will be only slightly affected by changes in shape. Absorption in the solar atmosphere above the flare is neglected; measured flare heights indicate that generally there will be little absorbing material above this height in the chromosphere. Although we can hope to gain only an estimate of conditions in flares by using such a simple model, elaborations hardly seem justified in view of the paucity of observational data.

2. Emission intensities in Hα and D₃.—If the model flare layer is optically very thick at a particular frequency, so that no photospheric radiation penetrates the flare, the emergent intensity \( I_\nu(\varpi, \mu) \) in a direction making an angle \( \cos^{-1}(\mu) \) with the outward normal can be found from the expression:

\[
I_\nu(\varpi, \mu) = \int_0^\infty \mathfrak{F}_\nu(\tau) e^{-\tau/\mu} d\tau/\mu
\]

where \( \mathfrak{F}_\nu(\tau) \) is a source function defined as the ratio of the monochromatic emission and extinction coefficients at the optical depth \( \tau \), i.e.,

\[
\mathfrak{F}_\nu(\tau) = E_\nu/\kappa_\nu
\]

For either coherent or noncoherent scattering in the medium, the emission coefficient \( E_\nu \) at the line centre can be written in the approximate form,*

\[
E_0 = (1 - \lambda) \frac{J_0 \kappa_0}{4\pi} + \epsilon_0
\]

where \( \lambda \) is a scattering parameter defined so that \( 1 - \lambda \) is the fraction of absorbed quanta which are subsequently re-emitted in the same line, \( \epsilon_0 \) is a "true" emission coefficient, i.e., it represents the emitted component which is independent of absorption of radiation in the particular line, and \( J_\nu \) is the total intensity defined by

\[
J_\nu = \int_{4\pi} I_\nu(\mu) \, d\omega.
\]

On the assumption that \( \epsilon/\kappa \) and \( \lambda \) are constant throughout the flare, an expression for \( J_0 \) may be found using the Eddington approximation to the transfer equation

\[
\frac{d^2 J}{dt^2} = \lambda J - 4\pi \epsilon/\kappa,
\]

* This follows, e.g., from the discussion given by Woolley and Stibbs (14) pp. 164 et seq.
where frequency subscripts have been omitted for convenience. Equation (5) has the solution, finite at large depths,

$$J = 4\pi e/\kappa\lambda + ae^{-\sqrt{3}\lambda}\tau.$$  \hspace{1cm} (6)

The integration constant $a$ in (6) may be obtained as usual from the surface condition $J = (2/3) dJ/d\tau$ and it is then easily shown from (3), (2) and (1) that

$$I(0, \mu) = \frac{e}{\kappa\lambda} \left[ 1 - (1 - \mu)/(1 + 2\sqrt{\lambda/3})(1 + \mu\sqrt{3\lambda}) \right].$$  \hspace{1cm} (7)

The quantities $e/\kappa\lambda$ and $\lambda$ may be evaluated quite readily using methods given by Giovanelli and Jefferies (15). Provided we know the various rates for collisional and radiative transitions between the atomic states. Collisional rates for any temperature may be found from appropriate cross section data and are given for H and He in the author's publications (16, 17). Radiative transitions are more troublesome since the intensity varies throughout the medium and is in any case only found by solving the equilibrium equations and the transfer equations. As pointed out in (15), a simplification is possible if the atmosphere is optically either thick or thin in the various spectral regions of interest. In the following the flare layer is assumed to be thick in the $L\alpha$ and $L\beta$ lines of H and in their counterparts in He, thin in the subsidiary continua, and thick for the principal (Lyman) continua unless otherwise specified. It can easily be shown from expressions for the optical thickness and ground state populations that, if the layer thickness is $\gtrsim 10^5$ cm and the helium abundance not improbably low, these conditions on the Lyman and Lyman type lines are met when $N_e \gtrsim 10^{12}$ cm$^{-3}$ and $T \lesssim 2.5 \times 10^4$ deg. K; anticipating the results of the present analysis it can be said that the derived conditions in flares conform to these limitations.

**Table I**

<table>
<thead>
<tr>
<th>$N_e$ (cm$^{-3}$)</th>
<th>$1.0 \times 10^4$</th>
<th>$1.5 \times 10^4$</th>
<th>$2.5 \times 10^4$</th>
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<tr>
<td>$1 \times 10^{11}$</td>
<td>0.24</td>
<td>0.35</td>
<td>0.48</td>
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**Table II**

<table>
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<th>$1.5 \times 10^4$</th>
<th>$2.5 \times 10^4$</th>
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<tbody>
<tr>
<td>$1 \times 10^{11}$</td>
<td>0.28</td>
<td>0.34</td>
<td>0.39</td>
<td>0.50</td>
</tr>
<tr>
<td>$1 \times 10^{12}$</td>
<td>0.99</td>
<td>1.1</td>
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<td>3.6</td>
<td>4.0</td>
<td>5.4</td>
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</table>

Knowing $e/\kappa\lambda$ and $\lambda$, (7) may be evaluated and the emergent flare intensity expressed as a fraction of the appropriate continuum intensity at the centre of the Sun's disk (Münch (19)). Values obtained for the central intensities of H$\alpha$ and D$\beta$ in this way and so applying only to flares optically thick in these lines, in accordance with (1), are given as functions of $N_e$ and $T$ in Tables 1 and 2. These

* Some collisional rates given in (16) are too large by a factor of two. Corrections are given by Jefferies and Giovanelli (18).
figures should be reliable to 50 per cent or better; at electron concentrations \( \gtrsim 10^{12} \text{ cm}^{-3} \) collisions generally predominate and errors in collision rates tend to cancel, due to corresponding errors in reverse transitions.

3. Optical thicknesses in H and He spectra.—The results given in Tables I and II apply to layers with high opacities at the line centres. Before making any comparison with observation, it is necessary to find expressions for the optical thicknesses of the layers in terms of their physical thicknesses \( z \).

At the centres of Hz, D\textsubscript{3} and \( \lambda 6678 \) the optical thicknesses are given, using a Doppler form absorption coefficient, by

\[
\begin{align*}
\tau(\text{Hz}) & = 4.9 \times 10^{-11} T^{-1/2} N_2(\text{H}) z \\
\tau(D_3) & = 6.3 \times 10^{-11} T^{-1/2} N_2^3(\text{He}) z \\
\tau(\lambda 6678) & = 8.6 \times 10^{-11} T^{-1/2} N_2^3(\text{He}) z
\end{align*}
\]

where it has been assumed that the hydrogen and helium fine structure states of a particular principal quantum number are distributed in the ratios of their statistical weights.

To evaluate the optical thicknesses from (8), values of the II-state populations are required. For He these are given in (17) and for H they can be found in a similar manner. Equation (8) then gives the results shown in Tables III, IV and V computed on the assumption of an H to He population abundance ratio of 5.

### Table III

<table>
<thead>
<tr>
<th>( N_e(\text{cm}^{-3}) )</th>
<th>( T(\text{deg. K}) )</th>
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</thead>
<tbody>
<tr>
<td>( 1.0 \times 10^4 )</td>
<td>( 1.5 \times 10^4 )</td>
</tr>
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<td>( 10^{12} )</td>
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### Table IV

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<td>( 10^{14} )</td>
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### Table V

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</thead>
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</tr>
<tr>
<td>( 10^{14} )</td>
<td>( 5.7 \times 10^{-7} )</td>
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</table>

4. Comparison with observation. (a) Emitted intensities.—The computed results given in the last two sections may now be compared with observational data. These are meagre and rather conflicting as far as simultaneous observations of Hz and D\textsubscript{3} are concerned. Thus Ellison (1) gives intensities for one flare only,
which in the above units are 3.0 for Hα and 1.2 for D₃. Richardson and Minkowski (7) recorded D₃ only for limb flares; there is evidence, however, that these observations did not include any very strong flares. Allen (5) made visual estimates of the total changes in the Fraunhofer lines during a flare but his results do not lend themselves readily to the present discussion of central intensities.

Tables III and IV indicate that, particularly at the lower temperatures, the optical thickness in Hα will generally be considerably greater than in D₃. It may be noted that, since an underlying sunspot is not usually seen through a flare viewed at the centre of Hα, the flare is normally optically thick at this wavelength. There is no observational evidence for the D₃ optical thickness; however, it is probably never very large since the line is observable only occasionally. Thus while the Hα central intensity generally corresponds to that for an optically thick model this is not the case for D₃, and so, while the central intensity of Hα should be as given in Table I, for D₃ the intensity should lie between unity—corresponding to radiation from the underlying photosphere—and the value given in Table II.

The closely similar values of the saturation intensities given in Tables I and II indicate that, if the D₃ line appears at all, Hα and D₃ should appear at the line centre either both in absorption or both in emission. The uncertainties in the computed data might allow for some departure from this expected behaviour, but it is interesting that Waldmeier (20) has found just this type of relationship between Hα and D₃, the transition from absorption to emission in D₃ occurring with an Hα central intensity of about 1.0.

It should also be pointed out that, with the noncoherent scattering mechanism probably applying for Hα, a double peaked profile will result—Jefferies (21)—as observed in most flares, and the maximum intensity across the line could exceed that at the centre by a considerable amount—a factor ~2 would not appear excessive. For D₃ this behaviour should not be so marked, since it will show a greater degree of coherence in scattering than Hα. Further, since the D₃ optical thickness is in general much smaller than Hα, the double peaks have less tendency to form.

The line λ6678, which is the singlet counterpart of the triplet D₃, is only seen in the brightest flares (1); presumably the flare layer is optically very thin in this line for all but the strongest flares, and possibly even for them.

According to Richardson and Minkowski (7), and Suemoto (8), the flare emission at the Balmer series limit is small. If this is accepted as applying to most flares it may be used as a criterion for limiting their possible range of physical conditions. Thus the emission per unit solid angle from an atmosphere thin in the Balmer continuum is, at the series limit,

\[ E_\nu = 2.1 \times 10^{-34} N_e^2 T^{-3/2} \text{ergs cm}^{-2} \text{sec}^{-1} (\text{c/s})^{-1} \]

while the background radiation at the centre of the disk is, Münch (19),

\[ I_\nu = 1.2 \times 10^{-5} \text{ergs cm}^{-2} \text{sec}^{-1} (\text{c/s})^{-1} \]

so that

\[ E_\nu / I_\nu = 1.8 \times 10^{-29} N_e^2 T^{-3/2}. \]

If this ratio is to be \(< 5 \times 10^{-2} \) for Balmer continuum to escape observation, then

\[ N_e^2 T^{-3/2} < 3 \times 10^{27} \text{cgs units}. \]
We now attempt to determine, using (10) and the results given in Tables I to V, the electron temperature and concentration for what might be considered a typical class 2–3 flare with low emission in λ6678 and in the Balmer continuum, and an Hα emission at the line centre ≥10 times the neighbouring continuum at the centre of the disk.

Fig. 1 shows the $T$, $N_e$ space for the region $5 \times 10^3 \leq T \leq 2 \times 10^4$ deg. K, $10^{11} \leq N_e \leq 10^{14}$ cm$^{-3}$, together with the zones prohibited to a model flare as giving theoretical results incompatible with observation. Two values of the model thickness, $z = 10^7$ and $10^8$ cm, are used. The left hand curve, corresponding to the restriction on the Hα central intensity, is uncertain for $T < 10^4$ deg. K as theoretical results are lacking; this part of the curve is therefore shown by a broken line. The intensity of and opacity in D$\alpha$ do not materially limit the allowed $T$ and $N_e$ values and so for clarity its contributions to the figure are omitted.

The main virtue of the results given in Fig. 1 is that they show approximate limits to values of $T$ and $N_e$; these values are subject to some uncertainty arising from errors in the basic data used in solving the equilibrium equations and also from possible error in the adopted helium abundance. For reasonable variations in these quantities, however, there will be only minor changes in the positions of the bounding curves.

(b) The Hα width.—So far we have not attempted to explain the great width of the Hα line observed in solar flares, although clearly this most characteristic feature must be accounted for on any valid model. Observational data on the Hα line width are quite plentiful and have been investigated, e.g., by Goldberg et al. (13) and by Bruzek (22). These authors have shown that the Hα width depends strongly on the position of the flare on the disk, wider lines being nearer the limb, as might be expected. It appears from their analyses that we need to account for lines ~4Å wide at the disk centre. The line widths referred to have been measured visually and so refer to the difference between the two wavelengths for which the contrast between flare and background is just visible.
The estimated line width of a given flare thus depends on the contrast sensitivity of the observer, assumed here to be 2 per cent.

In the Hα wings, the flare intensity is given by an expression of the form,

\[ I_v^{(p)} = I_v^{(p)}(1 - \tau_v) + \int \epsilon_v \, dz \]  \hspace{1cm} (11)

where \( I_v^{(p)} \) is the monochromatic photospheric intensity and \( \tau_v \) is the optical thickness in Hα measured up from the photosphere and through the flare. We shall write \( \tau = \tau_F + \tau^c \) where \( \tau_F \) is the opacity of the flaring region and \( \tau^c \) that of the material below the flare. Since Lyman radiation emitted by the flare will excite the underlying material, the contribution \( \tau^c \) may be quite considerable.

The contrast \( P \) between the flare and its surroundings follows from (11) as

\[ P_v = \frac{I_v^{(p)} - I_v^{(p)}}{I_v^{(p)}} = \frac{\int \epsilon_v \, dz}{I_v^{(p)}} - \tau_v, \]

which is readily shown to be equivalent to

\[ P_v = \tau_v \left\{ \frac{4N_3 \exp \left( \frac{X_{\text{Hα}}}{T} \right)}{9N_2 f_v} - 1 \right\} \]  \hspace{1cm} (12)

where \( (N_3/N_2) \) is an average population ratio taken over the emitting and absorbing region; \( X_{\text{Hα}} = hv/kT \), \( T \) being the radiation temperature (6150 deg. K), corresponding to the solar continuum near Hα, and \( 1 - f_v \), represents the fractional depression of the Hα wings below the continuum ; at \( \Delta \lambda = 2\AA, f_v \approx 0.8 \).

**Table VI**

<table>
<thead>
<tr>
<th>( N_e (\text{cm}^{-3}) )</th>
<th>( 1.0 \times 10^4 )</th>
<th>( 1.5 \times 10^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 10^{11} )</td>
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<td>( 3 \times 10^5 )</td>
</tr>
<tr>
<td>( 10^{13} )</td>
<td>( 1.5 \times 10^7 )</td>
<td>( 4 \times 10^6 )</td>
</tr>
</tbody>
</table>

For the flare models of interest here, the magnitude of the bracketed term in (13) is found to be only slightly dependent on \( N_e \) and to have values of about 2.5 and 5, respectively, for \( T = 10^4 \) and \( 1.5 \times 10^4 \) deg. K. Excitation by Lyman radiation will result in a similar value for this term in the region around a flare. To account for an Hα line of width 4\AA at the centre of the disk, we thus require,

\[ \tau = 8 \times 10^{-3}, \hspace{1cm} (T = 1.0 \times 10^4 \text{deg. K}) \]

\[ \tau = 4 \times 10^{-3}, \hspace{1cm} (T = 1.5 \times 10^4 \text{deg. K}) \]  \hspace{1cm} (14)

To check whether these are compatible with the models described by Fig. 1, consider first the broadening mechanism. Which of the two most likely causes, statistical Stark effect or radiation damping, is dominant depends on the ion concentration ; the former holds for \( N_e \geq 10^{13} \text{cm}^{-3} \), the latter for lower \( N_e \). The optical thicknesses follow from standard expressions as

\[ \tau_s = 1.5 \times 10^{-29} (\Delta \lambda)^{-5/2} \int N_e N_2 \, dz \]  \hspace{1cm} (15)

for statistical Stark and

\[ \tau_N = 5.0 \times 10^{-17} (\Delta \lambda)^{-2} \int N_2 \, dz \]

for natural broadening, \( \Delta \lambda \) being in Angströms.

The component \( \tau_F \) can be found using Table VI which gives, as a function of \( N_e \) and \( T \), values of \( N_2 \) obtained by the solution of the equilibrium equations.

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for the \( \Pi \)-state population of hydrogen. Table VII shows the resultant \( \tau^e \) for two values of the model thickness and various electron temperatures and concentrations. Those models not compatible with Fig. 1 are indicated by an asterisk.

The other component, \( \tau' \), of \( \tau \) can be found as follows. Neglecting the Lyman emission of the underlying material and treating it simply as a purely absorbing and scattering medium, it is readily shown that the total intensity \( J' \) at the centre of \( L \alpha \), say, below an optically thick flare is given by

\[
J' = (2\pi \varepsilon / \kappa \lambda) \exp \left[ - \sqrt{(3\lambda) \tau'} \right],
\]

where \( \tau' \) is optical depth in the underlying medium and it is supposed that the scattering parameter \( \lambda \) has the same value as in the flare. The population of the \( \Pi \)-state in the lower region follows from the usual equilibrium equation, which in the present case is simply

\[
A_{12}N_1 = A_{21}N_2,
\]

where \( A_{12}N_1 \) is the rate of absorption of \( L \alpha \) radiation directed down from the flare. The number of \( \Pi \)-state atoms in the line of sight is then given by

\[
\int N_2 \, dz = \int A_{12}N_1 \, dz / A_{21}. \tag{16}
\]

Supposing the \( L \alpha \) line to be flat, of width \( q \Delta \nu_d \), \( \Delta \nu_d \) being the Doppler frequency width, \( \int A_{12}N_1 \, dz \) is then given by

\[
\int A_{12}N_1 \, dz = \frac{q \Delta \nu_d (2\pi \varepsilon / \kappa \lambda)}{h\nu_0 \sqrt{(3\lambda)}}. \tag{17}
\]

**Table VII**

<table>
<thead>
<tr>
<th>( T ) (deg K)</th>
<th>( z = 10^6 \text{cm} )</th>
<th>( z = 10^8 \text{cm} )</th>
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<tr>
<td>( 1.5 \times 10^5 )</td>
<td>( 4 \times 10^{-5} )</td>
<td>( 1 \times 10^{-3}# )</td>
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</table>

For flare temperatures of \( 10^4 \) and \( 1.5 \times 10^4 \) deg. K, (17) has the approximate values \( 2 \times 10^{12}q \) and \( 2 \times 10^{20}q \) respectively, and these are only slightly dependent on the electron concentration of the flare. Taking \( q = 5 \), corresponding to a flare of optical thickness \( 10^5 \) at the centre of \( L \alpha \), we find, for \( T = 10^4 \) deg. K,

\[
\int N_2 \, dz = 2 \times 10^{11} \text{cm}^{-2} \tag{18}
\]

and for \( T = 1.5 \times 10^4 \) deg. K,

\[
\int N_2 \, dz = 2 \times 10^{13} \text{cm}^{-2}.
\]

The resultant values of \( \tau^e \) are given in Table VIII for natural and statistical Stark broadening. The electron concentration \( N'_e \) in this table is that of the region below the flare and has a value given by the equilibrium equation

\[
N'_e / N_1 \simeq \frac{P_{1e}}{P_{c1} + P_{c2}}, \tag{19}
\]

the \( P \)'s being transition rates to and from the continuum. The excitation rate \( P_{1e} \) is readily evaluated, knowing the \( L \alpha \) intensity, and is approximately \( 10^{-11} N'_e (\theta) \) or \( 10^{-8} N'_e (\theta) \) for flare temperatures of \( 10^4 \) or \( 1.5 \times 10^4 \) deg. K, respectively, \( N'_e (\theta) \) being the flare electron concentration. The spontaneous rate \( P_{c1} + P_{c2} \) is approximately \( 4 \times 10^{-13} N'_e \). At 500 km above the photosphere, \( N_1 \sim 10^{14} \text{cm}^{-3} \).
while, at the photosphere, $N_1 \sim 10^{16}$ cm$^{-3}$. If the $Lc$ radiation penetrates to these levels, it follows from (19) that the ionization in the underlying region will be almost complete for a flare in which $10^4 \leq T \leq 1.5 \times 10^4$ deg K and $10^{12} \leq N_e \leq 10^{18}$ cm$^{-3}$.

From Tables VII and VIII and the above discussion, it appears that in general for flare temperatures $\sim 1.5 \times 10^4$ deg K, we should expect $\tau^c > \tau^F$, while at lower temperatures $\tau^F > \tau^c$. In either case, it seems possible to find a flare model which satisfies the requirement (14) and is compatible with Fig. 1. However, consideration of the observed relationship between Hα width and central intensity and the variation of this width with time indicates that the $\tau^c$ component predominates at least in the initial stages of large flares, for the following reasons. At temperatures $\geq 10^4$ deg K, hydrogen is largely ionized and, as shown in Table VI, a decrease in temperature increases $N_2$ in the flare and so increases $\tau^F$. While the bracketed term in (13) decreases with decreasing temperature, the net result is that, if $\tau^F$ were the principal component of $\tau$, an increase—or at best a very slow decrease—of line width would accompany a reduction in temperature and so in central intensity. This is in disagreement with observation, and it is especially difficult to see how the observed rapid initial decrease in line width from the maximum value could be accounted for by Hα absorption and emission processes in the flare itself.

The extension by Lyman radiation of the flare excitation conditions into the surrounding material can, however, explain these emission characteristics in the Hα wings. As a flare cools from an electron temperature of $1.5 \times 10^4$ deg K, the Lyman radiation intensities decrease rapidly and so, according to these arguments, does the Hα line width. As the cooling progresses, excitation in the surroundings becomes too small to be of significance, and the Hα width is then controlled by excitation in the flare itself. It appears, then, that at the maximum phase of a large flare, the Hα line is Stark-broadened and, as the cooling progresses, natural broadening takes over since, from Fig. 1, an electron concentration $< 10^{13}$ cm$^{-3}$ is suggested. In this later phase, where the line width is controlled by Hα emission and absorption processes within the flaring region itself, one would expect only slow changes of line width with changing Hα intensity.

The Lyman spectrum and ionospheric disturbances.—The ionospheric effects of solar flares have often been ascribed to enhancement of emission in the Lyman series and continuum and it is thus of interest to compare the computed emission with estimates of the normal solar background in these frequencies.

The total intensity $J_\nu$ of the emergent radiation from a scattering and emitting medium of high opacity is given by the approximate relation:

$$J_\nu = \frac{2\sqrt{\lambda/3}}{1 + 2\sqrt{\lambda/3}} \cdot \frac{4\pi e}{\kappa\lambda}$$

For $L\alpha$ and $Lc$, (20) may be readily evaluated by the methods indicated in (15). Resultant values of $J_\nu$ at the centre of $L\alpha$ and at the head of the Lyman continuum are given in Tables IX and X.
From an assessment of results of rocket observations, Watanabe et al. (24) have concluded that the $L\alpha$ flux at the earth is $\sim 2 \times 10^{-1}$ ergs cm$^{-2}$ sec$^{-1}$. Byram et al. (25) have given more recent values which indicate a considerable temporal variation of this flux, figures as high as 9 ergs cm$^{-2}$ sec$^{-1}$ having been recorded in recent rocket flights. Taking the earlier figures as a lower limit and adopting a line width of 0.2 A (26), the equivalent black body temperature of the sun at $L\alpha$ is found to be $6.6 \times 10^3$ deg. K and $J \sim 2 \times 10^{-6}$ c.g.s. units. For $Lc$ no observations have been made, but for an effective black body temperature of $6.0 \times 10^3$ deg. K, $J \sim 1 \times 10^{-11}$ c.g.s. units.

Unpublished results of Gardner (27) on ionospheric observations at times of large solar flares show that the electron density at about 70 km can increase by a factor $\sim 20$. Since the equilibrium value of the electron density varies as the square root of the intensity of the ionizing radiations, an increase of $\sim 4 \times 10^3$ in the incident intensity is required if the same radiation is involved in causing normal and disturbed ionization. Since a large flare has area $\sim 10^{-3}$ of the visible hemisphere, the intensity of the incident radiation must then increase by a factor $\sim 4 \times 10^5$ at the flare. Comparison of Table IX with the background $L\alpha$ flux shows that such a large increase is not obtainable with models compatible with Fig. 1 unless the line width increases to an extreme value. For $Lc$, results cannot be definite since the background is unknown; it appears from Table X that, if this background corresponds to that from a black body at $6 \times 10^3$ deg. K, the $Lc$ flux from a flare may be some $10^5$ times the normal background.

**Table IX**

<table>
<thead>
<tr>
<th>$N_e$ (cm$^{-2}$)</th>
<th>$T$ (deg. K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{12}$</td>
<td>$1.0 \times 10^4$</td>
</tr>
<tr>
<td>$10^{13}$</td>
<td>$1.2 \times 10^7$</td>
</tr>
</tbody>
</table>

**Table X**

<table>
<thead>
<tr>
<th>$N_e$ (cm$^{-2}$)</th>
<th>$T$ (deg. K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{12}$</td>
<td>$1.0 \times 10^4$</td>
</tr>
<tr>
<td>$10^{13}$</td>
<td>$1.3 \times 10^7$</td>
</tr>
</tbody>
</table>

It seems from the above that we cannot ascribe both normal and enhanced D layer ionization to the absorption of $L\alpha$ nor by inference to any other of the Lyman lines. There is insufficient observational evidence to make any similar conclusion for $Lc$ radiation.

A possible alternative source of ionizing radiation might be the helium resonance lines and continua which, because of their higher excitation potentials, will be enhanced to an even greater degree than the corresponding hydrogen emission. There is, however, as shown by C. Warwick (28), an additional condition which the ionizing radiation must satisfy. Her analysis of the association between heights of limb flares and the occurrence of SID's suggested that the ionizing radiation responsible for the production of an SID is absorbed in tangential passage through the solar atmosphere unless formed at a height above about 15,000 km. From an investigation of the opacity of the chromosphere in various possible ionizing radiations—X-rays, $Lc$, $L\beta$, and $L\alpha$—Warwick concluded from the above criterion that $L\alpha$ is the most probable cause of the SID's.
Since, however, the first helium resonance line at $\lambda 584$ has an absorption coefficient almost equal to that of $L\alpha$ and since, also, the number of ground state He atoms at 15 000 km above the solar surface is at least as large as for hydrogen, the tangential opacity in $\lambda 584$ is at least as large as that of $L\alpha$. Warwick's observations, therefore, could be interpreted equally well in terms of the helium resonance line as the SID producing agent.

There are, however, difficulties associated with this interpretation. Firstly, the actual flux in the helium resonance lines is low even for a flare at 15 000 deg. K, although the flux increases rapidly with increasing temperature and may be sufficient to produce SID's with a large flare. Furthermore, published data on absorption cross-sections and atmospheric concentrations suggest that these resonance lines will be absorbed at $\sim 180$ km above the Earth's surface. The usual statement that the radiation cannot penetrate to the 70 km region is, however, not necessarily correct; the radiation is not all lost on absorption since, as the atom density is low, a good deal will be re-radiated by spontaneous recombination so that the radiation is effectively scattered and may penetrate to much lower levels than that at which it was first absorbed. However, excess ionization would be produced from the 180 km level down, and the fact that this is not observed argues against attributing SID's to radiation, such as the helium resonance lines, which is absorbed first at high levels in the atmosphere.

5. Conclusions.—Generally well verified observational results on the hydrogen and helium emissions of class 2-3 flares have been shown to be consistent with a uniform and static flare model whose electron temperatures and concentrations lie respectively in the ranges $1.0 \times 10^4$ to $1.5 \times 10^4$ deg. K and $5 \times 10^{11}$ to $\sim 10^{13}$ cm$^{-3}$.

It has been shown that excitation in lower regions due to Lyman radiation from the flare is adequate to explain the $H\alpha$ line width for a flare temperature $\sim 1.5 \times 10^4$ deg. K, the line being Stark-broadened at these levels. On this basis the observed relationship between $H\alpha$ central intensity and the line width may be accounted for, and the observed rapid decrease of line width away from the maximum receives a ready explanation. It has also been shown that the $L\alpha$ flux flare is insufficient to account for the increased ionization observed simultaneously in the D layer, and that there are difficulties in attributing it to the helium resonance lines $\lambda 584$.

For a complete analysis along the lines indicated in this work far more data are required on flare spectra. In particular, simultaneous measurements of the helium and hydrogen spectra for both limb and disk flares are urgently needed and observations in the region of the Balmer continuum could furnish very useful data for limiting the possible range of physical conditions in flares.

6. Acknowledgments.—I am indebted to Mr F. F. Gardner of the C.S.I.R.O. Division of Radiophysics for providing, in advance of publication, information on the ionospheric electron densities at times of large solar flares. I wish also to thank Dr R. G. Giovanelli for the benefit derived from discussions on the subject matter reported here.

The latter part of this work was carried out at the Harvard College Observatory and I am very grateful to the Director, Dr D. H. Menzel, for affording me the opportunity to work there.


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(22) A. Bruzek, Zs. f. Astrophys., 38, 1, 1955.
(27) F. F. Gardner (personal communication).

Notes added in Proof

1. The referee has drawn the author's attention to two interesting recent publications by Svestka* on the physical conditions in flares; the results of these are generally in agreement with those found above. It is interesting to note that he also has been forced to the conclusion that Lyman \( \alpha \) radiation into the layers below the flaring region must be considered to explain the \( \text{H} \alpha \) width. He does not, however, consider the influence of Lyman continuum in ionizing this material and so producing Stark broadening. Further if the \( \text{H} \alpha \) opacity arises largely from a region below the flare, then Svestka's analysis based on the assumption that the \( n = 10 \) state atoms—whose total number is derived from the \( \text{H} \alpha \) width—are distributed uniformly through the whole flare cannot be valid. Although, in general, Svestka recognizes the importance of departures from thermodynamic equilibrium, he implies in places an equality between kinetic and excitation temperatures. This is not acceptable and the run of temperature with depth derived this way is invalid since the excitation temperature must have an intrinsic increase with depth due to the increase in radiation intensity into the flare.

2. A recently completed investigation by the author on the influence of non-coherent scattering on the shape of emission lines indicates that the emergent Lyman \( \alpha \) line from a flare will be strongly self reversed and that, while the central intensity given here should be correct, in the wings the intensity may increase by a factor \( \sim 10^2 \). Even so it seems difficult to account for the observed effects in the \( \text{D} \) region in terms of excitation by Lyman \( \alpha \).