THE D₃ EMISION OF PROMINENCES

J. T. Jefferies
(Communicated by R. G. Giovanelli)
(Received 1956 April 30)

Summary

The computed emission of Hα and D₃ from an isothermal slab model prominence is shown to give agreement with observational data of Brück and Moss when the model has helium abundance one fifth that of hydrogen, thickness <2 × 10⁴ km, temperature between 1.1 × 10⁴ deg. K and 2.0 × 10⁴ deg. K and electron concentration between 10¹⁰ cm⁻³ to 5 × 10¹⁰ cm⁻³. Some evidence has been found that the higher temperatures are generally associated with lower electron concentrations. Comparison of the present computations with earlier results obtained by the author from an analysis of the contour of the Hα emission of prominences shows that the ratio of the abundances of helium and hydrogen atoms in solar prominences lies in the range 0.12 to 0.4. As for Hα, the self emission in D₃ is generally small compared with the diffusely reflected and transmitted component.

1. Introduction.—In a recent publication (1) the author has analysed observational data of Conway (2) and Ellison (3) on the central intensities and half-widths of the Hα emission lines of stable prominences in terms of a simple prominence model. It was shown there that the observational results were compatible with models having a thickness in the observed range, kinetic temperature T between 10⁴ deg. K and 2 × 10⁴ deg. K, and an electron concentration Nₑ between 10¹⁰ cm⁻³ and 5 × 10¹⁰ cm⁻³.

To compute the Hα contours on which the analysis of (1) was based, expressions were required for the excited state populations of hydrogen as a function of electron temperature and concentration. The author has recently (4) computed the corresponding data for helium, and an analysis similar to (1) could now be made for the D₃(3⁢S D−2⁢P) emission line. In general, however, observations of the D₃ emission have not been made in the necessary detail, the most extensive set of data being that of Brück and Moss (5, 6, 7), who made simultaneous measurements of the equivalent widths of Hα and D₃ at points in a considerable number of prominences. In this paper we compute the Hα and D₃ emission from isothermal slab model prominences having various physical properties. Comparison of these results with the observations of Brück and Moss gives information on the range of electron temperature and concentrations of solar prominences.

2. The equivalent width of Hα.—As shown in (1) the monochromatic specific intensity Iₜ(0, 1) of the Hα radiation emerging at right angles to the plane of a model prominence is easily obtained by means of an Eddington approximation. The value of Iₜ(0, 1) depends on the type of scattering adopted; and in (1) two cases were considered, namely a purely coherent and a partially coherent scattering...
mechanism. In the latter case—which we adopt here as being theoretically sounder—the source function $\mathcal{S}_\nu$ for the Hα radiation is written, following Woolley and Stibbs (8), in the form

$$\mathcal{S}_\nu = \frac{1 - \lambda}{4\pi} \left[ a J_\nu + b J_{\nu_0} \right] + \epsilon_\nu / \kappa_\nu,$$  

(1)

where $\lambda$ is a parameter defined so that $1 - \lambda$ is the probability that an absorbed quantum will be subsequently re-emitted in the same spectral line, $\epsilon_\nu$ and $\kappa_\nu$ are respectively the monochromatic emission and extinction coefficients. The total intensities $J_{\nu_0}$ and $J_\nu$ at the centre of the line and at a frequency $\nu$ respectively are defined by

$$J_\nu = \int I_\nu d\omega.$$  

The constants $a$ and $b$ are defined by

$$a = \frac{\delta_k}{\delta_j + \delta_k}; \quad b = \frac{\delta_j}{\delta_j + \delta_k},$$

$k$ and $j$ referring respectively to the upper and lower states involved in the transition and $\delta_j$ and $\delta_k$ being given by

$$\delta_j = \sum_{i<j} A_{ji}/4\pi; \quad \delta_k = \sum_{i<k} A_{ki}/4\pi,$$

the $A$'s being spontaneous transition rates.

The emergent specific intensity consists of a self-emitted component and one due to the radiation from the solar disk diffusely transmitted and reflected by the slab. The magnitude of this latter component depends on the law of darkening assumed for the disk radiation and we adopt, as in (1), a law of the form

$$I(\mu) = \frac{0.4 I(1)}{1 + 1.5 \mu}$$

(2)

where $\mu = \cos \theta$, $\theta$ being the angle between the normal to the solar surface and the direction of emergence of the radiation.

With the non-coherent scattering mechanism implied by (1), the intensity of radiation from the prominence at frequency $\nu$ may be expressed in terms of the disk intensities at the line centre and at a frequency $\nu$. An approximate expression for the specific intensity $I_\nu(\phi, \tau)$ of the normally emergent radiation can readily be obtained, as shown in (1), in the form

$$I_\nu(\phi, \tau) = \frac{\epsilon}{\kappa \lambda} A_\nu(\lambda, \tau_1) + I_{\nu_0} B_\nu(\lambda, \tau_1) + I_\nu C_\nu(\lambda, \tau_1)$$

(3)

where $\tau_1$ is the optical thickness of the model at frequency $\nu$ and $A_\nu$, $B_\nu$ and $C_\nu$ are rather lengthy expressions given explicitly in (1). The specific intensity $I_\nu$ of the normally emergent disk radiation will vary over an absorption line such as Hα; for $D_8$ however, which does not appear in the Fraunhofer spectrum, it is independent of frequency over the line. In (3) the first term represents the self-emission while the second and third are respectively the non-coherent and the coherent components of the diffusely scattered radiation.

For given electron concentrations and temperatures, expressions for $\epsilon/\kappa$ and $\lambda$ may be obtained in terms of the transition rates by general methods given by Giovanelli and Jefferies (9). To obtain the central intensities and half-widths on which the analysis in (1) was based, the contour of the line was computed from (3) for models of given optical thickness, electron concentration and
temperature. For the present work, areas under these profile curves have been measured and the equivalent width of Hα obtained on dividing these by the intensity in one angstrom of the central disk continuum near Hα, which, according to figures given by Munch (10), corresponds to that from a black body at 6150 deg. K.

3. The equivalent width of D₃.—The emission in the helium D₃ line depends on the electron concentration and temperature, the thickness and the helium abundance. In the absence of better knowledge, we shall suppose helium to be one-fifth as abundant as hydrogen. As we shall be interested only in temperatures above 10⁴ deg. K, where hydrogen is largely ionized, the hydrogen concentration will be effectively equal to the electron concentration.

In most cases of interest to us here, the optical depth in D₃ is small, and in this case the specific intensity $I_\nu(\nu, 1)$ given by (3) can be shown to be of the form

$$I_\nu(\nu, 1) = \left[ \frac{\epsilon}{\kappa} + \frac{1 - \lambda}{2} \delta I_\nu \right] \tau_1$$

where $\delta$ is a factor depending on the law of darkening adopted and in our case is 0.678. Values of $\epsilon/\kappa$ and $\lambda$, which are effectively frequency-independent, may be obtained as for Hα while, from standard expressions for the Doppler broadened extinction coefficient $\kappa$, we find

$$\tau_1(D_3) = 6.3 \times 10^{-11} N_{2\alpha} z T^{-1/2} \exp \left[ - \left( \frac{\epsilon \Delta \nu}{e\nu} \right)^2 \right]$$

where $N_{2\alpha}$ is the concentration, in cm⁻³, of the 2α state, $z$ is the thickness of the model in cm, $\Delta \nu = \nu - \nu_0$ and the mean atomic velocity $\nu = \sqrt{(2kT/M_{He})}$. Expressions for $N_{2\alpha}$ are given in (4).

Having computed $I_\nu(\nu, 1)$ from (3) or (4), integration over the spectral line gives the total D₃ emission. This is reduced to an equivalent width by dividing by the normally emergent continuum intensity in one angstrom near D₃, which corresponds to that from a black body at 6270 deg. K (10).

A difficulty arises, however, in that $N_1$ and so $N_{2\alpha} \kappa$ and $\tau_1(D_3)$, depends quite strongly on the intensity of the ultra-violet continuum of helium and thus on the opacity of the prominence to this radiation. On the assumption that external irradiation of the prominence is negligible in this spectral region, equivalent widths of D₃ have been computed for the limiting cases obtained by assuming either very high or very low opacities, providing limits between which the true result must lie. The range of applicability of each assumption may be checked using expressions for the opacity in terms of $N_1$ and $z$ (4); for $N_1 \sim 10^{10}$ cm⁻³ and reasonable thicknesses, the opacity is certainly high when $T \lesssim 10^4$ deg. K and low for $T \gtrsim 2.5 \times 10^4$ deg. K. For transitional optical thicknesses $5 \lesssim \sqrt{3} \lambda \tau_1 \lesssim 0.2$ approximate results may be obtained by interpolation between the limiting cases. This can give no more than a rough estimate, however, as the D₃ emission changes by a large factor between these limiting cases. Thus for $T = 1.5 \times 10^4$ deg. K, $N_1 = 5 \times 10^{10}$ cm⁻³, the transition occurs when the Hα equivalent width is $\sim 50$ mA, the D₃ widths found on the assumptions of low and high opacities then being $\sim 1$ mA and $\sim 60$ mA respectively. It is clear that the slope of an Hα versus D₃ equivalent width curve will be very small in the transition region but its exact form is difficult to determine.
4. Comparison with observation.—The relationships between the computed equivalent widths of Hα and D₃ are shown in Fig. 1 for various values of $N_e$ and $T$ and for an abundance ratio $\text{He}/\text{H} = 0.2$. Each point on a curve corresponds to a definite model thickness and, where applicable in the region graphed, the curves have been terminated for a thickness of $2 \times 10^4$ km. As this is a generous upper limit for the thickness of a solar prominence, parts of the computed curves lying beyond the terminal point—even if giving agreement with observational data on equivalent widths—cannot correspond to a valid prominence model.

For the $T = 10^4$ deg. K curves, the opacity in the ultra-violet continuum of helium is high except for negligibly small values of the $D_3$ equivalent width, while for $T = 2.5 \times 10^4$ deg. K the opacity is very small over the range of equivalent widths covered. The results obtained for $T = 1.5 \times 10^4$ deg. K, $N_e = 5 \times 10^{10}$ cm$^{-3}$ assuming low opacity in the ultra-violet helium continuum are shown by a continuous curve in Fig. 1. For these physical conditions, however, the opacity in the ultra-violet continuum becomes $\sim 1$ when the Hα equivalent width is $\sim 50$ mA and consequently near this point the true curve will depart from the low opacity curve and will rise monotonically to join the curve obtained by assuming high opacity in the ultra-violet continuum. As this latter curve could not be shown adequately with the scale used in Fig. 1, the coordinates are given of a point near the end of the transitional region. The estimated form of the curve in the transitional region is indicated by a broken line.

For $D_3$ the emission is almost entirely due to reflection and transmission of the disk radiation, the self-emitted component amounting at most to no more than $\sim 10$ per cent of the total. It is of interest to note that, in most cases, a similar conclusion applied for the Hα radiation from prominences (1).

Observational points given in the first two publications of Brück and Moss (5,6) are also plotted in Fig. 1. Points for some particular prominences are indicated separately in order to show the general form of the relation between the intensities for a single prominence.

Some general conclusions can be drawn at once. We note first that a reduction of the relative abundance of He would reduce the $D_3$ emission for a given Hα emission and so make the curves steeper. As it is unlikely that the relative helium abundance is much greater than that adopted, it seems that a temperature of $10^4$ deg. K is too low to account for any significant number of prominence observations. This applies for any value of $N_e$. It appears also that, for any $T$, $N_e$ must be greater than $10^{10}$ cm$^{-3}$ since a model with this electron concentration gives observed Hα and $D_3$ emissions only for excessive thicknesses if at all.

To assess the validity of the higher temperature models, we show in Fig. 2 the relationship between the computed $D_3$ intensity and model thickness. The broken lines again represent the estimated shapes of the curves in the transition regions. Since the majority of observations correspond to $D_3$ equivalent widths $< 6$ mA and the thickness could hardly be greater than $2 \times 10^4$ km there is a strong indication, when Figs. 1 and 2 are taken together, that few if any observations correspond to models with $T \leq 1.1 \times 10^4$ deg. K. An upper limit is probably $2 \times 10^4$ deg. K but here the unknown helium abundance makes estimates more tentative. It seems that in general $N_e$ is not much greater than $5 \times 10^{10}$ cm$^{-3}$ but definitely greater than $10^{10}$ cm$^{-3}$. 

© Royal Astronomical Society • Provided by the NASA Astrophysics Data System
5. Temperature variations in a single prominence.—A third set of observations by Brück and Moss (7) gives Hα and D$_3$ equivalent widths for a large number of points in a single prominence. These results are shown in Fig. 3 together with the computed curves extracted from Fig. 1. Points relating to different
regions are differentiated as explained in an insert and the Hα isophotes of the prominence, as given by Brück and Moss [7] are also shown. It is clear that there exist systematic differences and from the trend of the curves it could be inferred that the left-hand edge of the prominence has a rather higher temperature than the central regions. The alternative of a constant temperature and significantly lower electron concentration seems unlikely as it would require too great a thickness at the edge. On the right-hand edge, however, the behaviour is not as clear; the lower intensities of both Hα and D3 point to a smaller thickness or electron concentration, or both, but there is no clear indication of a temperature variation. The separation of the prominence into two distinct regions may be seen from Fig. 3 by comparison of points for the left- and right-hand sides of the prominence.

6. Discussion.—The conclusions reached here concerning the electron concentration and temperatures of an average prominence agree well with those found from previous analysis [1] of the Hα emission line contours and so suggest that the adopted helium abundance is probably not far out. In fact, by combination with the results found in [1], one may make an estimate of the relative helium abundance as follows.

For partially coherent scattering, it appears from [1] that the temperature range for all prominences is from about $1 \times 10^4$ deg. K to $2 \times 10^4$ deg. K.
Since a change in the relative helium abundance would change the $D_3$ equivalent width by an almost equal factor while leaving the H$\alpha$ intensity unchanged we see from Fig. 1 that, for $T = 2 \times 10^4$ deg. K, a reduction of the helium abundance by a factor of more than about 1.5 would certainly result in a computed curve lying above many of the points in Fig. 1. At the other end of the temperature range, an increase of the relative abundance by a factor greater than two would shift the curves for $T = 10^4$ deg. K, $N_e = 5 \times 10^{10}$ cm$^{-3}$ to a position such that some observational points lay to its left. Since the curves in Fig. 1 are based on an abundance ratio He/H = 0.2, the comparison of the present results with those found in (1) indicates that the helium abundance in prominences relative to that of hydrogen lies between the limits 0.4 and 0.12, and these limits have been generously estimated.

For the higher temperature models discussed here, certain cases seem to be excluded by observation. As may be seen from Fig. 2, a model with $T = 1.5 \times 10^4$ deg. K, $N_e = 5 \times 10^{10}$ cm$^{-3}$ gives $D_3$ emission of the right order of magnitude for thickness $\lesssim 4 \times 10^3$ km. However, as the thickness increases, the opacity in the ultra-violet continuum of helium becomes of the order of unity and the computed equivalent width of $D_3$ increases sharply to values $> 10^2$ mA, whereas the maximum value observed by Brück and Moss is 16 mA. There is evidently some fundamental reason for the non-appearance of the high $D_3$ emissions, e.g. that higher temperatures are associated with lower values of $N_e$. It is of interest to note that a similar relationship was suggested from the analysis of (1). As mentioned above, the observations shown in Fig. 3 seem to indicate a temperature decrease away from the left-hand edge of the prominence. The intensities of H$\alpha$ and $D_3$ also indicate that the thickness or electron concentration, or both, increase away from this edge. This is at least not incompatible with the suggested variation of temperature with electron concentration.

7. Conclusions.—By comparison of the observed and computed equivalent widths of the H$\alpha$ and $D_3$ emission of solar prominences it has been shown that a consistent interpretation of the observations is possible in terms of a slab model prominence whose temperature is between about $1.1 \times 10^4$ deg. K to $2.0 \times 10^4$ deg. K and electron concentration—and thus hydrogen abundance—between $10^{10}$ cm$^{-3}$ to $5 \times 10^{10}$ cm$^{-3}$ if the helium abundance is one-fifth that of hydrogen. These results are in good agreement with those obtained earlier from an analysis of the contours of the H$\alpha$ line emitted by prominences. When taken together, these two analyses indicate that the ratio of the helium to the hydrogen abundance in a solar prominence lies in the range 0.12 to 0.4.

It has been found that the $D_3$ prominence emission is made up almost entirely of diffusely scattered disk radiation.

8. Acknowledgment.—The author wishes to thank Dr R. G. Giovanelli for his continued interest and advice.

Division of Physics,
Commonwealth Scientific and Industrial Research Organization,
Sydney:
1956 April 23.
The \( D_\beta \) emission of prominences

References