THE Hα EMISSION OF PROMINENCES

J. T. Jefferies

(Communicated by R. G. Giovanelli)

(Received 1955 August 18)*

Summary

It is shown that Hα radiation from the solar disk is strongly reflected by prominences and is an important component of their radiation.

An analysis of their observed Hα emission at the limb indicates a range of kinetic temperatures of about \(10^4\) to \(2 \times 10^4\) deg. K and electron concentrations of about \(10^{10}\) to \(5 \times 10^{10}\) cm\(^{-3}\) for stable prominences.

Two alternative explanations are given for the double peaked profiles of Hα sometimes observed in solar prominences, depending on whether scattering is coherent or incoherent.

1. Introduction.—Measurements of the central intensities and half-widths of the Hα radiation emitted by stable prominences on the Sun's limb have been made by Conway (1) and by Ellison (2). In a later publication Conway (3) has interpreted some of her profile observations in terms of a theoretical expression for the emission from a model prominence and obtained kinetic temperatures in the range \(1.7 \times 10^4\) to \(1.8 \times 10^4\) deg. K. The absolute intensity measurements, however, require for their interpretation a knowledge of the excited state populations of hydrogen as a function of temperature and density, such as have been discussed recently by Jefferies and Giovanelli (4). On the basis of results obtained in (4) we present here an analysis of the observational results of (1) and (2) and deduce some of the physical properties of prominences consistent with the observational material.

To analyse the data given by Conway (1) and Ellison (2), we shall compute the Hα emission from a model hydrogen prominence in the form of a slab of uniform kinetic temperature, electron concentration and thickness. The slab is supposed to be perpendicular to the Sun's surface, oriented so that its minimum thickness is in the line of sight and illuminated by chromospheric and photospheric radiation incident from a hemisphere the diametrical plane of which is at right angles to the plane of the prominence. The intensity of the Hα radiation incident on the prominence may be related to that at the centre of the disk by a limb-darkening law \(I_0(\mu') = f I_0(1)[1 + \mu'^{-2}]\), where \(I_0(\mu')\) is the intensity at an angle \(\cos^{-1}\mu'\) to the normal to the Sun's surface. The present results are rather insensitive to departures from this law. The radiation from this model prominence will consist of two parts, one the diffusely reflected radiation and the other the emission of the material itself. These contributions depend on the degree of coherence in frequency of the scattered radiation. We shall consider here two cases such that the scattering is either completely or partially coherent; in each case we shall obtain expressions for the self-emitted and diffusely reflected radiation and compare the computed emission profiles with observational results.

* Received in original form 1955 March 29.
2. The equation of transfer.—The monochromatic specific intensity \( I_\nu(\tau, \mu) \) of the radiation in a plane parallel slab such as our model prominence is given by the equation of transfer

\[
\mu \frac{dI_\nu(\tau, \mu)}{d\tau} = I_\nu(\tau, \mu) - \mathfrak{F}_\nu(\tau, \mu),
\]

where \( \cos^{-1} \mu \) is the angle the radiation makes with the normal to the slab, \( \tau_\nu \) is the optical thickness and \( \mathfrak{F}_\nu \equiv f_\nu/\kappa_\nu \) is the source function, \( f_\nu \) and \( \kappa_\nu \) being respectively the monochromatic emission and absorption coefficients. For a medium which scatters coherently and isotropically, the source function may be written

\[
\mathfrak{F}_\nu(\tau) = \frac{\epsilon_\nu}{\kappa_\nu} + (1 - \lambda)J_\nu/4\pi
\]

where \( \epsilon_\nu \) is the true emission coefficient, \( J_\nu \) is the total monochromatic intensity \( \equiv \int I_\nu \, d\omega \) and \( \lambda \) is a scattering parameter defined so that \( (1 - \lambda) \) is the fraction of absorbed radiation which is subsequently scattered.

The specific intensity of the radiation emerging from the atmosphere follows on solution of (1) and is given, neglecting frequency subscripts, by

\[
I(0, \mu) = I(\tau_1, \mu) \exp (-\tau_1/\mu) + \int_0^{\tau_1} \mathfrak{F}(t, \mu) \exp (-t/\mu) \, dt/\mu,
\]

\( \tau_1 \) being the total optical thickness of the atmosphere.

To evaluate \( I \), we substitute in (2) an approximate value of \( J \) obtained as the solution of Eddington’s approximation to the equation of transfer,

\[
\frac{1}{3} \frac{dJ}{d\tau} = \lambda J - \frac{\epsilon}{\kappa}.\]

If \( \epsilon/\kappa \) and \( \lambda \) are independent of \( \tau \), the solution of (4) is

\[
J = \frac{4\pi \epsilon}{\kappa \lambda} + A \exp(\sqrt{3\lambda \tau}) + B \exp(-\sqrt{3\lambda \tau}),
\]

\( A \) and \( B \) being integration constants. Since the total intensities on each face of the slab are equal, it follows that \( B = A \exp(\sqrt{3\lambda \tau_1}) \).

The second boundary condition follows readily from the definitions of the total intensity \( J \), given above, and of the net flux \( F \equiv \int I_\mu \, d\omega \). Assuming that, at the surface of the slab, the specific intensity of the outwardly directed radiation is independent of \( \mu \) and that the inwardly directed radiation, coming from the solar disk, follows a law of darkening of the form given above, it follows readily that

\[
J - 2\pi'I = 2F
\]

where \( I' = 0.678I(1) \).

Using the Eddington approximation \( F = \frac{1}{3} dJ/d\tau \), the integration constants \( A \) and \( B \) are easily found and the total intensity takes the form

\[
J = \frac{4\pi \epsilon}{\kappa \lambda} \left[ 1 - \frac{\exp(-\sqrt{3\lambda \tau}) + \exp(\sqrt{3\lambda(\tau - \tau_1)})}{D} \right]
+ \frac{2\pi I'}{D} \left[ \exp(\sqrt{3\lambda(\tau - \tau_1)}) + \exp(-\sqrt{3\lambda \tau}) \right]
\]

where \( D = 1 + 2\sqrt{\lambda/3} + (1 - 2\sqrt{\lambda/3}) \exp(-\sqrt{3\lambda \tau_1}) \).
Substituting (7) into (2) and carrying out the integration in (3) it follows that
the specific intensity of the emergent radiation in the direction \( \mu = 1 \), i.e. at
right angles to the plane of the prominence, is

\[
I(\theta, 1) = \frac{e}{\kappa \lambda} \left[ 1 - \exp(-\tau_1) - F(\beta, \tau_1) \right] + \frac{I'}{2} F(\beta, \tau_1) \quad (8)
\]

where \( \beta = \sqrt{3} \lambda \) and

\[
F(\beta, \tau_1) = \frac{1 - \lambda}{D} \left\{ \exp(-\beta \tau_1) \left[ \frac{\exp\left[\left(\beta - 1\right)\tau_1\right] - 1}{\beta - 1} \right] + \frac{1 - \exp\left[-(\beta + 1)\tau_1\right]}{\beta + 1} \right\} .
\]

The term \( I(\tau_1, \mu) \exp(-\tau_1/\mu) \) appearing in (3) represents the radiation
transmitted directly (i.e. not diffusely) through the prominence. For a
prominence projecting beyond the limb this term is zero since there is no
incident radiation in the line of sight.

We shall be interested in the ratio \( R_v \) of the intensity \( I_v(\theta, 1) \) to the intensity \( I_w \)
of the continuous spectrum at the centre of the disk just outside the Ha line.
This ratio may be written

\[
R_v = \frac{\epsilon}{\kappa \lambda} \left[ 1 - \exp(-\tau_1) - F(\beta, \tau_1) \right] + 0.339r_v F(\beta, \tau_1) \quad (9)
\]

where \( \tau_v \) represents the contour of the Ha absorption line at the centre of the
disk. The two terms on the right of (9) represent respectively the emission
from the prominence itself and the diffusely reflected incident radiation. The
function \( F(\beta, \tau_1) \) differs by less than 5 per cent from the accurate expression
obtained by Chandrasekhar (5) for the diffuse reflection and transmission of
such a slab, at least over the range available for comparison, i.e. where
Chandrasekhar's X and Y functions have been tabulated. The function
\( F(\beta, \tau_1) \) for \( \lambda = 6 \times 10^{-8} \) is shown in Fig. 1.

\[
F(\beta, \tau_1)
\]

\[
\beta \quad \tau_1
\]

![Fig. 1.—The function \( F(\beta, \tau_1) \) for \( \lambda = 6 \times 10^{-8} \).](image-url)
2.1. **Mixed coherent and non-coherent scattering.**—According to Woolley and Stibbs (6) the redistribution of frequency on scattering results in a source function of the approximate form

\[ \mathfrak{S}_\nu = \frac{1 - \lambda}{4\pi} [aJ_\nu + bJ_{\nu^*}] + \epsilon_\nu / \kappa_\nu \]  

(10)

where \( a \) and \( b \) are defined by

\[ a = \frac{\delta_k}{\delta_j + \delta_k}, \quad b = \frac{\delta_j}{\delta_j + \delta_k}, \]

\( k \) and \( j \) referring respectively to the upper and lower states of the atom, and the quantities \( \delta_j \) and \( \delta_k \) being defined by

\[ \delta_j = \sum_{i<j} A_{ij} / 4\pi, \quad \delta_k = \sum_{i<k} A_{ki} / 4\pi, \]

the \( A \)'s being spontaneous transition rates.

The specific intensity \( I(\alpha, \mu) \) of the radiation emerging at an angle \( \cos^{-1} \mu \) to the normal to the prominence is given by (3) where \( \mathfrak{S}_\nu \) is now given by (10). Expressions for the total intensities \( J_\nu \) and \( J_{\nu^*} \) may be obtained from the approximate equation, analogous to (4)

\[ \frac{I}{3} \frac{d^2 J}{d\tau^2} = J - (1 - \lambda)(aJ + bJ_0) - 4\pi\epsilon / \kappa \]  

(11)

where \( J_0 \) represents the central intensity; remaining frequency subscripts have been omitted. The solution of (11) with the boundary conditions used above follows readily. Since, at the line centre, (11) is equivalent to (4), the solution for \( J_0 \) is the same as for the case of coherent scattering.

Substituting values of \( J \) and \( J_0 \) into (3) we find

\[ I(\alpha, 1) = \frac{\epsilon}{\kappa \lambda} \left[ 1 - \exp (-\tau_1) - \phi(\beta, \tau_1) \right] + \frac{I'_0}{2} \phi(\beta, \tau_1) \]

\[ + \left[ \left( \frac{I'}{2} - \frac{I'_0}{2} \right) x \frac{D_2}{D_1} \right] - \frac{\epsilon}{\kappa \lambda} \left( 1 - x \frac{D_2}{D_1} \right) \psi(\tau_1) \]  

(12)

where

\[ \phi(\beta, \tau_1) = \frac{(1 - \lambda)(b + ax)}{D_1} \left\{ \frac{1 - \exp \left[ -(\beta k + 1)\tau_1 \right]}{\beta k + 1} + \frac{\exp (-\tau_1) - \exp (-\beta k \tau_1)}{\beta k - 1} \right\}, \]

\[ \psi(\tau_1) = \frac{a(1 - \lambda)}{D_2} \left\{ \frac{1 - \exp \left[ -(q + 1)\tau_1 \right]}{q + 1} + \frac{\exp (-\tau_1) - \exp (-q \tau_1)}{q - 1} \right\}, \]

\[ k = \tau_0 / \tau, \quad q = \sqrt{3 [1 - a(1 - \lambda)]}, \quad x = \frac{b(1 - \lambda)}{b(1 - \lambda) - \lambda(k^2 - 1)}, \]

and

\[ D_1 = (1 + \frac{2}{3} \beta) + (1 - \frac{2}{3} \beta) \exp (-\beta k \tau_1), \]

\[ D_2 = (1 + \frac{2}{3} \beta k) + (1 - \frac{2}{3} \beta k) \exp (-\beta k \tau_1), \]

\[ D_3 = (1 + \frac{2}{3} q) + (1 - \frac{2}{3} q) \exp (-q \tau_1). \]

In this equation the terms multiplying \( \epsilon / \kappa \lambda \) represent the self-emitted component, those multiplying \( I' \) and \( I'_0 \) the diffusely reflected and transmitted component.

The diffusely reflected radiation is by no means negligible across the line for either coherent or partially coherent scattering; indeed, it predominates over the self-emission of the prominence in many cases, and as a result considerably modifies the central intensity and half-width. In view of this it may be necessary to reconsider the interpretation of other prominence spectral lines taking into account the effects of diffuse reflection of chromospheric radiation.
3. The Hα intensity and contour.—The central intensity and contour of the Hα radiation emitted by a model prominence of given $\tau_0^0$—the optical depth at the line centre—may be computed, depending on the type of scattering considered, from either (9) or (12) if we know the values of the parameters and their frequency dependence. Expressions for the quantities $e/\kappa$ and $\varphi$ are given in (4), equations (7.5) and (7.6); $r_\nu$ is taken from Fig. 5 of the author's paper (7) and is the mean of four sets of observations of the Hα contour at the centre of the disk. The value of $\varphi$ is almost constant for the various temperatures and electron concentrations used here and for simplicity is taken to be constant and equal to $6 \times 10^{-3}$. The frequency dependence of $\tau_1$ is taken as the standard Doppler expression

$$\tau_1 = \tau_0^0 \exp \left[ -\left(\frac{c \Delta \nu}{\nu \varphi}\right)^2 \right]$$

where the mean atomic velocity $v = \sqrt{2kT/M}$. Typical computed profiles, showing the magnitudes of the reflected components, are given in Figs. 2(a) and 2(b). Table I gives the self-emitted and diffusely reflected components of the intensity at the centre of Hα for a series of values of $\tau_0^0$ for each of the temperatures and electron concentrations considered. The profiles of the

---

**Figs. 2(a) and 2(b).—Hα contours of model prominences for values of $\beta_0\tau_0^0$ shown in the top right of each diagram. The diffusely scattered component is shown as a broken curve. The figure relates to a model with $T=10^4$ deg. K, $N_e=10^{16}$ cm$^{-3}$ and with low optical thickness in the Lyman continuum. Fig. 2 (a) is for the coherent and 2 (b) for the partially coherent case.**
emergent radiation may be obtained from these values using the above frequency dependence of \( \tau_1^0 \) to find \( \tau_1 \).

### Table I

Relative Contributions of the Self Emitted and Diffusely Reflected Radiation to the \( \text{H}_\alpha \) Intensity*

<table>
<thead>
<tr>
<th>( N_e = 10^{10} \text{ cm}^{-3} )</th>
<th>( N_e = 5 \times 10^{10} \text{ cm}^{-3} )</th>
<th>( N_e = 10^{10} \text{cm}^{-3} )</th>
<th>( N_e = 5 \times 10^{10} \text{cm}^{-3} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta \tau \rangle \text{Ha} )</td>
<td>( \beta \tau \rangle \text{Le} \ll 1 )</td>
<td>( \beta \tau \rangle \text{Le} \approx 1 )</td>
<td>( \beta \tau \rangle \text{Le} \gg 1 )</td>
</tr>
<tr>
<td>( T = 10^4 \text{deg. K} )</td>
<td>( T = 1.5 \times 10^4 \text{deg. K} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4.57</td>
<td>6.14</td>
<td>12.1</td>
</tr>
<tr>
<td>2</td>
<td>4.79</td>
<td>4.85</td>
<td>9.61</td>
</tr>
<tr>
<td>0.5</td>
<td>5.25</td>
<td>1.40</td>
<td>2.78</td>
</tr>
<tr>
<td>0.1</td>
<td>2.93</td>
<td>0.19</td>
<td>0.38</td>
</tr>
<tr>
<td>0.01</td>
<td>0.40</td>
<td>0.01</td>
<td>0.03</td>
</tr>
</tbody>
</table>

* The self-emitted component is given in columns 3 to 8; the diffusely reflected component shown in column 2 is constant owing to the assumed constancy of \( \lambda \). Results are given for each of the two limiting cases \( \langle \beta \tau \rangle \text{Le} \ll 1 \) and \( \langle \beta \tau \rangle \text{Le} \gg 1 \) for \( T = 10^4 \text{ deg. K} \). The units of intensity are such that the continuum near \( \text{H}_\alpha \) at the centre of the solar disk equals 100.

As shown in (4), \( c/\kappa \) depends on the opacity of the slab to Lyman continuous radiation. As a general treatment taking this into account seems unnecessarily complex, we have computed the radiation characteristics for the two limiting cases obtained by assuming the models to be optically thick or optically thin in \( L_e \). From known expressions for this optical thickness we can estimate where the transition region lies between these limiting cases and, from the results in the limiting cases, estimate the radiation characteristics for the transition region. This complication only arises for the models with electron temperature \( T = 10^4 \text{ deg. K} \). For the other temperature considered, \( T = 1.5 \times 10^4 \text{ deg. K} \), the optical thickness in \( L_e \) is always small in the range of \( \text{H}_\alpha \) optical thicknesses used.

### 4. Comparison with Observation

As the observational results of Conway (1) and those of Ellison (2) are rather different in emphasis, and as we wish to consider some of the latter in detail, separate comparisons of these observations with the computed curves have been made.

Figs. 3(a) and 3(b) show for coherent and partially coherent scattering respectively a comparison of Conway’s observational points with the computed curves of half-width against central intensity found by taking a series of values of \( \tau_1^0 \) for each value of \( T \) and \( N_e \). On the whole, this comparison indicates, for stable prominences, a temperature of about \( 10^4 \) to \( 1.5 \times 10^4 \text{ deg. K} \) and an electron concentration of about \( 10^{10} \) to \( 5 \times 10^{10} \text{ cm}^{-3} \).

The information is insufficient to establish very definitely the values of temperature and electron concentration. There is an indication that a model with lower \( T \) and \( N_e \) may also fit the observations quite well, while, for the partially coherent case at least, a higher temperature model would be required to fit some of the lower intensity observations. Considerations of the physical thicknesses of the models, to be made later, will help to place limits on the values of \( N_e \) and \( T \).

Ellison’s measurements were made on a small number of prominences; for each one he recorded observations at a number of heights above the solar limb. These results are compared with the theoretical curves in Figs. 4(a) and 4(b). The notation used is the same as in (2), but points corresponding to double-peaked or flat profiles are shown with a subscript F for future reference.
The extent to which any one set of observational points follows the trend of the computed curves gives an idea of the uniformity of $N_e$ and $T$ at various heights in a prominence. There is some indication of such a uniformity for some prominences, but much more extensive observations would be required to confirm this.

Figs. 3 (a) and 3 (b).—Comparison of computed central intensity–half-width curves with Conway's observations. The broken parts of the curves refer to the region of transition between low and high optical thickness in the Lyman continuum. Fig. 3 (a) refers to the coherent and 3 (b) to the partially coherent case.
For the partially coherent case shown in Fig. 4(b) a higher temperature model would be required to fit some of the observations. For \( T = 2.0 \times 10^4 \) deg. K, the limiting half-width is 0.72 A and the general shapes of the curves suggest that, for some prominences, temperatures may be as high as this, electron concentrations still being of the order of \( 10^{10} \) to \( 5 \times 10^{10} \) cm\(^{-3}\).

**Figs. 4(a) and 4(b).**—Comparison of computed central intensity-half-width curves with Ellison's observations. The heavy parts of the curves refer to computed contours with flat-topped or double-peaked profiles. The letter F next to an observational point indicates that this observation showed a flat or doubled profile. Fig. 4(a) refers to the coherent, 4(b) to the partially coherent case.
Although the ranges of $T$ and $N_e$ found above seem to give agreement with observation, there is another parameter, the thickness, whose value must be considered. Each point on the curves corresponds to a certain optical thickness in Hα and so to some physical thickness of the model. Unless the thicknesses which correspond to the bulk of the observations are of the order observed for prominences, the models will be unsatisfactory. A reasonable range of thicknesses may be based on the observations of M and Mme D’Azambuja (8) which show that about 90 per cent of stable prominences have thicknesses between $5 \times 10^3$ and $8 \times 10^3$ km as measured on the Sun’s disk. This range is rather narrow for a consideration of measurements on the limb since, owing to the filamentary structure of prominences, some observations will be made through quite small thicknesses. On the other hand, observations will not in general be made at right angles to the plane of the prominence and some allowance for this should be made. We shall adopt a thickness range of $2 \times 10^3$ to $2 \times 10^4$ km. The observations indicate that the Hα lines from a majority of prominences have half-widths in the range 0.80 to 1.00 Å. If our models are to be satisfactory, those with thicknesses in the range $2 \times 10^3$ to $2 \times 10^4$ km should give half-widths about 0.80 to 1.00 Å. Reference to Fig. 5(a) shows that in this respect the range of $N_e$ and $T$ seems to meet the requirements. The general trend of the curves indicates that a lower $T$ would require a higher $N_e$ to fit the observations; however, such a model would not fit the results of Figs. 3(a) and 4(a).

If the scattering is of the partially coherent type used here, Fig. 5(b) suggests that an electron concentration of $10^{10}$ cm$^{-3}$ would be too low to give agreement with observation unless the temperature is about $2 \times 10^4$ deg. K or more. This would not be inconsistent with the results shown in Figs. 3(b) and 4(b).

In summary, if the atomic scattering is coherent, the observational results of the Hα intensities and half-widths can only be explained by kinetic temperatures in the range about $10^4$ to $1.5 \times 10^4$ deg. K and electron concentrations about $10^{10}$ to $5 \times 10^{10}$ cm$^{-3}$, the lower values of $N_e$ being probably associated with higher $T$, and vice versa. For the partially coherent type of scattering adopted, the temperature range is about $10^4$ to $2 \times 10^4$ deg. K, the electron concentration again being in the range about $10^{10}$ to $5 \times 10^{10}$ cm$^{-3}$; the lower values of $N_e$ are, however, consistent only with the higher values of $T$.

5. Flat and double-peaked profiles.—Ellison (2) describes a number of Hα prominence profiles which are flat or double-peaked. He suggests an explanation of these in terms of a prominence model consisting of a core at about 20,000 deg. K surrounded by a layer at 10,000 deg. K, pointing out that such a model could give rise to the observed profiles. While this is so, it seems more likely on physical grounds that any temperature gradient would be in the reverse direction, in which case no doubling would occur.

Alternative explanations arise from the present analysis. If the scattering is coherent, it is seen from (9) that, while the term representing the prominence self-emission decreases away from the line centre, the reflection term $r_v F(\beta, \tau_1)$ may not. Depending on the relative magnitudes of the emitted and reflected radiation, there results a normal, flat-topped or double-peaked profile as shown in Fig. 2(a).

For the mixed coherent and non-coherent scattering considered here, double-peaked profiles may also appear, as illustrated in Fig. 2(b), but for a
quite different reason. Radiation in the wings comes, on the average, from
greater depths than in the centre of the line, the relative emitted intensities
being proportional to the total intensities at the appropriate average depths.
However, disregarding incident radiation, the total intensity increases with

\[ \text{HAWF WIDTH (ANGSTROEMS)} \]

\[ \text{THICKNESS (Km)} \]

\[ 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \]

(a)

\[ \text{HAWF WIDTH (ANGSTROEMS)} \]

\[ \text{THICKNESS (Km)} \]

\[ 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \]

(b)

Figs. 5 (a) and 5 (b).—Computed curves of half-width of the emergent radiation against
thickness of the model. The broken parts of the curves, as in Figs. 3 and 4, refer to the region
of transition between low and high optical depths in the Lyman continuum. Fig. 5 (a) refers
to the coherent, and 5 (b) to the partially coherent case.
depth at a rate which decreases away from the line centre. Non-coherent scattering causes some of the radiation absorbed near the line centre to be re-emitted in the wings, thus enhancing the total wing intensities and, as a result, the emitted intensity there may exceed that near the centre and a double-peaked profile result.

It follows then that for either scattering mechanism, double-peaked profiles will result if the atmosphere is sufficiently thick.

In Figs. 4(a) and 4(b) the subscript "F" indicates the flat or doubled profiles mentioned by Ellison, and the region over which the computed curves show the same behaviour is indicated by a heavier line. The agreement suggests that one of these explanations accounts for at least part of the phenomenon.

6. Limb and disk intensities.—Further evidence of the importance of diffuse reflection is obtained by considering the specific intensities of a prominence when observed at the limb and on the disk of the Sun. For simplicity we confine attention to an optically thick prominence; the conclusions, however, are not essentially altered by this restriction, nor, since we shall deal only with the line centre, is it dependent on the type of scattering considered, i.e. coherent or partially coherent.

The specific intensity, at the centre of Hα, of the radiation emerging at an angle \( \cos^{-1} \mu \) to the normal to a thick model prominence is easily found to be given by

\[
I(0, \mu) = \frac{\epsilon}{\kappa \lambda} \left[ 1 - \frac{1 - \lambda}{(1 + 2 \beta)(1 + \mu \beta)} \right] + \frac{I'}{2} \frac{(1 + \lambda)}{(1 + 2 \beta)(1 + \mu \beta)}.\]

(10)

We shall consider two extreme cases such that (i) the reflected component is negligible, and (ii) the self-emitted component is negligible.

In case (i) it is easily seen from (10) that, when \( \lambda = 6 \times 10^{-3} \),

\[
\frac{I(0, 1)}{I(0, 0)} \approx \frac{5}{2(1 + \beta)} = 2.2
\]

(11)

and so the specific intensity of a prominence observed at the limb (\( \mu = 1 \)) is over twice as great as when observed on the disk (\( \mu = 0 \)).

In case (ii) it follows simply that,

\[
\frac{I(0, 1)}{I(0, 0)} = \frac{1}{1 + \beta} = \frac{1}{1.13}
\]

(12)

so that the specific intensity for a limb observation is slightly less than that for a disk observation.

From observation, Ellison (2) found that the Hα limb and disk intensities of an individual prominence are much the same, from which it follows that diffuse reflection plays a considerable part in the Hα emission of prominences.

7. Conclusions.—Observational values of intensities and half-widths of the Hα emission of stable prominences have been analysed in terms of simple prominence models. It has been shown that agreement between observation and theory is obtained for models with thicknesses of the order observed and with kinetic temperatures in the range \( 10^4 \) deg. K to \( 2 \times 10^4 \) deg. K and electron concentrations between \( 10^{10} \) cm\(^{-3} \) and \( 5 \times 10^{10} \) cm\(^{-3} \). The theoretical results have been found to be comparatively insensitive to the assumption of either fully or partially coherent scattering and the observational data are inadequate to decide between them, although on theoretical grounds the partially coherent mechanism is preferable.
It has been shown that diffuse reflection of Hα disk radiation by prominences constitutes a considerable part of their total emission and is a factor in accounting for the observed equality of limb and disk intensities of an individual prominence.

The work described in this paper was carried out as part of the programme of the Division of Physics of the Commonwealth Scientific and Industrial Research Organization, Australia.

8. Acknowledgments.—I should like to express my thanks to Dr R. G. Giovanelli for his constant interest and for the benefit of his valuable comments.

Division of Physics,
National Standards Laboratory,
Commonwealth Scientific and Industrial Research Organization,
Sydney:
1955 August.

References