TRANSITION PROBABILITIES OF FORBIDDEN LINES

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ABSTRACT

Transition probabilities are given for a number of forbidden lines of astrophysical interest. Also, from the computed transition probabilities, expected equivalent widths of [Ca II] and [S I] forbidden absorption lines in the solar spectrum are calculated.

I. INTRODUCTION

A knowledge of the transition probabilities of forbidden lines is important not only for making identifications but also for discussing the physical conditions in the objects which emit these lines. In this paper the transition probabilities are given for a number of forbidden lines of astrophysical interest. Besides the transition probabilities of the observed lines, probabilities are computed for the other transitions which compete with them in removing atoms from the excited levels. In each case the computations have been carried out by the methods of Shortley, calculating the line strengths first and then, from them, the transition probabilities.

II. THE Ca II 4^2S — 3^2D DOUBLET

The Ca II 4^2S — 3^2D doublet has been identified and studied in v Sagittarii by Merrill and by Greenstein and Merrill. These lines occur only by quadrupole transitions, and, from the simple formulae for the case of one electron outside a closed shell, the line strengths are found to be

\[ S_q (3^2S_{1/2}, 4^2D_{5/2}) = 36 s_q (4d, 3s), \]

\[ S_q (3^2S_{1/2}, 4^2D_{3/2}) = 24 s_q (4d, 3s), \]

in terms of the radial integral

\[ s_q (4d, 3s) = -\frac{1}{3\sqrt{5}} \int_0^{\infty} r^2 R(4d) R(3s) \, dr. \]

This integral was evaluated numerically from the Hartree wave functions with exchange for Ca II and was found to have the value \( s_q = 10.83/3\sqrt{5} = 1.61 \) atomic units. The cancellation between positive and negative portions of the integral, the importance of which for judging the reliability of the results has been emphasized by Bates, is negligible, for the last 4s loop contributes 10.896, while the second-last loop contributes -0.068, and the first two loops do not affect the result at all. Transition probabilities, computed using the observed energy difference, are given in Table 1.

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1 Phys. Rev., 57, 225, 1940. Definitions of line strengths and all other quantities used are given in Shortley's paper and therefore are not repeated here.


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The magnetic dipole strength of the transition $4^2\text{D}_{3/2} \leftarrow 4^2\text{D}_{5/2}$ is $1^2$, and the corresponding transition probability is given in Table 1. This transition can also occur by quadrupole radiation, but the probability is smaller by a factor of about $10^{-6}$ and has therefore not been calculated. The smallness of the $2^2\text{D}_{3/2} \leftarrow 2^2\text{D}_{5/2}$ transition probability shows that the $2^2\text{D}_{5/2}$ level is not drained by this transition.

<table>
<thead>
<tr>
<th>Ion</th>
<th>Transition</th>
<th>Coupling Parameter $x$</th>
<th>Wave Length</th>
<th>Transition Probability (Sec$^{-1}$)</th>
<th>Electric Quadrupole</th>
<th>Magnetic Dipole</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ca II</td>
<td>$4s-3d \quad 2^2\text{S}<em>{1/2} \leftarrow 2^2\text{D}</em>{3/2}$</td>
<td>-</td>
<td>7291.46</td>
<td>-</td>
<td>1.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$3d \quad 2^2\text{S}<em>{1/2} \leftarrow 2^2\text{D}</em>{5/2}$</td>
<td>-</td>
<td>7323.88</td>
<td>-</td>
<td>1.3</td>
<td>3.8</td>
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<tr>
<td>Fe XV</td>
<td>$3s3p \quad 2^1\text{P}_1 \leftarrow 2^1\text{P}_0$</td>
<td>0.270</td>
<td>7059.62</td>
<td>2.4x10$^{-6}$</td>
<td></td>
<td>1.6x10$^{-3}$</td>
</tr>
<tr>
<td>A XI</td>
<td>$2p^4 \quad 2^1\text{P}_1 \leftarrow 2^1\text{P}_0$</td>
<td>0.300</td>
<td>6919</td>
<td>3.0x10$^{-4}$</td>
<td>67</td>
<td></td>
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<td></td>
<td>$2p^3-3p^4 \quad 2^1\text{P}_1 \leftarrow 2^1\text{P}_0$</td>
<td>0.1369</td>
<td>5414</td>
<td>1.5x10$^{-3}$</td>
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<tr>
<td>K VI</td>
<td>$3p^4 \quad 2^1\text{D}_2 \leftarrow 2^1\text{S}_0$</td>
<td>0.1701</td>
<td>3688</td>
<td>0.300</td>
<td>8.1</td>
<td>0.300</td>
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<tr>
<td>Ca VII</td>
<td>$3p^5 \quad 2^1\text{D}_2 \leftarrow 2^1\text{S}_0$</td>
<td>0.0517</td>
<td>7724.7</td>
<td>0.530</td>
<td>4.1</td>
<td>0.530</td>
</tr>
<tr>
<td>S I</td>
<td>$3p^4 \quad 1^3\text{P}_2 \leftarrow 1^3\text{S}_0$</td>
<td>0.0719</td>
<td>4589.0</td>
<td>4.1</td>
<td>1.3</td>
<td>4.1</td>
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<td>Cl II</td>
<td>$3p^4 \quad 1^3\text{P}_1 \leftarrow 1^3\text{S}_0$</td>
<td>0.0517</td>
<td>4506.9</td>
<td>0.015</td>
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<td>A III</td>
<td>$3p^4 \quad 1^3\text{P}_2 \leftarrow 1^3\text{S}_0$</td>
<td>0.0959</td>
<td>3675.0</td>
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<tr>
<td>K IV</td>
<td>$3p^4 \quad 1^3\text{P}_1 \leftarrow 1^3\text{S}_0$</td>
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<td>3583.0</td>
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<td>Kr III</td>
<td>$4p^4 \quad 2^1\text{D}_2 \leftarrow 2^1\text{D}_2$</td>
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<td>Xe III</td>
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<td>-</td>
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<tr>
<td></td>
<td>$3p^5 \quad 2^1\text{D}_2 \leftarrow 2^1\text{D}_2$</td>
<td>-</td>
<td>4311.0</td>
<td>-</td>
<td>10</td>
<td>-</td>
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</tbody>
</table>

* Wave lengths from the RMT are given for lines in the observable regions.

### III. THE [Fe XV] 3s3p $^3\text{P}_0 ^3\text{P}_2$ TRANSITION

The [Fe XV] 3s3p $^3\text{P}_0 ^3\text{P}_2$ transition is the only coronal line identified by Edlén as arising from a highly excited state (E.P. = 31.9 volts) and not from a transition within the ground configuration. In pure LS coupling the magnetic dipole line strength of a $^3\text{P}_1 ^3\text{P}_2$ transition is $S_m = S_\pi = \frac{2}{3}$; but, if the breakdown of coupling is taken into account, the $^3\text{P}_0$ and $^1\text{P}_1$ levels are mixed. If we write $\psi(^3\text{P}_0)$ and $\psi(^1\text{P}_1)$ as the actual eigenfunctions, and $\psi(^3\text{P}_1)$ and $\psi(^1\text{P}_0)$ as the LS eigenfunctions, then

$$\psi(^3\text{P}_0) = a\psi(^3\text{P}_1) + b\psi(^1\text{P}_0),$$
$$\psi(^1\text{P}_0) = -b\psi(^3\text{P}_0) + a\psi(^1\text{P}_1),$$

$^6$ Zs. f. A., 22, 30, 1943.
and the magnetic dipole line strength is \( S_m(\text{3P}_1, \text{3P}_0) = \frac{3}{8} a^2 \). The \( sp \) configuration is a special case of the \( sl \) configuration, which has been worked out completely, and in which all results can be expressed in terms of the single coupling parameter \( \chi = 3\bar{\gamma}/4G_0 \). In terms of this parameter,

\[
a = 1 - \frac{1}{8}\chi^2 + \frac{2}{27}\chi^3 + \frac{5}{162}\chi^4 + \ldots .
\]

Only the asymptotic expansion for small \( \chi \) is given here, because all the experimental data fall in this range. For evaluating \( \chi \), only the \( \text{3P} \) splitting is available in the case of \( \text{Fe xv} \), and the ratio \( \frac{(\text{3P}_2 - \text{3P}_1)}{(\text{3P}_1 - \text{3P}_0)} = 2.43 \) gives \( \chi = 0.202 \), which, in turn, gives \( a = 0.996 \) and \( S_m = 2.48 \). The resulting transition probability, calculated with the observed coronal wave number, is given in Table 1.

Similarly, the quadrupole line strengths of the transitions from the \( \text{3P}_2 \) level are found to be

\[
S_q(\text{3P}_2, \text{3P}_1) = \frac{15}{16} a^2 s_q(3p, 3p),
\]

\[
S_q(\text{3P}_0, \text{3P}_1) = \frac{15}{81} s_q(3p, 3p),
\]

from the formulae applying for the two-electron case. Here

\[
s_q(3p, 3p) = -\frac{2}{5} \int_0^\infty r^2 R(3p) R(3p) \, dr.
\]

The value of this radial integral has been estimated by taking the effective charge acting on the 3p electron to be 10\( \frac{1}{2} \) atomic units and using the hydrogen-like approximation. This procedure gives a value \( s_q(3p, 3p) = 0.30 \) atomic units. The wave number for the \( \text{3P}_0 - \text{3P}_2 \) transition was obtained by using the laboratory value of the ratio \( \frac{(\text{3P}_2 - \text{3P}_1)}{(\text{3P}_1 - \text{3P}_0)} \) and the coronal wave number of the \( \text{3P}_2 - \text{3P}_1 \) transition; and the resulting transition probabilities are tabulated in Table 1. The extreme smallness of the quadrupole transition probabilities in comparison with the magnetic dipole transition probabilities has been emphasized many times in previous work on forbidden lines.

IV. THE \( \ell^p \) FORBIDDEN TRANSITIONS

Fairly recently a number of forbidden lines whose transition probabilities were not given either by Pasternack or by Edlén have been identified. These transition probabilities may easily be calculated from the tables published by Shortley, Aller, Baker, and Menzel, but it has seemed worth while to give the final numerical results here. Transition probabilities are given only for the \( \ell^p \) and \( \ell^4 \) configurations, and in these cases the effects of the spin-spin and spin-other-orbit matrix elements on the transition probabilities are small and can safely be neglected. The estimates of the radial integrals as given by Pasternack have been used in calculating the quadrupole probabilities, and for \( A \chi \) the value \( s_2 = \frac{5}{2} s_3 = 0.124 \) was estimated from the extrapolated \( F_2 \) by his method. In the \( 4\ell^4 \) and \( 5\ell^4 \) configurations the quadrupole moments can hardly be found.

\]

\[ ^8 \text{B. Edlén, Zs. f. Phys., 103, 536, 1936.}
\]

\[ ^9 \text{Ap. J., 92, 129, 1940.}
\]

\[ ^10 \text{See P. Swings, J. Opt. Soc. America, 41, 153, 1951, and references given there.}
\]

\[ ^11 \text{Ap. J., 93, 178, 1941.}
\]

\]
in this way, because of the different regions in the ion effective in determining $F_2$ and $s_q$, and so only the magnetic dipole transition probabilities are given for these cases. The quadrupole transition probabilities are very low anyhow, in comparison with the magnetic dipole probabilities, except for the $^1D_2-^1S_0$ transition, which is not likely to be observed because of the high excitation energy required. In all cases the value of the coupling parameter $\chi$ was calculated from the ratio $(^3P_2-^3P_0)/(^3D_2-^3P_2)$, and the line strengths were obtained either from the tables or, for low $\chi$, from their asymptotic expressions. In the $3p^4$ configurations the energies of the $^1S_0$ levels, as located by Edlén and given in the table of atomic energy levels were used, while in the $3p^5$ configuration the extrapolated positions as given in the RMT were used. For $\text{A XI}$, the extrapolated values of $\chi$ and $^3P_2-^3P_1$ (which has been confirmed by the identification of this line in the post-maximum spectrum of RS Ophiuchi) given by Edlén have been used. The numerical results are collected in Table 1.

V. FORBIDDEN LINES IN ABSORPTION

As Bowen has pointed out, forbidden transitions may be observed in absorption as weak lines. Using the transition probabilities computed above, expected equivalent widths in the solar spectrum of the $[\text{Ca II}]$ and $[\text{S I}]$ lines have been calculated by Minnaert's detailed theory of weak lines. The abundances assumed, together with the calculated widths, are given in Table 2. If the $[\text{S I}]$ 7725 line could be observed in a star, say HD 45677, so that an accurate wave length could be determined, it might be possible to observe this line in absorption in the sun and determine a value for the solar abundance of sulphur. There are atmospheric $O_2$ lines at $\lambda$ 7724.59 and $\lambda$ 7724.88.

I wish to thank Professor S. Chandrasekhar for reading the manuscript and discussing it with me, and Professor B. Strömgren for discussing the problem of weak solar lines. Also I wish to thank the Atomic Energy Commission for fellowship support during the time this work was done.

<table>
<thead>
<tr>
<th>Ion</th>
<th>Assumed Abundance Relative to Hydrogen</th>
<th>Wave Length (Å)</th>
<th>Equivalent Width (mA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{Ca II}$</td>
<td>1.6×10^{-6}</td>
<td>7291.46, 7323.88</td>
<td>10.5, 7.2</td>
</tr>
<tr>
<td>$\text{S I}$</td>
<td>2×10^{-4}</td>
<td>7724.7, 4589.0</td>
<td>1.9, 0.5</td>
</tr>
</tbody>
</table>

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