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λ 3365 is probably the same as that recently suspected by Wares,1 while the ones at λ 3103 and λ 3160 probably have not been observed previously.

An attempt will be made to observe the spectrum of Comet Encke when it becomes brighter, in order that faint bands may be separated from the auroral lines and that any other bands existing in the ultraviolet may be found.2

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November 10, 1937

PARTIALLY DEGENERATE STELLAR CONFIGURATIONS

As the author has shown,1 the structure of completely degenerate configurations can be fully described by the differential equation

$$\frac{1}{\eta^2} \frac{d}{d\eta} \left( \eta^2 \frac{d\varphi}{d\eta} \right) = - \left( \varphi^2 - \frac{1}{\eta^2} \right)^{1/2}.$$

This equation describes exactly the structure of stellar configurations based on the exact equation of state of a degenerate electron gas, and in particular takes into account the gradual change of the equation of state from the law $p = K_s \rho^{5/3}$ to the law $p = K_s \rho^{4/3}$, with increasing density. On the theory of white dwarfs based on equation (I), the gaseous fringe is neglected as being insignificant, as may indeed be readily verified.2 However, for such stars in the interior of which we have only partial degeneracy at not too high densities, the relativistic effects can be properly neglected; and the problem now is to take into account the gradual change of the equa-

1 Unpublished; verbally reported at the meeting of the American Astronomical Society in September, 1937.
2 Such attempts were made on November 21 and 23, when the comet was brighter, but at a considerably lower altitude. The band at λ 3365 is present on both plates; λ 3160 is suspected on the first one only.

1 M.N., 95, 207, 1934.
tion of state from the law \( p = K_1 \rho^{5/3} \) to the law \( p = (k/\mu H) \rho T \).

The exact equation of state, neglecting relativity effects, can be written as

\[
\rho = 2 \frac{(2\pi mkT)^{1/2}}{h^3} \mu H U_{1/2}(A),
\]

\[
\dot{\rho} = 2 \frac{(2\pi mkT)^{1/2}}{h^3} kT U_{3/2}(A),
\]

where \( U_p(A) \) is a function of \( A \) defined by

\[
U_p = \frac{1}{\Gamma(p + 1)} \int_0^\infty \frac{u^p du}{e^u + 1}.
\]

To be able to study the structure of stellar configurations based on the equation of state parametrically expressed by (1) and (2), we should have to make an assumption concerning the circumstances determining the temperature gradient and the energy-source distribution. We shall consider two cases: (a) an isothermal gas sphere, and (b) the standard model.

(a) We can show that the structure of an isothermal gas sphere based on (1) and (2) is governed by the differential equation

\[
\frac{1}{\xi^2} \frac{d}{d\xi} \left( \xi^2 \frac{d\psi}{d\xi} \right) = -U_{1/2}(\xi^4),
\]

where

\[
\psi = \log A
\]

and \( \xi \) is the radius vector in a suitably chosen scale. We easily verify that, as \( A \to 0 \), equation (II) reduces to the usual Emden isothermal equation. On the other hand, if \( \log A \gg 1 \), equation (II) reduces to the Emden equation of index \( n = 3/2 \). A study of the configurations based on (II) should be of importance in following up certain cosmological speculations by Eddington.\(^3\)

b) For the "standard model" ($p_{\text{gas}} : p_{\text{rad}} = \beta : 1 - \beta = \text{Constant}$), we find that the structure will be governed by solutions of the differential equation

$$\frac{1}{\xi^2} \frac{d}{d\xi} \left( \xi^3 U_{3/2} \frac{d\psi}{d\xi} \right) = - U_{3/2} U_{1/2},$$

(III)

where $\psi$ is defined as in (4) and $\xi$ in a suitably chosen scale is the radius vector. Again one can show that, as $A \to 0$, equation (III) reduces to the Emden equation of index 3; while for $\log A \gg 1$, equation (III) reduces to the Emden equation of index $n = 3/2$. Equation (III), then, generalizes the usual standard model for stars of small mass and with incipient degeneracy in the central regions.

Finally, we may consider an isothermal gas sphere at such high temperatures that, if and when degeneracy sets in, it is already relativistic. For such configurations we can show that the differential equation governing the structure is

$$\frac{1}{\xi^2} \frac{d}{d\xi} \left( \xi^2 \frac{d\psi}{d\xi} \right) = - U_2.$$

(IV)

Configurations described by (IV) may be of importance in considerations relating to the ultimate fate of massive stars.

The problems suggested in this preliminary note will be examined in detail in a future communication.

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November 1937